NATIONAL EXAMINATIONS

December 2019

16-MEC-B6 FLUID MACHINERY

Three hours duration

Notes to Candidates

1. This is a **Closed Book** examination.

2. Examination consists of two Sections: **Section A is Calculative (5 questions)** and **Section B is Descriptive (3 questions).**

3. **Do four (4) questions (including all parts of each question) from Section A (Calculative) and two (2) questions from Section B (Descriptive).**

4. **Six questions constitute a complete paper.** (Total 60 marks).

5. **All questions are of equal value.** (Each 10 marks).

6. If doubt exists as to the interpretation of any question, the candidate is urged to submit, with the answer paper, a clear statement of any assumptions made.

7. Read the entire question before commencing the calculations and take note of hints or recommendations given.

8. If any initial parts of a multi-part question cannot be solved the remaining parts may be worked by making appropriate assumptions for the first parts from the technical data given.

9. Candidates may use one of the approved **Casio** or **Sharp** calculators.

10. **Reference data** for particular questions are given in the Attachments on pages 12 to 17. **All pages from which data has been obtained or on which answers have been written are to be returned with the answer booklet to show any working. Candidate's names must be on these sheets.**

11. **Reference formulae and constants** are given on pages 18 to 22.

12. **Drawing Instruments** (scale ruler, protractor and sharp pencil) are required for vector diagrams. While calculation of velocities by trigonometric ratios with reference to a sketch is acceptable it is longer and more time consuming.

SECTION A CALCULATIVE QUESTIONS

Show all steps in the calculations and state the units for all intermediate and final answers.

QUESTION 1 NUMBER OF STAGES FOR GAS TURBINE

Refer to the Examination Paper Attachments Page 12 **Gas Turbine for ARC-100 Nuclear Plant** for orientation only and to Page 13 **Gas Turbine Velocity Diagram** for vector nomenclature.

The proposed ARC-100 Small Modular Nuclear Reactor has been designed to employ a regenerative gas cycle for energy conversion. The working fluid is to be carbon dioxide which receives heat from the nuclear reactor via two sodium loops. An estimate of the size and number of stages of the turbine is required.

Technical parameters are as follows:

Carbon dioxide pressure at turbine inlet	21 MPa
Carbon dioxide pressure at turbine exhaust	7 MPa
Carbon dioxide temperature at turbine inlet	490˚C
Carbon dioxide temperature at turbine exhaust	341˚C
Carbon Dioxide mass flow rate	1 937 kg/s
Turbine speed	3 600 rev/min

Assume the following parameters for the working fluid:

Carbon dioxide specific heat c_p	0.844 kJ/kg˚C	(constant pressure)
Carbon dioxide specific heat c_v	0.655 kJ/kg˚C	(constant volume)

Assume that the parameters for carbon dioxide remain the same at elevated temperatures as at ambient conditions as given.

For basic design purposes assume the following guidelines:

- Inlet blade height $h = ¼ r$ where r is the blade root radius (at turbine inlet).
- Pure impulse blades (no acceleration in moving blades).
- No friction losses in the blades.
- Axial gas velocity V_A = 100 m/s at the inlet and throughout the turbine. *Note that V_A is in the same direction and of the same magnitude as V_0 at each stage in the turbine.*
- Gas velocity entering the moving blades $V_{S1} = 2.33 \times V_B$ where V_B is the blade velocity.

This question is continued on the next page

QUESTION 1 Continued

A preliminary design of the turbine (excluding the compressor) is to be devloped.

Calculate the following:

(a) Required gas flow area at the turbine inlet. (2)

(b) Diameter of the rotor at mid-height of the blades at the turbine inlet. (2)

(c) Blade velocity V_B and gas velocity V_{S1}. (2)

(d) Power output of the turbine (excluding the compressor). (1)

(e) Number of stages required. (3)

[10 marks]

QUESTION 2 VELOCITY DIAGRAM FOR GAS TURBINE

Refer to the Examination Paper Attachments Page 12 **Gas Turbine for ARC-100 Nuclear Plant** for orientation only and to Page 13 **Gas Turbine Velocity Diagram** for vector nomenclature.

The proposed ARC-100 Small Modular Nuclear Reactor has been designed to employ a regenerative gas cycle for energy conversion. The working fluid is to be carbon dioxide which receives heat from the nuclear reactor via two sodium loops. From basic design parameters the blade angles need to be defined by drawing a velocity diagram of the gas flow through the blades for the first stage.

Technical parameters are as follows:

Carbon dioxide pressure at turbine inlet	21 MPa
Carbon dioxide pressure at turbine exhaust	7 MPa
Carbon dioxide temperature at turbine inlet	490°C
Carbon dioxide temperature at turbine exhaust	341°C
Carbon Dioxide mass flow rate	1 937 kg/s
Turbine speed	3 600 rev/min
Rotor diameter at blade mid-height	0.617 m
Number of stages	4

Assume the following parameters for the working fluid:

Carbon dioxide specific heat c_p	0.844 kJ/kg°C	(constant pressure)
Carbon dioxide specific heat c_v	0.655 kJ/kg°C	(constant volume)

Assume that the parameters for carbon dioxide remain the same at elevated temperatures as at ambient conditions as given.

This question is continued on the next page

QUESTION 2 Continued

Do the following assuming that the axial gas velocity at the inlet of the turbine V_0 and throughout the turbine V_A is 100 m/s. *Note that V_A is in the same direction and of the same magnitude as V_0 at each stage in the turbine.*

(a) Calculate the enthalpy drop in the turbine per stage and hence the gas velocity entering the moving blades. (2)

(b) Calculate the blade velocity and draw to scale the velocity diagram for the first stage of the turbine. (5)

(c) Measure the blade angles of the fixed blade exit and moving blade entrance and exit. (1)

(d) Measure the gas velocities and calculate the power output of the whole turbine assuming that all stages have similar velocity diagrams. (2)

Note: The scale drawing should be to a large enough scale for accurate measurements (a scale of 50 mm = 100 m/s is suggested). While calculation of velocities by trigonometric ratios is acceptable it is longer and more time consuming.

[10 marks]

QUESTION 3 PUMP PERFORMANCE

Refer to the Examination Paper Attachments Page 14 **Pump Velocity Diagram.**

The attached diagram clarifies the nomenclature to be used in answering the question.

The picture below the velocity diagram shows a pump impeller of a radial flow centrifugal pump for pumping water. The key dimensions are as follows:

Blade inner diameter	D_1 = 130 mm	
Blade outer diameter	D_2 = 300 mm	
Blade inner height	h_1 = 20 mm	(in axial direction)
Blade outer height	h_2 = 10 mm	(in axial direction)
Blade inlet angle	β_1 = 20°	
Blade outlet angle	β_2 = 25°	
Pump speed	N = 1 750 rev/min	
Water flow rate	Q = 0.030 m^3/s	
Hydraulic head	H = 35 m	

For the given speed and flow rate draw to a scale the velocity diagrams at inlet and outlet and determine the following neglecting the vane thickness:

(a) Tangential blade velocities at inlet and outlet. (2)

(b) Radial water velocity at inlet and outlet. (2)

(c) Tangential water velocity at inlet and outlet. (2)

(d) Torque and power required to drive the impeller. (2)

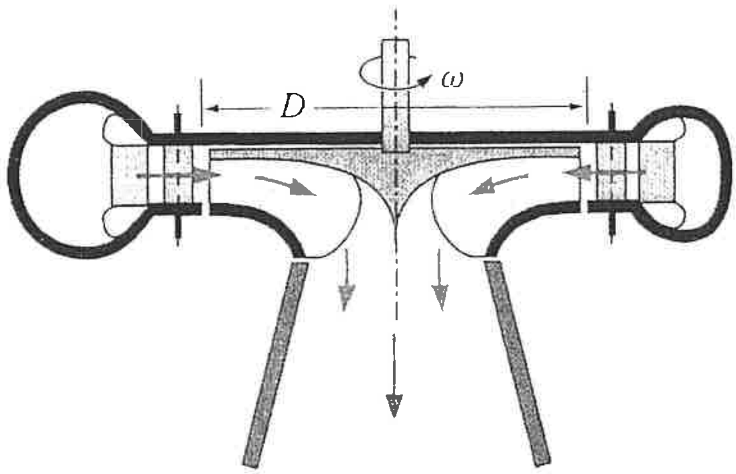
(e) Hydraulic power and efficiency of pump. (2)

Note: The scale drawing should be to a large enough scale for accurate measurements (a scale of 10 mm = 2 m/s is suggested). While calculation of velocities by trigonometric ratios is acceptable it is longer and more time consuming.

[10 marks]

QUESTION 4 HYDRO TURBINE DESIGN

Refer to the Examination Paper Attachments Page 15 **Hydro Turbine Parameters**.



In the preliminary stages in the design of a hydro power plant some basic dimensions need to be established. The conditions suggest a turbine of the Francis type as shown in the figure above. It has the following basic parameters:

Specific speed	$N_s = 0.9$	
Effective (net) head	$H = 160$ m	
Required speed	$N = 750$ rev/min	
Hydraulic efficiency	$\eta = 0.94$	
Runner blade inlet angle	$\alpha_1 = 18°$	(relative to tangent to runner)
Runner tip speed	$U_1 = 0.7\, V_{jet}$	

Note that V_{jet} is the free jet velocity when subject to the given head.

(a) Calculate the turbine power output. (2)

(b) Calculate the water flow rate. (1)

(c) Calculate the runner diameter D. (2)

(d) Sketch the velocity diagram at the runner inlet and, by scale drawing or calculation, determine the radial flow velocity. (3)

(e) Calculate the runner height at the inlet. (2)

[10 marks]

QUESTION 5 HYDRO TURBINES

PART I PELTON WHEEL

Refer to the Examination Paper Attachments Page 16 **Bridge River Plant**.

Note: Convert the given data to SI units using the conversions below and solve in SI units.

1 HP = 746 watts
1 inch = 25.4 mm
1 foot = 304.8 mm

Data:	
Gross head	1 226 ft
Net head	1 118 ft
Power output	62 000 HP
Rotational speed	300 rpm
Pitch diameter	95 in
Number of nozzles	6

Determine the following:

(a) Ratio of actual blade velocity to anticipated jet velocity. (3)

(b) Deviation as a percentage of the ratio calculated in (a) above from the ideal ratio and give a possible reason for this deviation. (1)

(c) Volume flow rate required to give the specified output. (1)

(5 marks)

This question is continued on the next page

QUESTION 5 Continued

PART II TURBINE SETTING

Refer to the Examination Paper Attachments Page 17 **Critical Cavitation Parameter**.

Vanderkloof Hydro Power Station has the following technical parameters:

Electrical generator design output	120 MW
Electrical generator voltage	11 kV
Speed of turbine-generator	125 rev/min
Type of hydro turbine	Francis
Design head on turbine	65 m
Maximum water consumption (at lower head)	217 m^3/s
Inlet diameter to spiral casing	7 m
Turbine runner diameter	5 462 mm
Turbine runner material	Stainless cast steel

(a) Calculate the specific speed of the turbine. (2)

(b) From the graph determine the Thoma cavitation parameter σ. (1)

(c) Calculate the setting (maximum elevation) of the turbine runner relative to the tailrace water level based on the critical cavitation parameter (Thoma coefficient). (2)

Show on the attached diagram on Page 17 where data has been obtained and return this page with your answer booklet.

(5 marks)

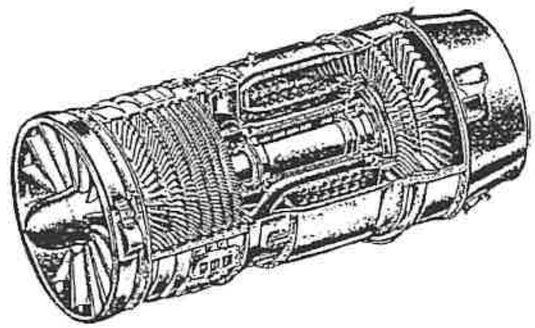
[10 marks]

SECTION B DESCRIPTIVE QUESTIONS

Note that a ten mark question requires an answer of approximately two full pages of complete explanations with sketches if appropriate to support the explanation.

QUESTION 6 GAS TURBINE NUMBER OF STAGES

The adjoining picture of an aircraft gas turbine engine shows the number of stages in the compressor and the number of stages in the turbine. Explain why the compressor has many more stages than the turbine. Give the reasons for selecting the appropriate number of stages for both the compressor and the turbine. Explain what requires the compressor to have a certain minimum number of stages and why the turbine must have multiple stages.



Answer the question by considering the limiting operating conditions for each part (compressor and turbine) of the machine.

[10 marks]

QUESTION 7 TURBINE BLADE CHARACTERISTICS

PART I IMPULSE AND REACTION

Explain the difference between an impulse turbine and a reaction turbine. In particular refer to the changes in velocity in both the fixed and moving blades. Clarify how the forces developed are created and how they influence the transfer of energy from the fluid to the blades. Show the difference between impulse and reaction in sketches of velocity diagrams for an axial flow gas or steam turbine.

(5 marks)

PART II OPTIMUM BLADE EFFICIENCY

With respect to a Pelton turbine show graphically in a sketch (efficiency versus blade velocity) <u>how</u> and explain <u>why</u> the efficiency varies with turbine blade velocity (wheel rotational speed) when the water jet velocity remains constant. Consider the whole range of possibilities from a blade velocity of zero to a blade velocity equal to that of the jet. If appropriate, draw velocity diagrams to illustrate the explanation.

(5 marks)

[10 marks]

QUESTION 8 CENTRIFUGAL PUMPS

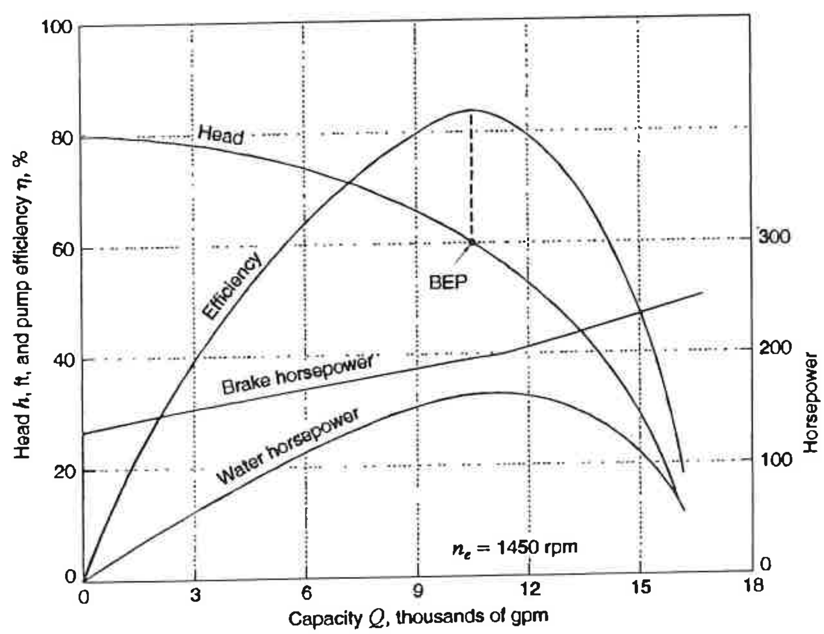
PART I EFFICIENCY AND LOSS

With reference to the figure below explain the following:

(a) Why the hydraulic efficiency (water horsepower) rises from zero to a peak and then declines towards zero

(b) Why the difference between the hydraulic power (water horsepower) and the mechanical power (brake horsepower) decreases to a low value and then increases to a value greater than the initial value.

(5 marks)



Characteristic curves for a typical mixed-flow centrifugal pump.

PART II NUMBER OF VANES

Centrifugal pump impellers are usually designed for an optimum number of vanes. Explain the effect on performance of a pump having both too many vanes or too few vanes. Clarify how the number of vanes influences the flow through the pump. Explain the reasons for these effects.

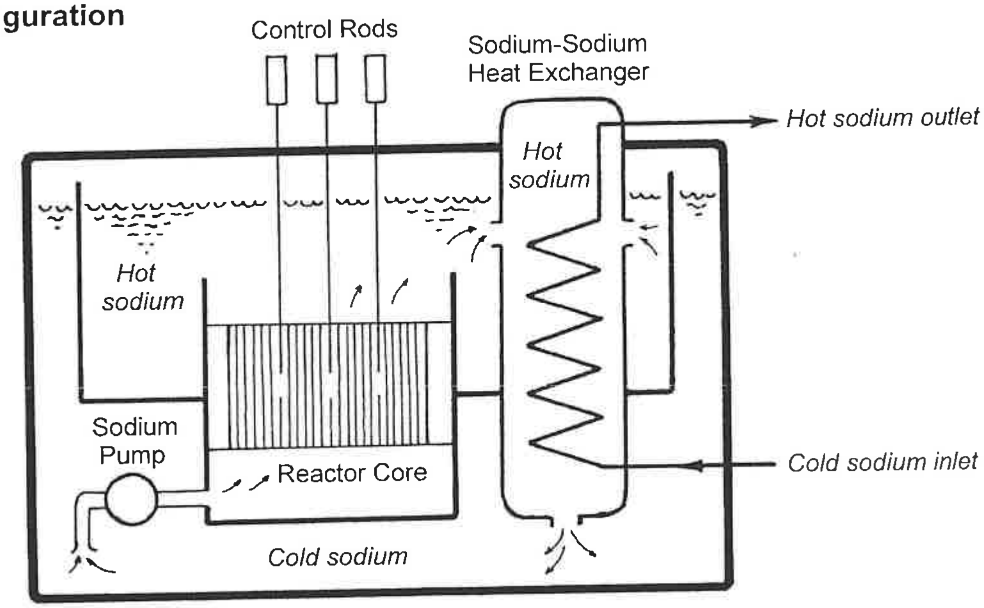
(5 marks)

[10 marks]

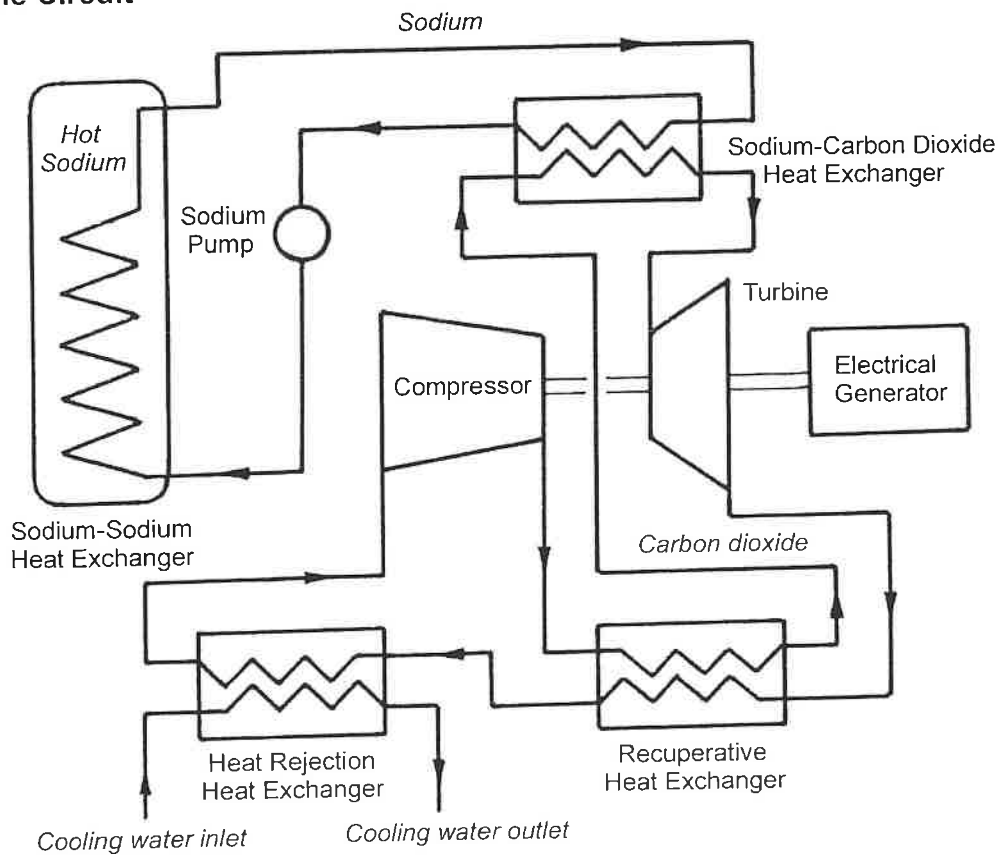
EXAMINATION PAPER ATTACHMENTS

QUESTION 1 & QUESTION 2 GAS TURBINE FOR ARC-100 NUCLEAR PLANT

Reactor Configuration



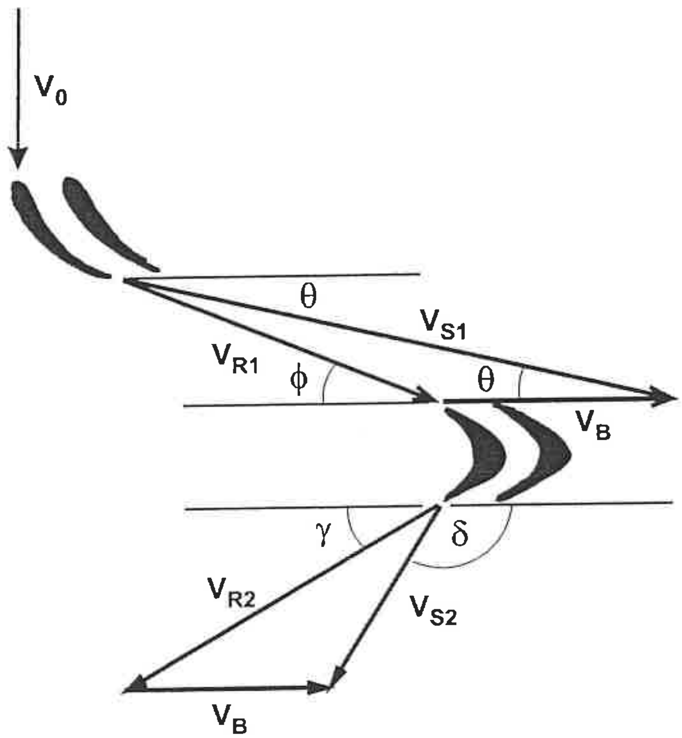
Gas Turbine Circuit



EXAMINATION PAPER ATTACHMENTS

QUESTION 1 & QUESTION 2 GAS TURBINE VELOCITY DIAGRAM

Nomenclature for velocity vectors and angles



V_0 Absolute gas velocity entering turbine

V_{S1} Absolute gas velocity entering moving blades

V_{R1} Relative gas velocity entering moving blades

V_B Moving blade velocity

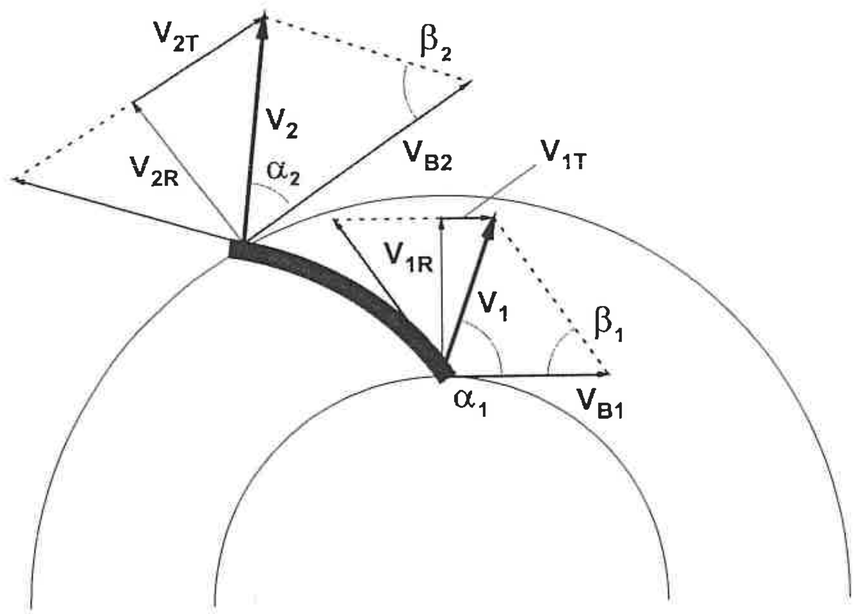
V_{R2} Relative gas velocity leaving moving blades

V_{S2} Absolute gas velocity leaving moving blades

EXAMINATION PAPER ATTACHMENTS

QUESTION 3 PUMP VELOCITY DIAGRAM

Nomenclature for velocity vectors and angles

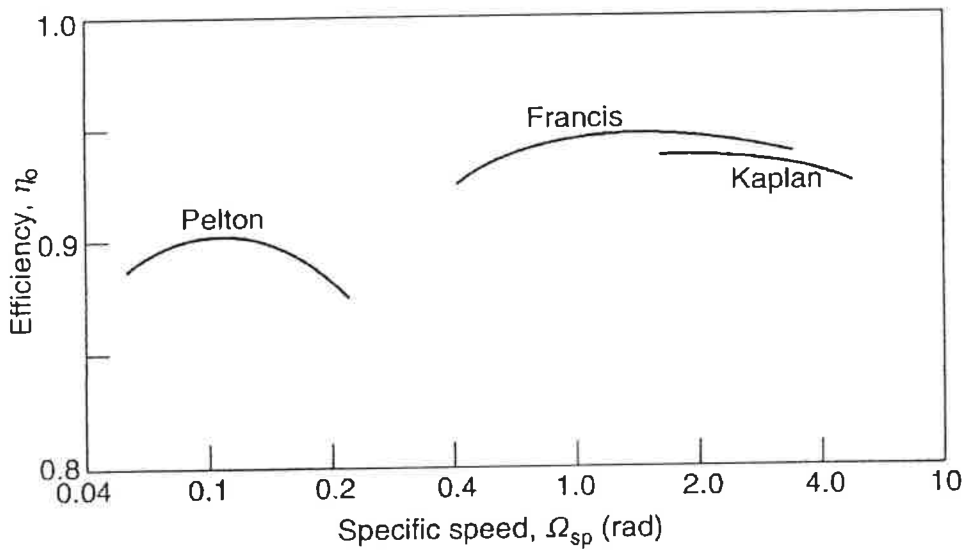


V_1	Absolute water velocity at inlet
V_{B1}	Blade velocity at inlet
V_{1R}	Radial water velocity at inlet
V_{1T}	Tangential water velocity at inlet
V_2	Absolute water velocity at outlet
V_{B1}	Blade velocity at outlet
V_{2R}	Radial water velocity at outlet
V_{2T}	Tangential water velocity at outlet

EXAMINATION PAPER ATACHMENTS

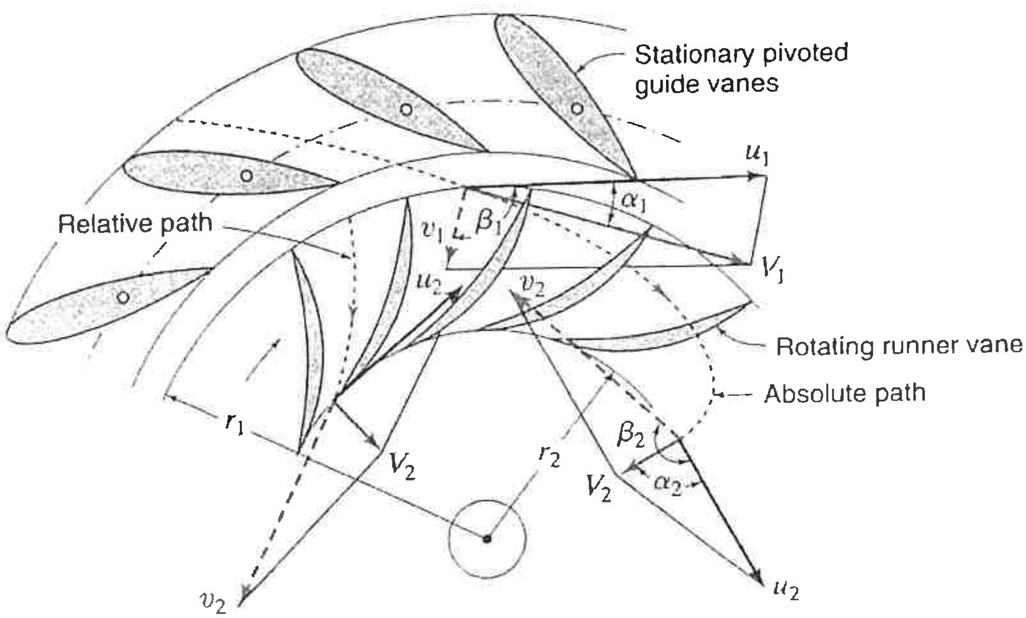
QUESTION 4 HYDRO TURBINE PARAMETERS

Turbine Efficiencies



Typical design point efficiencies for Pelton, Francis and Kaplan turbines

Velocity Diagram



Velocity vectors for radial inward flow hydraulic turbine (Francis type)

EXAMINATION PAPER ATTACHMENTS

QUESTION 5 PART I BRIDGE RIVER PLANT

484 *15 Impulse Turbines*

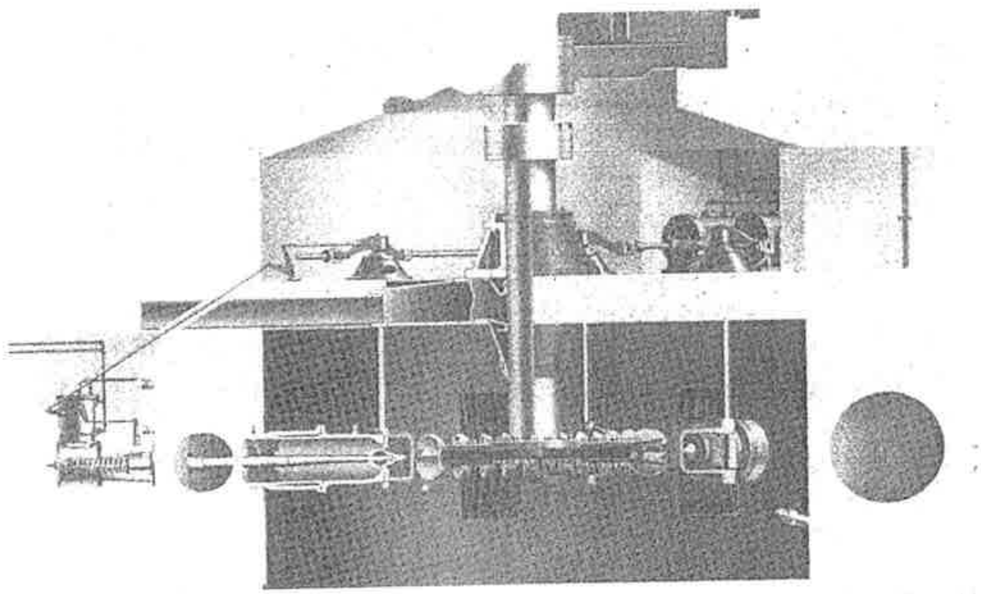
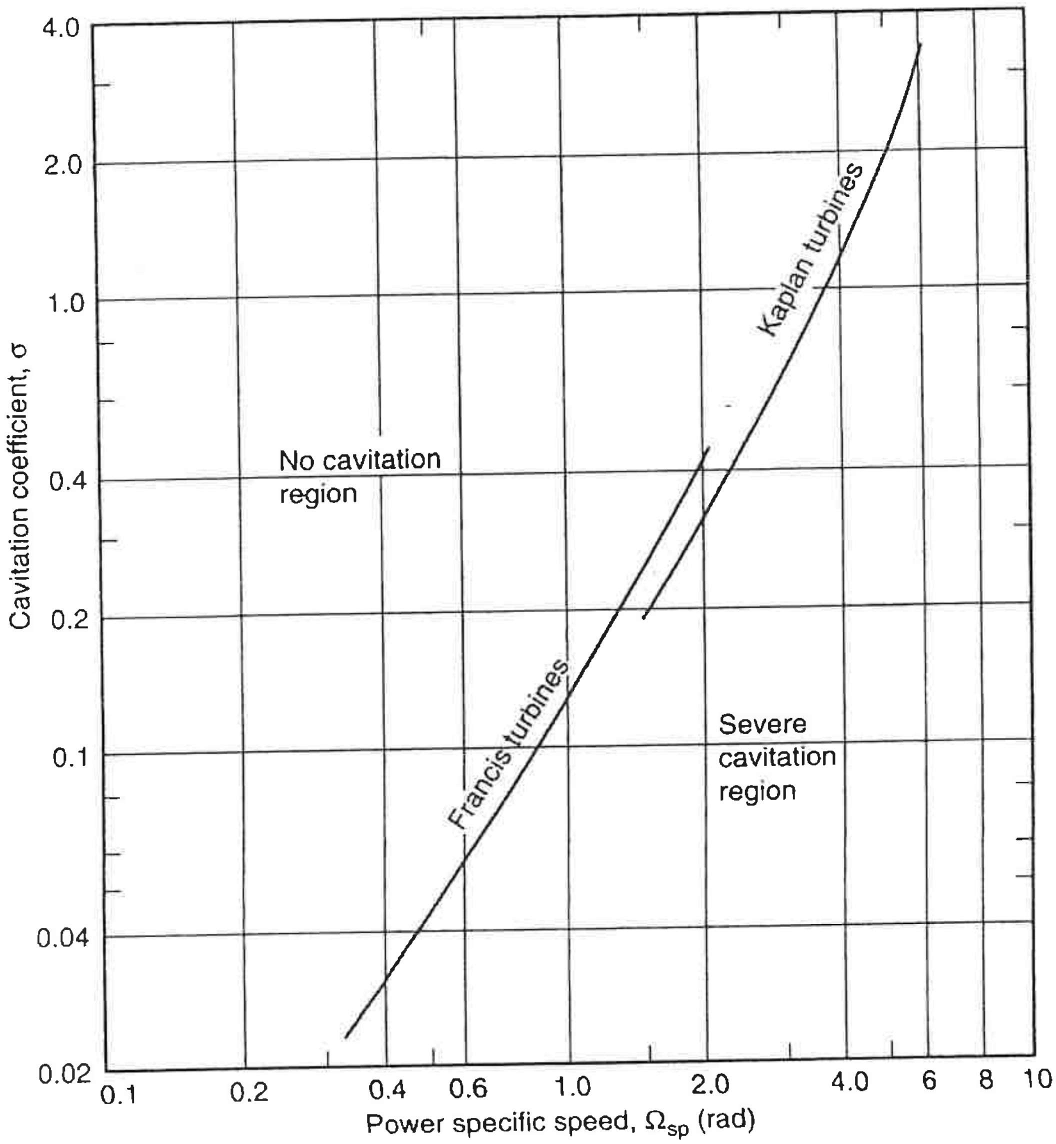


Figure 15.2 Vertical-shaft impulse turbine with six nozzles at Bridge River plant in British Columbia. Gross head = 1,226 ft, net head = 1,118 ft, 62,000 hp, $n = 300$ rpm, pitch diameter = 95 in.

EXAMINATION PAPER ATTACHMENTS

NAME ....

QUESTION 5 PART II CRITICAL CAVITATION PARAMETER



Variation of critical cavitation parameter with non-dimensional specific speed in SI units for Francis and Kaplan turbines

EXAMINATION REFERENCE MATERIAL

NOMENCLATURE FOR REFERENCE EQUATIONS (SI UNITS)

a	Sonic velocity	m/s
A	Flow area, Surface area	m^2
c_p	Specific heat at constant pressure	J/kg°C
c_v	Specific heat at constant volume	J/kg°C
b	Width	m
C	Velocity	m/s
D	Diameter	m
E	Energy	J
F	Force	N
g	Gravitational acceleration	m/s^2
h	Specific enthalpy	J/kg
h	System head	m
h_L	Head loss	m
H	Pump or turbine head	m
k	Ratio of specific heats	
L	Length	m
m	Mass	kg
M	Mass flow rate	kg/s
M_A	Mach number	
N	Rotational speed	rev/min
N_S	Specific Speed	
p	Pressure	Pa (N/m^2)
P	Power	W (J/s)
q	Heat transferred	J/kg
Q	Heat	J
Q	Flow rate	m^3/s
r	Radius	m
R	Specific gas constant	J/kg K
s	Entropy	J/kg K
T	Temperature	K
u	Specific internal energy	J/kg
U	Internal Energy	J
U	Velocity	m/s
v	Specific volume	m^3/kg
V	Velocity	m/s
w	Specific work	J/kg
W	Work	J
W	Velocity	m/s
x	Length	m

z	Elevation	m
α	Pump blade angle	°
α	Compressor blade angle	°
β	Pump blade angle	°
β	Compressor blade angle	°
γ	Turbine blade angle	°
φ	Turbine blade angle	°
δ	Turbine blade angle	°
η	Efficiency	
θ	Nozzle angle	°
μ	Dynamic viscosity	Ns/m^2
ν	Kinematic viscosity	m^2/s
ρ	Density	kg/m^3
σ_C	Critical cavitation parameter	
τ	Thrust	N
τ	Torque	Nm
φ	Peripheral velocity factor	
ω	Rotational speed	rad/s
Ω	Heat transfer rate	J/s

GENERAL CONSTANTS

Use unless otherwise specified

Acceleration due to gravity:	$g = 9.81\ m/s^2$	
Atmospheric pressure:	$p_{atm} = 100$ kPa	
Water vapour pressure:	$p_{vapour} = 1.71$ kPa	(at 15°C)
Water vapour pressure:	$p_{vapour} = 2.34$ kPa	(at 20°C)
Density of water:	$\rho_{water} = 1000\ kg/m^3$	
Density of air:	$\rho_{air} = 1.21\ kg/m^3$	(at 15°C)
Density of air:	$\rho_{air} = 1.19\ kg/m^3$	(at 20°C)
Specific heat of air:	$c_p = 1.005$ kJ/kg°C	
Specific heat of air:	$c_v = 0.718$ kJ/kg°C	
Specific heat of water:	$c_p = 4.19$ kJ/kg°C	

GENERAL REFERENCE EQUATIONS

Basic Thermodynamics

First Law:	$dE = \delta Q - \delta W$
Enthalpy:	$h = u + pv$
Continuity:	$\rho VA = \text{constant}$
Potential Energy:	$E_{PE} = mgz$
Kinetic Energy:	$E_{KE} = V^2/2$
Internal Energy:	$E_{IN} = U$
Flow Work:	$w = \Delta(pv)$
Energy Equation:	$zg + V^2/2 + u + pv + \Delta w + \Delta q = \text{constant}$

Ideal Gas Relationships

Gas Law:	$pv = RT$
Specific Heat at Constant Pressure:	$c_p = \Delta h/\Delta T$
Specific Heat at Constant Volume:	$c_v = \Delta u/\Delta T$
Specific Gas Constant:	$R = c_p - c_v$
Ratio of Specific Heats	$k = c_p / c_v$
Isentropic Relations:	$p_1/p_2 = (v_2/v_1)^k = (T_1/T_2)^{k/(k-1)}$

FLUID MACHINERY REFERENCE EQUATIONS

Fluid Mechanics

Pressure	$p = \rho gh$
Continuity Equation	$\rho_1 V_1 A_1 = \rho_2 V_2 A_2 = M$
Bernoulli's Equation	$p_1/\rho g + z_1 + V_1^2/2g = p_2/\rho g + z_2 + V_2^2/2g$
Momentum Equation	$F = p_1 A_1 - p_2 A_2 - \rho VA(V_2 - V_1)$ (one dimensional)

Energy Equation

Pump and Turbine	$p_1/\rho g + z_1 + V_1^2/2g + w_{in}/g = p_2/\rho g + z_2 + V_2^2/2g + w_{out}/g$
Pipe Flow	$p_1/\rho g + z_1 + V_1^2/2g = p_2/\rho g + z_2 + V_2^2/2g + h_L$

Compressible Flow

Mach Number	$M_A = V/a$
Sonic Velocity	$a = [kRT]^{1/2}$

Steam Turbines

Nozzle Equation:	$h_1 - h_2 = (V_2^2 - V_1^2) / 2$
Work:	$w = [(V_1^2{}_{absolute} - V_2^2{}_{absolute}) + (V_2^2{}_{relative} - V_1^2{}_{relative})] / 2$
Work:	$w = (V_{S1}\cos\theta - V_{S2}\cos\delta)\, V_{blade}$
Power:	$P = wM$
Force on Blades:	$F = M\,(V_{S1}\cos\theta - V_{S2}\cos\delta)$
Power to Blades:	$P = M\,(V_{S1}\cos\theta - V_{S2}\cos\delta)\, V_B$
Power to Blades:	$P = M\,[(V_{S1}^2 - V_{S2}^2) + (V_{R2}^2 - V_{R1}^2)] / 2$

Gas Turbines

State Equation:	$pv = RT$
Isentropic Equation:	$(T_2/T_1) = (p_2/p_1)^{(k-1)/k}$
Enthalpy Change:	$h_1 - h_2 = c_p\,(T_1 - T_2)$ (ideal gas)
Nozzle Equation:	$h_1 - h_2 = (V_2^2 - V_1^2) / 2$
Work:	$w = (C_1\sin\alpha_1 + C_2\sin\alpha_2)\, U$
Work:	$w = [(C_1^2 - C_2^2) + (W_2^2 - W_1^2)] / 2$
Power:	$P = wM$

Compressors

Work	$w = U\,(C_{Y2} - C_{Y1})$
Rotor Enthalpy Change	$h_1 + \frac{1}{2}W_1^2 = h_2 + \frac{1}{2}W_2^2$
Stator Enthalpy Change	$h_2 + \frac{1}{2}C_2^2 = h_3 + \frac{1}{2}C_3^2$
Isentropic Equation	$(T_3/T_1) = (p_3/p_1)^{(k-1)/k}$

Hydraulic Machines

Similarity Equations:	$Q_M/Q_P = (\omega_M/\omega_P)\,(D_M/D_P)^3$
	$H_M/H_P = (\omega_M/\omega_P)^2\,(D_M/D_P)^2$
	$P_M/P_P = (\rho_M/\rho_P)\,(\omega_M/\omega_P)^3\,(D_M/D_P)^5$
Pump Specific Speed:	$N_S = \omega\, Q^{1/2} / (gH)^{3/4}$
Turbine Specific Speed:	$N_S = \omega\, P^{1/2} / [\rho^{1/2}\,(gH)^{5/4}]$
Critical Cavitation Parameter:	$\sigma = [\{(p_{atmosphere} - p_{vapour}) / \rho g\} - \Delta z] / H$
Moody Efficiency Relationship:	$\eta_P = 1 - (1 - \eta_M)\,(D_M/D_P)^{1/4}\,(H_M/H_P)^{1/10}$
Power:	$P = \rho g Q H$

Pumps

Hydraulic Torque:	$\tau = \rho Q\ (r_2 V_{2T} - r_1 V_{1T})$
Hydraulic Torque:	$\tau = \rho Q\ (r_2 V_2 \cos\alpha_2 - r_1 V_1 \cos\alpha_1)$
Power:	$P = 2\pi N \tau$
Net Positive Suction Head:	$NPSH = [(p_{atmosphere} - p_{vapour}) / \rho g] - \Delta z - h_L$
Peripheral Velocity Factor:	$\varphi = V_{B2} / (2gh)^{1/2}$
Critical Cavitation Parameter:	$\sigma_C = NPSH / H$
Approximate Moody Efficiency:	$(1 - \eta_P) / (1 - \eta_M) \approx (D_M/D_P)^{1/5}$

Hydro Turbines

Power	$P = \tau\omega$
Power	$P = \rho Q\omega\ (r_1 V_1 \cos\alpha_1 - r_2 V_2 \cos\alpha_2)$
Power	$P = \rho Q\ (U_1 V_1 \cos\alpha_1 - U_2 V_2 \cos\alpha_2)$
Peripheral Runner Velocity	$U_1 = \varphi\ (2gH)^{1/2}$
Absolute Water Velocity	$V_1 = C_1\ (2gH)^{1/2}$
Tangential Velocity	$U = \omega r$
Angular Velocity	$\omega = 2\pi N / 60$

Jet Propulsion

Thrust:	$\tau = M\ (V_{jet} - V_{aircraft})$
Thrust Power:	$\tau V_{aircraft} = M\ (V_{jet} - V_{aircraft}) V_{aircraft}$
Jet Power:	$P = M\ (V_{jet}^2 - V_{aircraft}^2) / 2$
Propulsion Efficiency:	$\eta_p = 2\ V_{aircraft} / (V_{jet} + V_{aircraft})$

Wind Turbines

Maximum Ideal Power:	$P_{max} = 8\ \rho A V_1^3 / 27$