National Exams December 2016

98-Ind-B1, Applied Probability & Statistics

3 hours duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.

2. This is a closed book exam. Candidates are permitted to use one of the two permitted calculators (Sharp or Casio models).

3. Candidates are permitted to have an aid sheet consisting of one 8.5" x 11.0" sheet of paper. Writing is permitted on both sides of the paper.

4. This exam consists of three sections (A→C). Within each section, candidates will be given a choice of questions to answer. Please read the instructions for each section carefully. A breakdown of questions and marks is as follows:

Section A:	Do 2 of 4 Questions.	Total marks: 30
Section B:	Do 2 of 3 Questions.	Total marks: 30
Section C:	Do 2 of 4 Questions.	Total marks: 40
Exam:	6 Questions.	Total marks: 100

4. The value of each question is listed in the exam. Remember to check the instructions for each section. DO NOT ATTEMPT TO DO ALL QUESTIONS.

5. Statistical tables are provided.

Section A – Basic Concepts: Complete two of the following four questions. This section is worth a total of 30 marks. Do not attempt all questions.

1. Two random variables (X,Y) have a joint probability distribution. 100 observations are taken from a process in which x and y are selected according to f(x,y) and the following results obtained:

Observations		x = 1	x = 2	x = 3
y	1	15	10	10
	2	10	15	20
	3	5	10	5

   a. Find the marginal probability mass function of X.
   b. Find the marginal probability mass function of Y.
   c. Find μ_X and σ_X.
   d. Find the covariance of X and Y.
   e. Find the correlation of X and Y.
   f. Are X and Y independent? Why or why not?

15 Marks

2. In her spare time, an Industrial Engineering professor likes to brew beer in her basement. Over the last six batches made, she has recorded the temperature in the basement (on the day of brewing) and the length of time required for a batch to fully ferment.

Temp (°C)	Time (Days)
14	19
18	18
21	19
19	20
16	18
17	19

   a. Calculate the mean and variance of both the basement temperature and the time required to ferment.
   b. Calculate the covariance of the temperature and fermentation time.
   c. Calculate the correlation between temperature and fermentation time. Does basement temperature correlate to fermentation time? What does this mean?
   d. Calculate a 95% Confidence Interval for the mean fermentation time.
   e. Calculate a 95% Prediction Interval for the basement temperature.

15 Marks

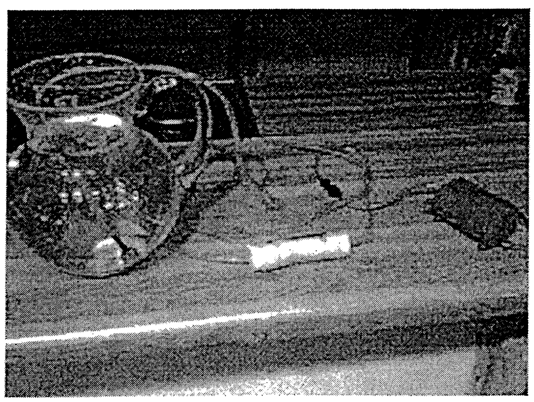
3. A function f(x) given below:

$$f(x) = \begin{cases} \alpha & 0 \leq x < 1 \\ \frac{2}{3} - \frac{1}{3}x & 1 \leq x < 2 \\ 0 & Otherwise \end{cases}$$

a. What value must α be for f(x) to be a valid probability function? Why?
b. Find the mean of the function, under the assumption that it is a valid pdf, using the value for α you discovered in part a.
c. Find the variance of the function using the value for α you discovered in part a.

15 Marks

4. The same professor from Q2 recently read an article describing the benefits of inducing oxygen into wort (the raw beer) before pitching yeast to start fermentation. The authors claim by raising the O_2 content in a chilled wort by 2 ppm, average fermentation time can be reduced by 1.5 days. Accordingly, the professor built her own aeration equipment, consisting of a small pump that you might see in a fish tank, a diffuser, and a filter to trap particulate matter from exiting the pump. See below for an example setup.



Assume that the professor uses this new setup for the next six batches of beer and, that being the good engineer she is, again records the basement temperature on the day that she brews and the total time it takes for a batch to ferment.

Original Setup	
Temp (°C)	Time (Days)
14	19
18	18
21	19
19	20
16	18
17	19

New Setup	
Temp (°C)	Time (Days)
23	17
20	20
18	18
19	16
21	14
22	17

Is there any evidence to suggest that fermentation times have dropped by 1.5 days after the introduction of aeration (New Setup) as compared to the data in Question 2 (Original Setup)? You may assume that fermentation time is normally distributed for both data sets. Please take very good care to explain the rationale and the method for making your inference.

15 Marks

Section B – Intermediate Concepts: Complete **two** of the following three questions. This section is worth a total of 30 marks. Do not attempt all questions.

1. The target volume of whole blood collected from a donor is 480 ml. When blood is collected a machine, called a shaker, both agitates the blood and monitors the mass of the material collected, from which volume may be estimated. Assume that a new shaker is being trialled. A random sample of 10 different units of whole blood were taken using the existing shaker (Shaker A) and the new shaker (shaker B):

Shaker A	Shaker B
486	494
474	479
471	486
473	477
471	483
478	484
476	487
486	484
466	488
483	482

a. Are the variances of the two samples the same? Use an $\alpha = 0.05$ to make your determination. Assume the samples are normally distributed.
b. Identify, at a 95% significance level, whether the volume of blood collected by the two shakers differs.
c. Test, at a 95% significance level, whether the volume of blood collected by Shaker A is less than 480 ml.
d. Provide a 95% confidence interval for the mean volume collected using Shaker B.
e. If, during the course of ordinary operations, a unit of blood collected using Shaker B is found to have a volume of 502 ml, could we say that this is an anomaly (i.e. not due to random chance)? Provide a test and a convincing argument supporting your hypothesis.
f. Provide a *brief* argument, without using any analysis, to support the assumption that the samples are drawn from normally distributed populations.

15 Marks

2. As part of a recent study of patients undergoing a particular medical treatment, researchers recorded the demographics of patients in their study, specifically looking at the relationship between body mass index (BMI) and age. The data is below:

Patient	1	2	3	4	5	6	7	8	9
BMI	19.92	20.59	29.02	20.78	25.97	20.39	23.29	17.27	35.24
Age	45.5	34.6	40.6	32.9	28.2	30.1	52.1	33.3	47.0

a. Fit a regression model for age (x) versus BMI (y) of the form $y = ax + b$
b. Test for the significance of the regression using $\alpha = 0.05$.
c. Determine whether the coefficient for x is significant.
d. Give the expected BMI for a 49.0 year old individual and determine if we would consider a 49.0 year old with a BMI of 32.5 to be a statistical anomaly.

15 Marks

3. A recent report by New Sunderland Power Inc (NSPI) suggests that the number of power outages in the Province of New Sunderland is not statistically different from those experienced by most people living in the remainder of the nation of Phantasia. The NSPI report notes that power failures in Phantasia are Poisson distributed with a mean $\lambda = 0.05$ failures per day. NSPI furthermore notes that in the past 39 billing periods there have been a total of 69 outages in New Sunderland. (A billing period is 28 days – there are 13 billing periods in a year). A summary of the number of power outages per billing is given below:

Outages	Num Billing Periods
0	8
1	12
2	9
3	5
4	2
5	2
6	1

For instance, there were 8 billing periods with no failures, 11 billing periods with 1 failure, ..., 2 billing periods with 6 failures.

Determine, using a χ^2 Goodness of Fit test, whether the data supports NSPI's assertion that the number of outages can be adequately modelled as a Poisson distribution with $\lambda = 0.05$ failures per day. Assume $\alpha = 0.05$. Please document any assumptions you make.

15 Marks

Section C – Advanced Concepts: Complete <u>two of the following four</u> questions. This section is worth a total of 40 marks. Do not attempt all questions.

1. An IE is conducting a study of machines used to fill frozen pies at a Kitchener area food company. The study consists of 3 randomly chosen types of pie (apple, strawberry, and blueberry) and three different speeds (slow, normal, fast) on an agitator (i.e. a shaker) used in the process to ensure that fruit particles do not adhere to the hopper delivering the filling to the pie shell. Assume that the target mass of the resulting pie is 400 grams.

		Pie Type					
		Apple		*Strawberry*		*Blueberry*	
Speed	*Fast*	395	390	392	392	402	405
		401	400	394	401	399	399
	Normal	396	399	390	392	404	403
		400	402	395	420	400	399
	Slow	410	408	404	406	415	412
		408	407	401	400	413	415

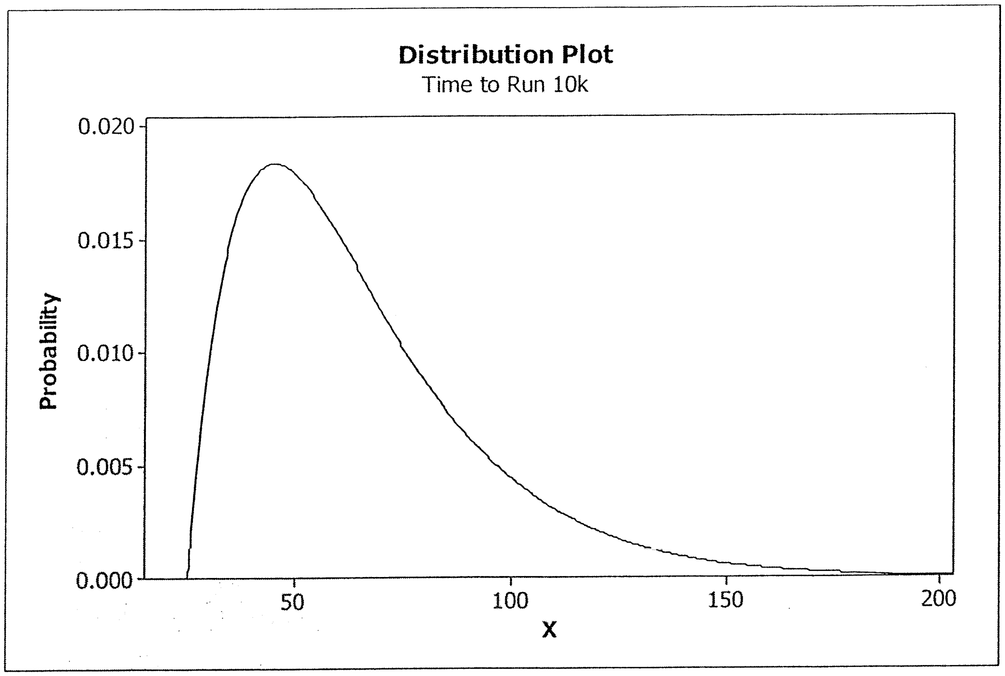
a. Is there any evidence to show that pie type or agitator speed significantly affects the resulting mass of the pie? Use a confidence level of 95% and select an appropriate test.
b. Are there particular settings of the shaker that are tied to pie type? Interpret these results with respect to the firm's operating procedures.

To assist you in your calculations, you may assume the grand sum for this data set is 14469 and that the grand mean for the data set is 401.9167. The variance for the data set is 54.82143. You may assume that the data is normally distributed and for ease of calculations that SSE is 813.25.

20 Marks

2. Being an engineer and a runner, I like to keep statistics on how my training is going. Each morning (well, say six days a week), I run 10km. I usually run the same route and mostly go at the same time of day. Weather conditions, being what they are in my home town of Halifax, can be variable. I record my times in an Excel sheet that I use for analysis. I note the date and time and usually a few words about the weather. When I get a new pair of shoes, I note this specifically, because I think there may be a relationship between the type of shoe that I wear and my training time.

   a. I would like to perform a statistical test of running shoe types, but there are issues with the data. For instance, because the shoes don't always last the same amount of time, the sample sizes between shoes is different. In addition, there is ample reason to assume that my training times are not Normally distributed:



Assume that I have five pairs of running shoes that I wish to evaluate (A-E). Assume also that I have between 30 and 60 weeks of data for each type of shoe. You may assume that each week contains either 5 or 6 data points. Describe what you might do with the data so that you can determine which shoe is better and still meet all of the underlying assumptions of your selected statistical test

b. Assume that the method for "fixing" the data you suggested in part (a) has been a success and we now have data that is Normally distributed. Determine if running shoe does indeed impact training time. Use an appropriate value of alpha and justify your choice.
c. Determine, again using an appropriate value of alpha, which running shoes (if any) yield significantly shorter times when compared with all other shoes.
d. Determine, again using an appropriate value of alpha, which running shoe (if any) is best when compared against all others.

A	B	C	D	E
49.3	50.0	46.8	51.2	54.7
51.2	51.8	47.2	51.3	54.3
51.6	54.1	49.9	45.1	48.5
48.6	53.6	50.3	53.6	51.5
49.5	51.7	47.8	48.2	49.1

20 Marks

3. After running several marathons, a runner has become convinced that her performance (as measured in time to complete the race) is a function of three

elements: x1 – total number of kilometres *100 run in the six weeks prior to the race; x2 – the longest single run completed during the six weeks prior to the race; and x3 – the expected temperature on race day. To test this hypothesis, the runner has collected the following dataset:

x1 Total km (*100)	x2 Max Dist	x3 Temp	y Time (Hrs)
4.6	27	11	4.3
4.5	31	10	3.8
4.0	30	25	3.5
4.1	25	22	3.6
4.4	28	14	3.5
4.2	32	17	3.4

The relationship between completion time and (x_1, x_2, x_3) is assumed to be linear in the region of interest.

a. Build the (X'X) matrix.

b. A partial table of $(X'X)^{-1}$ is given below, where * , **, and *** remain to be determined. Please determine what these values should be.

3186.5127	*	-10.0932	-22.9764
-585.1664	108.4063	1.7075	4.2294
-10.0932	1.7075	0.0562	**
-22.9764	***	0.0685	0.1706

c. Assume that $(X'X)^{-1}(X'Y)$ is:

-16.29491
4.1060
0.0093
0.1245

Calculate SSE, SSR, and SST.

d. Complete the ANOVA table for this regression model ($\alpha = 0.05$) and determine whether the linear model is a significant predictor of y.

e. Determine if any of the coefficients in your model is significant at the 95% level.

f. Based on your analysis, is the model a good predictor of race finishing time?

20 Marks

4. The following dataset is part of a 2^3 designed experiment to determine the effects of cutting speed (A), tool geometry (B), and cutting angle (C) on the life of a machine tool. Two levels of each factor are chosen and duplicates were run at each design point.

Run	A	B	C	Rep 1	Rep 2
1	-1	-1	-1	32	31
2	1	-1	-1	15	43
3	-1	1	-1	35	34
4	1	1	-1	35	47
5	-1	-1	1	44	45
6	1	-1	1	40	37
7	-1	1	1	60	50
8	1	1	1	39	41

a. Complete the rest of the design matrix for the four interactions
b. Compute the contrast and mean effects for the A factor, the AB factor, and the ABC factor.
c. Calculate the sum of squares for the A factor, the AB factor, and the ABC factor.
d. Assuming the following Contrasts and Sums of Squares, calculate the ANOVA table for this model. Assume $\alpha = 0.05$.

Contrasts

A	B	C	AB	AC	BC	ABC
-34	*	84	**	-50	-6	***

Sum of Squares

SS(A)	SS(B)	SS(C)	SS(AB)	SS(AC)	SS(BC)	SS(ABC)	SST
*	182.25	**	0	***	2.25	81	1457

e. Fit a linear regression model to the data and determine which linear and interaction terms are significant (if any).

20 Marks

Areas Under the Normal Curve

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.40	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.30	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.20	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.10	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.00	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.90	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.80	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.70	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.60	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.50	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.40	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.30	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.20	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.10	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.00	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.90	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.80	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.70	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.60	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.50	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.40	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.30	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.20	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.10	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.00	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.90	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.80	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.70	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.60	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.50	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.40	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.30	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.20	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.10	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.00	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Areas Under the Normal Curve

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.00	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.10	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.20	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.30	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.40	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.50	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.60	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.70	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.80	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.90	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.00	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.10	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.20	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.30	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.40	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.50	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.60	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.70	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.80	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.90	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.00	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.10	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.20	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.30	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.40	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.50	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.60	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.70	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.80	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.90	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.00	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.10	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.20	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.30	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.40	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Critical Values of the t-Distribution

ν	α						
	0.40	0.30	0.20	0.15	0.10	0.05	0.025
1	0.325	0.727	1.376	1.963	3.078	6.314	12.706
2	0.289	0.617	1.061	1.386	1.886	2.920	4.303
3	0.277	0.584	0.978	1.250	1.638	2.353	3.182
4	0.271	0.569	0.941	1.190	1.533	2.132	2.776
5	0.267	0.559	0.920	1.156	1.476	2.015	2.571
6	0.265	0.553	0.906	1.134	1.440	1.943	2.447
7	0.263	0.549	0.896	1.119	1.415	1.895	2.365
8	0.262	0.546	0.889	1.108	1.397	1.860	2.306
9	0.261	0.543	0.883	1.100	1.383	1.833	2.262
10	0.260	0.542	0.879	1.093	1.372	1.812	2.228
11	0.260	0.540	0.876	1.088	1.363	1.796	2.201
12	0.259	0.539	0.873	1.083	1.356	1.782	2.179
13	0.259	0.538	0.870	1.079	1.350	1.771	2.160
14	0.258	0.537	0.868	1.076	1.345	1.761	2.145
15	0.258	0.536	0.866	1.074	1.341	1.753	2.131
16	0.258	0.535	0.865	1.071	1.337	1.746	2.120
17	0.257	0.534	0.863	1.069	1.333	1.740	2.110
18	0.257	0.534	0.862	1.067	1.330	1.734	2.101
19	0.257	0.533	0.861	1.066	1.328	1.729	2.093
20	0.257	0.533	0.860	1.064	1.325	1.725	2.086
21	0.257	0.532	0.859	1.063	1.323	1.721	2.080
22	0.256	0.532	0.858	1.061	1.321	1.717	2.074
23	0.256	0.532	0.858	1.060	1.319	1.714	2.069
24	0.256	0.531	0.857	1.059	1.318	1.711	2.064
25	0.256	0.531	0.856	1.058	1.316	1.708	2.060
26	0.256	0.531	0.856	1.058	1.315	1.706	2.056
27	0.256	0.531	0.855	1.057	1.314	1.703	2.052
28	0.256	0.530	0.855	1.056	1.313	1.701	2.048
29	0.256	0.530	0.854	1.055	1.311	1.699	2.045
30	0.256	0.530	0.854	1.055	1.310	1.697	2.042
40	0.255	0.529	0.851	1.050	1.303	1.684	2.021
50	0.255	0.528	0.849	1.047	1.299	1.676	2.009
60	0.254	0.527	0.848	1.045	1.296	1.671	2.000
70	0.254	0.527	0.847	1.044	1.294	1.667	1.994
80	0.254	0.526	0.846	1.043	1.292	1.664	1.990
90	0.254	0.526	0.846	1.042	1.291	1.662	1.987
100	0.254	0.526	0.845	1.042	1.290	1.660	1.984
110	0.254	0.526	0.845	1.041	1.289	1.659	1.982
120	0.254	0.526	0.845	1.041	1.289	1.658	1.980
130	0.254	0.526	0.844	1.041	1.288	1.657	1.978
∞	0.253	0.524	0.842	1.036	1.282	1.645	1.960

Critical Values of the t-Distribution

	α						
ν	**0.02**	**0.015**	**0.01**	**0.0075**	**0.005**	**0.0025**	**0.0005**
1	15.895	21.205	31.821	42.433	63.657	127.321	636.619
2	4.849	5.643	6.965	8.073	9.925	14.089	31.599
3	3.482	3.896	4.541	5.047	5.841	7.453	12.924
4	2.999	3.298	3.747	4.088	4.604	5.598	8.610
5	2.757	3.003	3.365	3.634	4.032	4.773	6.869
6	2.612	2.829	3.143	3.372	3.707	4.317	5.959
7	2.517	2.715	2.998	3.203	3.499	4.029	5.408
8	2.449	2.634	2.896	3.085	3.355	3.833	5.041
9	2.398	2.574	2.821	2.998	3.250	3.690	4.781
10	2.359	2.527	2.764	2.932	3.169	3.581	4.587
11	2.328	2.491	2.718	2.879	3.106	3.497	4.437
12	2.303	2.461	2.681	2.836	3.055	3.428	4.318
13	2.282	2.436	2.650	2.801	3.012	3.372	4.221
14	2.264	2.415	2.624	2.771	2.977	3.326	4.140
15	2.249	2.397	2.602	2.746	2.947	3.286	4.073
16	2.235	2.382	2.583	2.724	2.921	3.252	4.015
17	2.224	2.368	2.567	2.706	2.898	3.222	3.965
18	2.214	2.356	2.552	2.689	2.878	3.197	3.922
19	2.205	2.346	2.539	2.674	2.861	3.174	3.883
20	2.197	2.336	2.528	2.661	2.845	3.153	3.850
21	2.189	2.328	2.518	2.649	2.831	3.135	3.819
22	2.183	2.320	2.508	2.639	2.819	3.119	3.792
23	2.177	2.313	2.500	2.629	2.807	3.104	3.768
24	2.172	2.307	2.492	2.620	2.797	3.091	3.745
25	2.167	2.301	2.485	2.612	2.787	3.078	3.725
26	2.162	2.296	2.479	2.605	2.779	3.067	3.707
27	2.158	2.291	2.473	2.598	2.771	3.057	3.690
28	2.154	2.286	2.467	2.592	2.763	3.047	3.674
29	2.150	2.282	2.462	2.586	2.756	3.038	3.659
30	2.147	2.278	2.457	2.581	2.750	3.030	3.646
40	2.123	2.250	2.423	2.542	2.704	2.971	3.551
50	2.109	2.234	2.403	2.519	2.678	2.937	3.496
60	2.099	2.223	2.390	2.504	2.660	2.915	3.460
70	2.093	2.215	2.381	2.494	2.648	2.899	3.435
80	2.088	2.209	2.374	2.486	2.639	2.887	3.416
90	2.084	2.205	2.368	2.480	2.632	2.878	3.402
100	2.081	2.201	2.364	2.475	2.626	2.871	3.390
110	2.078	2.199	2.361	2.471	2.621	2.865	3.381
120	2.076	2.196	2.358	2.468	2.617	2.860	3.373
130	2.075	2.194	2.355	2.465	2.614	2.856	3.367
∞	2.054	2.170	2.327	2.433	2.576	2.808	3.291

Critical Values of the Chi-Squared Distribution

ν	α 0.995	0.99	0.98	0.975	0.95	0.90	0.80	0.75	0.70	0.50
1	0.000	0.000	0.001	0.001	0.004	0.016	0.064	0.102	0.148	0.455
2	0.010	0.020	0.040	0.051	0.103	0.211	0.446	0.575	0.713	1.386
3	0.072	0.115	0.185	0.216	0.352	0.584	1.005	1.213	1.424	2.366
4	0.207	0.297	0.429	0.484	0.711	1.064	1.649	1.923	2.195	3.357
5	0.412	0.554	0.752	0.831	1.145	1.610	2.343	2.675	3.000	4.351
6	0.676	0.872	1.134	1.237	1.635	2.204	3.070	3.455	3.828	5.348
7	0.989	1.239	1.564	1.690	2.167	2.833	3.822	4.255	4.671	6.346
8	1.344	1.646	2.032	2.180	2.733	3.490	4.594	5.071	5.527	7.344
9	1.735	2.088	2.532	2.700	3.325	4.168	5.380	5.899	6.393	8.343
10	2.156	2.558	3.059	3.247	3.940	4.865	6.179	6.737	7.267	9.342
11	2.603	3.053	3.609	3.816	4.575	5.578	6.989	7.584	8.148	10.341
12	3.074	3.571	4.178	4.404	5.226	6.304	7.807	8.438	9.034	11.340
13	3.565	4.107	4.765	5.009	5.892	7.042	8.634	9.299	9.926	12.340
14	4.075	4.660	5.368	5.629	6.571	7.790	9.467	10.165	10.821	13.339
15	4.601	5.229	5.985	6.262	7.261	8.547	10.307	11.037	11.721	14.339
16	5.142	5.812	6.614	6.908	7.962	9.312	11.152	11.912	12.624	15.338
17	5.697	6.408	7.255	7.564	8.672	10.085	12.002	12.792	13.531	16.338
18	6.265	7.015	7.906	8.231	9.390	10.865	12.857	13.675	14.440	17.338
19	6.844	7.633	8.567	8.907	10.117	11.651	13.716	14.562	15.352	18.338
20	7.434	8.260	9.237	9.591	10.851	12.443	14.578	15.452	16.266	19.337
21	8.034	8.897	9.915	10.283	11.591	13.240	15.445	16.344	17.182	20.337
22	8.643	9.542	10.600	10.982	12.338	14.041	16.314	17.240	18.101	21.337
23	9.260	10.196	11.293	11.689	13.091	14.848	17.187	18.137	19.021	22.337
24	9.886	10.856	11.992	12.401	13.848	15.659	18.062	19.037	19.943	23.337
25	10.520	11.524	12.697	13.120	14.611	16.473	18.940	19.939	20.867	24.337
26	11.160	12.198	13.409	13.844	15.379	17.292	19.820	20.843	21.792	25.336
27	11.808	12.879	14.125	14.573	16.151	18.114	20.703	21.749	22.719	26.336
28	12.461	13.565	14.847	15.308	16.928	18.939	21.588	22.657	23.647	27.336
29	13.121	14.256	15.574	16.047	17.708	19.768	22.475	23.567	24.577	28.336
30	13.787	14.953	16.306	16.791	18.493	20.599	23.364	24.478	25.508	29.336

Critical Values of the Chi-Squared Distribution

	α									
v	**0.3**	**0.25**	**0.2**	**0.1**	**0.05**	**0.025**	**0.02**	**0.01**	**0.005**	**0.001**
1	1.074	1.323	1.642	2.706	3.841	5.024	5.412	6.635	7.879	10.828
2	2.408	2.773	3.219	4.605	5.991	7.378	7.824	9.210	10.597	13.816
3	3.665	4.108	4.642	6.251	7.815	9.348	9.837	11.345	12.838	16.266
4	4.878	5.385	5.989	7.779	9.488	11.143	11.668	13.277	14.860	18.467
5	6.064	6.626	7.289	9.236	11.070	12.833	13.388	15.086	16.750	20.515
6	7.231	7.841	8.558	10.645	12.592	14.449	15.033	16.812	18.548	22.458
7	8.383	9.037	9.803	12.017	14.067	16.013	16.622	18.475	20.278	24.322
8	9.524	10.219	11.030	13.362	15.507	17.535	18.168	20.090	21.955	26.124
9	10.656	11.389	12.242	14.684	16.919	19.023	19.679	21.666	23.589	27.877
10	11.781	12.549	13.442	15.987	18.307	20.483	21.161	23.209	25.188	29.588
11	12.899	13.701	14.631	17.275	19.675	21.920	22.618	24.725	26.757	31.264
12	14.011	14.845	15.812	18.549	21.026	23.337	24.054	26.217	28.300	32.909
13	15.119	15.984	16.985	19.812	22.362	24.736	25.472	27.688	29.819	34.528
14	16.222	17.117	18.151	21.064	23.685	26.119	26.873	29.141	31.319	36.123
15	17.322	18.245	19.311	22.307	24.996	27.488	28.259	30.578	32.801	37.697
16	18.418	19.369	20.465	23.542	26.296	28.845	29.633	32.000	34.267	39.252
17	19.511	20.489	21.615	24.769	27.587	30.191	30.995	33.409	35.718	40.790
18	20.601	21.605	22.760	25.989	28.869	31.526	32.346	34.805	37.156	42.312
19	21.689	22.718	23.900	27.204	30.144	32.852	33.687	36.191	38.582	43.820
20	22.775	23.828	25.038	28.412	31.410	34.170	35.020	37.566	39.997	45.315
21	23.858	24.935	26.171	29.615	32.671	35.479	36.343	38.932	41.401	46.797
22	24.939	26.039	27.301	30.813	33.924	36.781	37.659	40.289	42.796	48.268
23	26.018	27.141	28.429	32.007	35.172	38.076	38.968	41.638	44.181	49.728
24	27.096	28.241	29.553	33.196	36.415	39.364	40.270	42.980	45.559	51.179
25	28.172	29.339	30.675	34.382	37.652	40.646	41.566	44.314	46.928	52.620
26	29.246	30.435	31.795	35.563	38.885	41.923	42.856	45.642	48.290	54.052
27	30.319	31.528	32.912	36.741	40.113	43.195	44.140	46.963	49.645	55.476
28	31.391	32.620	34.027	37.916	41.337	44.461	45.419	48.278	50.993	56.892
29	32.461	33.711	35.139	39.087	42.557	45.722	46.693	49.588	52.336	58.301
30	33.530	34.800	36.250	40.256	43.773	46.979	47.962	50.892	53.672	59.703

Critical Values of the F Distribution

$f_{0.05(v1,v2)}$

v2	v1: 1	2	3	4	5	6	7	8	9	10
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54	241.88
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08
50	4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13	2.07	2.03
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83

Critical Values of the F Distribution

$f_{0.05(v1,v2)}$

	v1									
v2	10	12	15	20	24	30	40	60	120	∞
1	241.88	243.91	245.95	248.01	249.05	250.10	251.14	252.20	253.25	254.30
2	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50
3	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.37
6	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.41
12	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
26	2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69
27	2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
28	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
29	2.18	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64
30	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
50	2.03	1.95	1.87	1.78	1.74	1.69	1.63	1.58	1.51	1.44
60	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.26
∞	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.01

Critical Values of the F Distribution

$f_{0.01(v1,v2)}$

v2	v1: 1	2	3	4	5	6	7	8	9	10
1	4052.18	4999.50	5403.35	5624.58	5763.65	5858.99	5928.36	5981.07	6022.47	6055.85
2	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39	99.40
3	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35	27.23
4	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66	14.55
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16	10.05
6	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87
7	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62
8	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81
9	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10
14	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	3.94
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69
17	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68	3.59
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17
25	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22	3.13
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09
27	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15	3.06
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03
29	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09	3.00
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80
50	7.17	5.06	4.20	3.72	3.41	3.19	3.02	2.89	2.78	2.70
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47
∞	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32

Critical Values of the F Distribution

$f_{0.01(v1,v2)}$

	v1									
v2	**10**	**12**	**15**	**20**	**24**	**30**	**40**	**60**	**120**	**∞**
1	6055.85	6106.32	6157.28	6208.73	6234.63	6260.65	6286.78	6313.03	6339.39	6365.55
2	99.40	99.42	99.43	99.45	99.46	99.47	99.47	99.48	99.49	99.50
3	27.23	27.05	26.87	26.69	26.60	26.50	26.41	26.32	26.22	26.13
4	14.55	14.37	14.20	14.02	13.93	13.84	13.75	13.65	13.56	13.46
5	10.05	9.89	9.72	9.55	9.47	9.38	9.29	9.20	9.11	9.02
6	7.87	7.72	7.56	7.40	7.31	7.23	7.14	7.06	6.97	6.88
7	6.62	6.47	6.31	6.16	6.07	5.99	5.91	5.82	5.74	5.65
8	5.81	5.67	5.52	5.36	5.28	5.20	5.12	5.03	4.95	4.86
9	5.26	5.11	4.96	4.81	4.73	4.65	4.57	4.48	4.40	4.31
10	4.85	4.71	4.56	4.41	4.33	4.25	4.17	4.08	4.00	3.91
11	4.54	4.40	4.25	4.10	4.02	3.94	3.86	3.78	3.69	3.60
12	4.30	4.16	4.01	3.86	3.78	3.70	3.62	3.54	3.45	3.36
13	4.10	3.96	3.82	3.66	3.59	3.51	3.43	3.34	3.25	3.17
14	3.94	3.80	3.66	3.51	3.43	3.35	3.27	3.18	3.09	3.01
15	3.80	3.67	3.52	3.37	3.29	3.21	3.13	3.05	2.96	2.87
16	3.69	3.55	3.41	3.26	3.18	3.10	3.02	2.93	2.84	2.75
17	3.59	3.46	3.31	3.16	3.08	3.00	2.92	2.83	2.75	2.65
18	3.51	3.37	3.23	3.08	3.00	2.92	2.84	2.75	2.66	2.57
19	3.43	3.30	3.15	3.00	2.92	2.84	2.76	2.67	2.58	2.49
20	3.37	3.23	3.09	2.94	2.86	2.78	2.69	2.61	2.52	2.42
21	3.31	3.17	3.03	2.88	2.80	2.72	2.64	2.55	2.46	2.36
22	3.26	3.12	2.98	2.83	2.75	2.67	2.58	2.50	2.40	2.31
23	3.21	3.07	2.93	2.78	2.70	2.62	2.54	2.45	2.35	2.26
24	3.17	3.03	2.89	2.74	2.66	2.58	2.49	2.40	2.31	2.21
25	3.13	2.99	2.85	2.70	2.62	2.54	2.45	2.36	2.27	2.17
26	3.09	2.96	2.81	2.66	2.58	2.50	2.42	2.33	2.23	2.13
27	3.06	2.93	2.78	2.63	2.55	2.47	2.38	2.29	2.20	2.10
28	3.03	2.90	2.75	2.60	2.52	2.44	2.35	2.26	2.17	2.07
29	3.00	2.87	2.73	2.57	2.49	2.41	2.33	2.23	2.14	2.04
30	2.98	2.84	2.70	2.55	2.47	2.39	2.30	2.21	2.11	2.01
40	2.80	2.66	2.52	2.37	2.29	2.20	2.11	2.02	1.92	1.81
50	2.70	2.56	2.42	2.27	2.18	2.10	2.01	1.91	1.80	1.68
60	2.63	2.50	2.35	2.20	2.12	2.03	1.94	1.84	1.73	1.60
120	2.47	2.34	2.19	2.03	1.95	1.86	1.76	1.66	1.53	1.38
∞	2.32	2.18	2.04	1.88	1.79	1.70	1.59	1.47	1.32	1.01

Critical Values for Bartlett's Test

	$b_k(0.01;n)$								
	Number of populations, *k*								
n	2	3	4	5	6	7	8	9	10
3	0.1411	0.1672							
4	0.2843	0.3165	0.3475	0.3729	0.3937	0.4110			
5	0.3984	0.4304	0.4607	0.4850	0.5046	0.5207	0.5343	0.5458	0.5558
6	0.4850	0.5149	0.5430	0.5653	0.5832	0.5975	0.6100	0.6204	0.6293
7	0.5512	0.5787	0.6045	0.6248	0.6410	0.6542	0.6652	0.6744	0.6824
8	0.6031	0.6282	0.6518	0.6704	0.6851	0.6970	0.7069	0.7153	0.7225
9	0.6445	0.6676	0.6892	0.7062	0.7197	0.7305	0.7395	0.7471	0.7536
10	0.6783	0.6996	0.7195	0.7352	0.7475	0.7575	0.7657	0.7726	0.7786
11	0.7063	0.7260	0.7445	0.7590	0.7703	0.7795	0.7871	0.7935	0.7990
12	0.7299	0.7483	0.7654	0.7789	0.7894	0.7980	0.8050	0.8109	0.8160
13	0.7501	0.7672	0.7832	0.7958	0.8056	0.8135	0.8201	0.8256	0.8303
14	0.7674	0.7835	0.7985	0.8103	0.8195	0.8269	0.8330	0.8382	0.8426
15	0.7825	0.7977	0.8118	0.8229	0.8315	0.8385	0.8443	0.8491	0.8532
16	0.7958	0.8101	0.8235	0.8339	0.8421	0.8486	0.8541	0.8586	0.8625
17	0.8076	0.8211	0.8338	0.8436	0.8514	0.8576	0.8627	0.8670	0.8707
18	0.8181	0.8309	0.8429	0.8523	0.8596	0.8655	0.8704	0.8745	0.8780
19	0.8275	0.8397	0.8512	0.8601	0.8670	0.8727	0.8773	0.8811	0.8845
20	0.8360	0.8476	0.8586	0.8671	0.8737	0.8791	0.8835	0.8871	0.8903

Values for Tukey's Test
Upper Percentage Points of the Studentized Range Distribution: Values of q(0.05,k, υ)

Degrees of Freedom	Number of Treatments *k*								
υ	2	3	4	5	6	7	8	9	10
1	18.00	27.00	32.80	37.20	40.50	43.10	15.10	47.10	49.10
2	6.09	5.33	9.80	10.89	11.73	12.43	13.03	13.54	13.99
3	4.50	5.91	6.83	7.51	8.04	8.47	8.85	9.18	9.46
4	3.93	5.04	5.76	6.29	6.71	7.06	7.35	7.60	7.83
5	3.64	4.60	5.22	5.67	6.99	6.80	6.58	6.33	6.03
6	3.46	4.34	4.90	5.30	6.49	6.32	6.12	5.90	5.63
7	3.34	4.16	4.68	5.06	6.16	6.00	5.82	5.61	5.36
8	3.26	4.04	4.53	4.89	5.92	5.77	5.60	5.40	5.17
9	3.20	3.95	4.41	4.76	5.74	5.59	5.43	5.24	5.02
10	3.15	3.88	4.33	4.65	5.60	5.46	5.30	5.12	4.91
11	3.11	3.82	4.26	4.57	5.49	5.35	5.20	5.03	4.82
12	3.08	3.77	4.20	4.51	5.39	5.27	5.12	4.95	4.75
13	3.06	3.73	4.15	4.45	5.32	5.19	5.05	4.88	4.69
14	3.03	3.70	4.11	4.41	5.25	5.13	4.99	4.83	4.64
15	3.01	3.67	4.08	4.37	5.20	5.08	4.94	4.78	4.59
16	3.00	3.65	4.05	4.33	5.15	5.03	4.90	4.74	4.56
17	2.98	3.63	4.02	4.30	5.11	4.99	4.86	4.70	4.52
18	2.97	3.61	4.00	4.28	5.07	4.96	4.82	4.67	4.49
19	2.96	3.59	3.98	4.25	5.04	4.92	4.79	4.65	4.47
20	2.95	3.58	3.96	4.23	5.01	4.90	4.77	4.62	4.45
24	2.92	3.53	3.90	4.17	4.92	4.81	4.68	4.54	4.37
30	2.89	3.49	3.85	4.10	4.82	4.72	4.60	4.46	4.30
40	2.86	3.44	3.79	4.04	4.73	4.63	4.52	4.39	4.23
60	2.83	3.40	3.74	3.98	4.65	4.55	4.44	4.31	4.16
120	2.80	3.36	3.68	3.92	4.56	4.47	4.36	4.24	4.10
∞	2.77	3.31	3.63	3.86	4.47	4.39	4.29	4.17	4.03

Wilcoxon Rank-Sum Test

One Tailed Test at $\alpha = 0.025$ or Two-Tailed Test at $\alpha = 0.05$

n_1 \ n_2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1																		
2						0	0	0	0	1	1	1	1	1	2	2	2	2
3			0	1	1	2	2	3	3	4	4	5	5	6	6	7	7	8
4		0	1	2	3	4	4	5	6	7	8	9	10	11	11	12	13	13
5			2	3	5	6	7	8	9	11	12	13	14	15	17	18	19	20
6				5	6	8	10	11	13	14	16	17	19	21	22	24	25	27
7					8	10	12	14	16	18	20	22	24	26	28	30	32	34
8						13	15	17	19	22	24	26	29	31	34	36	38	41
9							17	20	23	26	28	31	34	37	39	42	45	48
10								23	26	29	33	36	39	42	45	48	52	55
11									30	33	37	40	44	47	51	55	58	62
12										37	41	45	49	53	57	61	65	69
13											45	50	54	59	63	67	72	76
14												55	59	64	67	74	78	83
15													64	70	75	80	85	90
16														75	81	86	92	98
17															87	93	99	105
18																99	106	112
19																	113	119
20																		127

One Tailed Test at $\alpha = 0.05$ or Two-Tailed Test at $\alpha = 0.1$

n_1 \ n_2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1																	0	0
2		0	0	0	1	1	1	1	2	2	3	3	3	3	4	4	4	4
3	0	0	1	2	2	3	4	4	5	5	6	7	7	8	9	9	10	11
4		1	2	3	4	5	6	7	8	9	10	11	12	14	15	16	17	18
5			4	5	6	8	9	11	12	13	15	16	18	19	20	22	23	25
6				7	8	10	12	14	16	17	19	21	23	25	26	28	30	32
7					11	13	15	17	19	21	24	26	28	30	33	35	37	39
8						15	18	20	23	26	28	31	33	36	39	41	44	47
9							21	24	27	30	33	36	39	42	45	48	51	54
10								27	31	34	37	41	44	48	51	55	58	62
11									34	38	42	46	50	54	57	61	65	69
12										42	47	51	55	60	64	68	72	77
13											51	56	61	65	70	75	80	84
14												61	66	71	77	82	87	92
15													72	77	83	88	94	100
16														83	89	95	101	107
17															96	102	109	115
18																109	116	123
19																	123	130
20																		138