National Exams December 2014

98-Ind-A1

Operations Research

3 hours duration

NOTES:

1 If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.

2 Any non-communicating calculator is permitted. This is an Open Book exam. Note to candidates: You must indicate the type of calculator being used, i.e. write the name and model designation of the calculator, on the first left hand sheet of the exam workbook.

3 There may be more questions than you are able to answer in the allotted time. Although the total value of the questions is 150, any marks achieved will be considered toward the 100 total requirements.

15

1. A company has one item in its inventory that requires special storage. The company estimates the storage cost at $2.00 per unit per year. The demand for the item is 40,000 units per year, and the ordering cost is $16.00 per order.

   a. Assuming shortages are not allowed, write an expression for the total yearly cost in terms of the order quantity Q, and determine the optimal order quantity.

   b. Assuming shortages are allowed and cost $4.00 per unit per year, write an expression for the total yearly cost in terms of the order quantity Q and the maximum shortage level s, and determine the optimal order quantity.

15

2. A single overhead crane attends to re-load ten machines. When a machine finishes its load, the overhead crane is called to unload the machine and to provide it with a new load from an adjacent storage area. The machine time per load is assumed exponential with mean 30 minutes. The time from the moment the crane moves to service a machine until a new load is installed is also exponential with mean 10 minutes.

   a. What percentage of time is the crane idle?

   b. What is the expected number of machines waiting for crane service?

15

3. A particular project consists of nine tasks. The crash times, normal times, bi,j values giving the cost increase associated with a unit time saving for activity (i,j), and precedence relationships are given below:

Task	Min Task Time	Normal Task Time	bi,j	Immediate Predecessors
A	1	3	4	none
B	2	4	1	none
C	0.5	2	1	A
D	2	5	1	A
E	1	6	3	B, C
F	1	2	7	D, E
G	3	4	9	D, E
H	2	3	5	F
I	4	5	8	G

a) Draw an appropriate project diagram, and identify the tasks on the critical path.

b) Write, but do not solve, an LP model for the problem of determining optimal activity times for completing the project by a specified deadline.

4 A company has contracted to sell certain quantities of a particular product over the next four months. Because of variations in the size of the labour force, the production capacity and costs vary from month to month. Storage costs are incurred on any item carried over to a later month. (If the item is sold in the same month as production, no storage cost is incurred.) These costs are summarized below: 15

Month	Contracted Sales	Production Capacity	Production Cost per Unit	Storage Cost per Unit
1	60	90	70	2
2	70	60	72	1
3	90	80	70	1
4	70	100	65	3

Formulate, but do not solve, the linear programming model to minimize costs

5 Solve the following integer LP problem using the branch and bound method. 15

Maximize $x1 + 2x_2$

subject to

$$x_1 + 3x_2 \leq 16$$
$$x_1 + x_2 \leq 7$$
$$x_1 \geq 0, x_2 \geq 0,$$
x_1 and x_2 integers

Note: The graphical LP method may be used within the branch and bound solution method.

6 An electric utility is considering 5 possible locations to build additional power plants over the next 20 years. The cost of building a plant at each site, the annual operating cost and the energy/year (in MWh) that can be provided at each site are given, as well as the total energy (in MWh) requirement for each year of the planning horizon. Assume that at most one plant can be built at each site, that at most one plant can be placed into service in a given year, and that it can produce its full energy contribution starting the year it is placed in service. The company can currently generate 500,000 MWh per year, using its existing resources. Develop an integer programming model to determine the expansion plan that will minimize overall construction and operating costs over the 20 years. 15

15 7 Consider the following problem

Maximize $z = 21x_1 + 9x_2 + 4x_3$ (profit)

Subject to

$2x_1 + x_2 + x_3 \leq 31$ (resource constraint 1)
$3x_1 + 2x_2 + x_3 \leq 60$ (resource constraint 2)
$x_1 + 2x_2 + x_3 \geq 50$ (requirement constraint)
$x_1 \geq 0$
$x_2 \geq 0$

The simplex method yields the following final set of equations

$z + (1/2)x_3 + (2/3)x_4 + x_6 = 291$
$x_1 + (1/3)x_3 + (2/3)x_4 + (1/3)x_6 = 4$
$x_2 + (1/3)x_3 - (1/3)x_4 - (2/3)x_6 = 23$
$x_5 - (2/3)x_3 - (4/3)x_4 + (1/3)x_6 = 2$

where x_4 is the slack variable for resource constraint 1, x_5 is the slack variable for resource constraint 2, and x_6 is the slack variable for the requirement constraint.

a) What is the optimal solution, the maximum profit, the marginal values of resources 1 and 2, and the marginal cost of the requirement?

b) How much can the coefficient of x_2 in the objective function vary without affecting the optimal solution?

c) By how much would the profit be increased if 5 more units of resource 1 where available? What would be the new solution?

15 8 A company buys tractors at a cost of $6,500 and sells them for $10,000. A charge of $2500 is incurred for each order of tractors, regardless of the size of the order. The company estimates the holding charge for one tractor to be $500 per tractor per month.

a. If there is a monthly demand for 15 tractors what is the optimal ordering policy, and the resulting monthly inventory (i.e. holding and ordering) cost?

b. If instead the demand is not constant and the forecast for the next 4 months is 20, 25, 12 and 3, use dynamic programming to determine the optimal ordering policy and the minimal cost over the four-month period.

15 9 Hearts Dog Food Company is in fierce competition with Corporal for the country's dog biscuit market. Every dog owner buys one box of dog biscuits per month. If last month he bought a box of Hearts biscuits, then there is an 80% chance that next month's purchase will also be for Hearts. If last month's purchase was for a box of Corporal biscuits then there is a 90% chance that next month he will again purchase Corporal. Hearts sells a box of biscuits for $1 which it costs $0.80 to produce.

a) If there are 40 million dog owners in the country what is Hearts' annual expected profit?

b) If Hearts is willing to reduce its selling price by x cents, it can increase the probability that a Hearts customer in one month will again purchase a Hearts box next month by x%. By how much should Hearts reduce its price to maximize its expected profit?

15 10 Every time a machine breaks down it requires 1, 2, or 3 hours to fix it according to the following probability distribution.

Repair time (hours)	Probability
1	0.30
2	0.50
3	0.20

The number of machine breakdowns per day is given by

Machine breakdowns per day	Probability
0	0.30
1	0.30
2	0.40

a. Draw a flowchart for a procedure that will estimate the average amount of work required in a day to fix machines.

b. Using the following random number list, carry out the simulation for two days operation, and compare the simulated average with the theoretical average amount of repair work per day.

[13516048662961142804223666665439975]