National Exams – December 2019

16-Mec-A6 Advanced Fluid Mechanics

3 hours duration

NOTES:

1. If doubt exists as to the interpretation of any question the candidate is urged to submit with the answer paper a clear statement of the assumptions made.
2. Candidates may use any approved Sharp/Casio calculator.
   The exam is OPEN BOOK.
3. Any FIVE (5) out of the 6 questions constitute a complete exam paper for a total of 100 MARKS.
   The first five questions as they appear in the answer book will be marked.
4. Each question is of equal value (20 marks) and question items are marked as indicated.
5. Clarity and organization of the answer are important.

(20) **Question 1**

Air ($\gamma = 1.4$, $R = 287$ J/(kg K)) flows from a very large reservoir through a convergent-divergent nozzle. The air in the reservoir is kept at a constant temperature of T_0 =300 K. The exit area is A_E = 10 cm^2 and the throat area is A_T = 6.95 cm^2. When the back pressure is P_b = 100 kPa (absolute), the pressure distribution as measured along the centre-line shows a discontinuity at the exit as shown schematically in Figure 1. The flow may be assumed adiabatic for all conditions and isentropic in the absence of shocks.

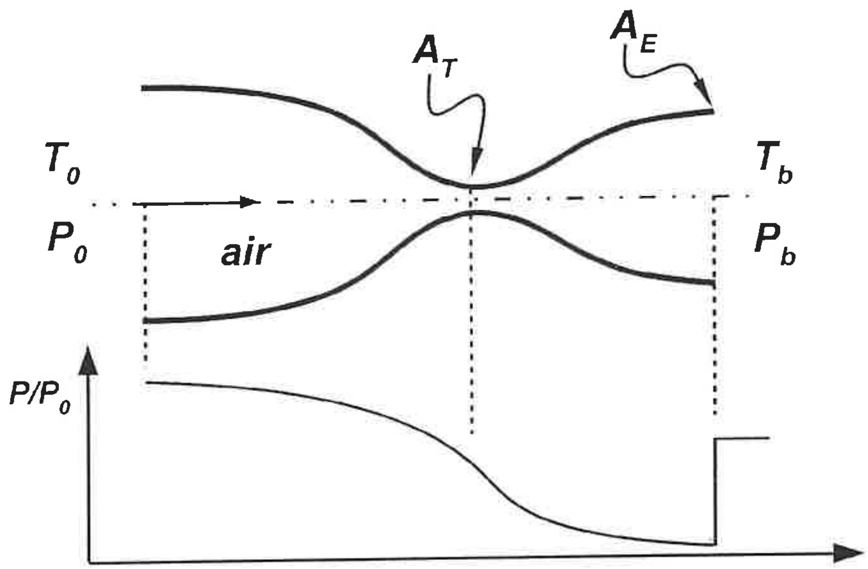


Figure 1: Top: Convergent-divergent nozzle attached to a large air reservoir. Bottom: Pressure distribution along the nozzle centre-line for a back pressure of P_b.

(5) (a) What is the total pressure P_0 in the reservoir.

(5) (b) What is the speed of the flow and the temperature directly downstream of the exit?

(5) (c) What is the mass flow rate?

(5) (d) What is the lowest back pressure for which the flow will be subsonic throughout the channel? What will be the mass flow rate at this back pressure?

(20) **Question 2**

Consider the ideal flow given by the velocity potential function

$$\phi = \frac{-A}{2\pi} \ln r$$

where A is a positive constant.

(5) (a) Determine the stream function ψ.

(5) (b) Sketch the equipotential lines and the stream lines of this flow.

(5) (c) Calculate the radial velocity V_r and identify the flow pattern.

(5) (d) Give the physical meaning of the constant A.

(20) **Question 3**

A large reservoir is connected near its base to a 40 m long (L) annulus (radii of $a = 4$ cm and $b = 6$ cm) made of commercial steel ($e = 0.046$ mm). Consider $\rho = 1000$ kg/m^3 and $\nu = \mu/\rho = 1.02 \cdot 10^{-6}$ m^2/s for water. Also, assume $D_{eff}/D_h = 0.670$ in the annulus.

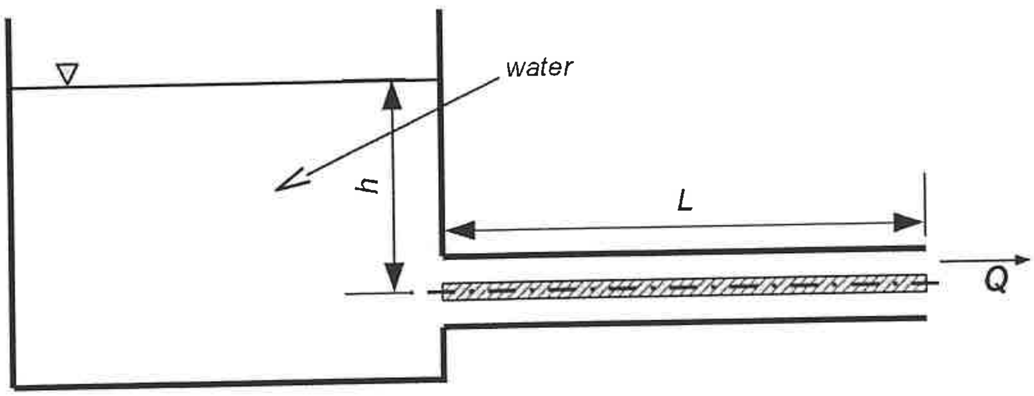


Figure 2: Annular pipe attached to a reservoir.

(15) (a) What should the water level h in the reservoir be to maintain a flow of $Q = 0.01$ m^3/s? Neglect initially the entrance effects at the annulus.

(5) (b) Estimate the effect in this case of a well-designed entrance, compared with a sharp-edged entrance that can be considered to have a loss coefficient of $K = 0.5$.

(20) **Question 4**

Consider the steady 2D flow between parallel plates, where the driving force for fluid motion is supplied by:

i) a moving upper plate, which translates from left to right at the constant speed U, and

ii) a constant pressure gradient $(\mathrm{d}p/\mathrm{d}x)$, which applies in the x-direction.

Consider the separation between the two plates in the y-direction as $2b$.

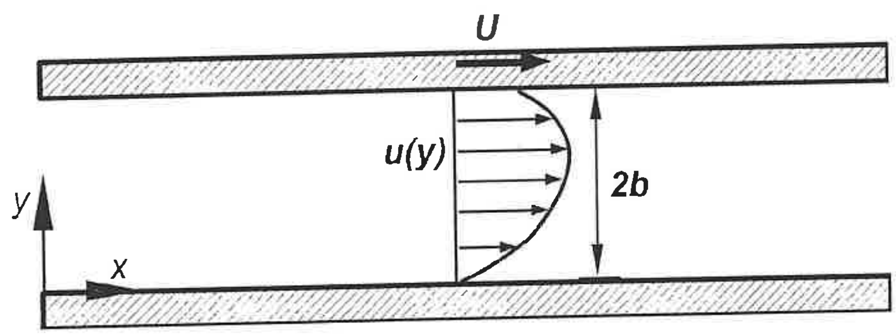


Figure 3: Schematic of flow between parallel plates.

(4) (a) You can think of this flow as a combination of two classical flows. Name these two flows and give a brief description or a schematic.

(8) (b) Simplify the 2D Navier-Stokes equation in the x-direction as it applies to this flow. Justify your simplifications.

(8) (c) Integrate the simplified Navier-Stokes equation and apply boundary conditions to find the corresponding velocity function $u(y)$.

(20) **Question 5**

At a sudden contraction in a pipe the diameter changes from D_1 to D_2. The pressure drop, Δp, which develops across the contraction is a function of D_1 and D_2, as well as the velocity U in the larger pipe, and the fluid density ρ and viscosity μ.

(15) (a) Using the Buckingham Pi theorem, and using D_1, U, and μ as repeating variables, determine a suitable set of dimensionless parameters.

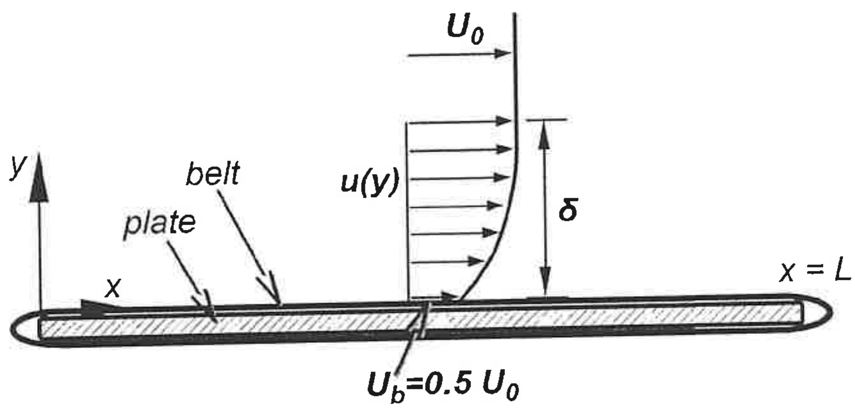
(5) (b) Explain why would it be incorrect to include the velocity in the smaller pipe as an additional variable?

(20) **Question 6**

A plate with a rolling belt around it (width W and length L) is exposed to a uniform free stream flow of constant speed U_0. The fluid is Newtonian with constant properties (ρ is the density, μ is the dynamic viscosity, and $\nu = \mu/\rho$ is the kinematic viscosity).

A laminar boundary layer develops over the belt and causes the belt to move. The belt moves towards the right (in the same direction as the free stream) at a constant speed of $U_b = 0.5\,U_0$, as shown in the figure. The plate and belt are very wide compared to the boundary layer thickness, δ, such that $W/\delta \gg 10$.

It is desired to find an approximation of the friction of the belt using the integral boundary layer equations. It can be assumed that at $x = 0$, $\delta = 0$. It can also be assumed that the free stream flow is irrotational.



(5) (a) Given the trial function of the form:

$$\frac{u}{U_0} = a + b\,\frac{y}{\delta} + c\left(\frac{y}{\delta}\right)^2 ,$$

determine suitable values for the coefficients a, b, and c.

(5) (b) Determine the expression for the boundary layer thickness, δ, as a function of the position x.

(5) (c) What is the local wall shear stress coefficient $C_{fx} = \tau_w/(\rho U_0^2/2)$?

(5) (d) What is the average wall shear stress, $\overline{\tau_w}$, between $x = 0$ and $x = L$?