Professional Engineers of Ontario

Annual Examinations – December 2016

07-Elec-B3
Digital Communication Systems

3 Hours Duration

Notes:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper a clear statement of any assumptions made.
2. This is a closed book exam. A PEO-approved non-programmable calculator is permitted; any Casio or Sharp approved model.
3. There are **5 questions** on this exam. **Any 4 questions constitute a complete paper.** Only the first 4 questions as they appear in your answer book will be marked.
4. Marks allocated to each question are noted in the left margin. A complete paper is worth 100 marks.

(25 marks) **Question 1.** This question concerns link budgeting.

(10 marks) a. Consider a wireless system with transmitter power of 10 W, antenna gains of 6 dB, receiver losses of 9 dB, receiver noise figure of -174 dBm/Hz, a bandwidth of 10 MHz, and a fading margin requirement of 6 dB. Aside from free-space losses, no other gains or losses are present other than path loss. If the receiver requires a signal-to-noise ratio of at least 6 dB, what is the maximum allowed path loss (in dB)?

(10 marks) b. Using a free-space path loss of $30 \log_{10}(4 \pi df/c)$, where d represents the distance from transmitter to receiver, f represents the carrier frequency, and c represents the speed of light ($c = 3.0 \times 10^8$ m/s), and assuming a carrier frequency of 1.5 GHz, is the signal-to-noise criterion satisfied when d = 200 m? Show all work.

(5 marks) c. In part b, what is the path loss exponent of the system? (Explain in 1 sentence how you got it.)

(25 marks) **Question 2.** This question concerns source coding.

(15 marks) a. You are given a source with eight letters: A, B, C, D, E, F, G, H. The probabilities of these letters are: Pr(A) = 0.29; Pr(B) = 0.04; Pr(C) = 0.11; Pr(D) = 0.25; Pr(E) = 0.08; Pr(F) = 0.12; Pr(G) = 0.10; Pr(H) = 0.01. Find a Huffman code for this source.

(5 marks) b. What is the entropy of the source in part a?

(5 marks) c. If a vendor promised a compression scheme less than your answer from part b, would you buy the product? Explain in 2-3 sentences.

(25 marks) **Question 3.** This question concerns error-control coding.

(5 marks) a. Consider a binary code with the following parity check matrix. Find the corresponding generator matrix.

$$H = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

(5 marks) b. Using the result from part a, give the codeword for the information sequence: 0 1 0 1

(10 marks) c. Using an example, illustrate how the code from part a can correct a single bit error.

(5 marks) d. Is it possible for this code to correct two errors? Can it detect two errors (without correcting them)? Explain (in 2-3 sentences).

(25 marks) **Question 4.** This question concerns signal modulation and detection.

(5 marks) a. Consider signals $s_0(t)$ and $s_1(t)$, which are used to modulate the binary symbols "0" and "1", respectively, where

$$s_1(t) = \begin{cases} \sin(2\pi t/T), & 0 \le t \le T \\ 0 & \text{elsewhere} \end{cases}$$

and $s_0(t) = 0$. Sketch the two signals, and sketch the impulse response of the matched filter $m(t)$, assuming the filter is matched to $s_1(t)$, and assuming the filter output is sampled at time T.

(5 marks) b. In the absence of noise, what is the matched filter output at time T, if $s_1(t)$ is sent? Trig identity if you need it: $\sin^2 x = (1 - \cos 2x)/2$

(5 marks) c. At the sampling instant (time T), the matched filter output is corrupted by additive Gaussian noise with zero mean and variance σ^2. Give the optimal decision rule assuming that 0 and 1 are equiprobable.

(10 marks) d. Given that

$$\frac{1}{2}\text{erfc}\left(\frac{t-\mu}{\sqrt{2\sigma^2}}\right) = \int_t^\infty \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) dx$$

and given your decision rule from part c, express the probability of error *given than a 0 was sent* in terms of erfc.

(25 marks) **Question 5.** This question concerns sampling and D/A conversion.

(5 marks) a. CD-quality audio has a sampling frequency of 44.1 kHz. Using the Nyquist sampling criterion, what is the maximum signal bandwidth in order to reconstruct the signal exactly?

(5 marks) b. Briefly explain pulse code modulation (PCM). If PCM is used to encode the signal from part a with 16 bits per sample, what is the required data rate to represent the signal? (If you didn't get an answer for part a, assume a value.)

(5 marks) c. Give an example of "aliasing" (2-3 sentences).

(5 marks) d. Suppose 16-bit PCM is used to sample a signal restricted between -5 V and +5 V. What is the maximum quantization error?

(5 marks) e. The data rate of MP3-quality audio is much less than your answer from part b. Give one reason why.