

National Exam May 2019

**04-BS-06: Mechanics of Materials**

Duration: 3 hours

**Notes:**

1. If doubt exists as to the interpretation of any questions, the candidate is urged to submit with the answer paper a clear statement of any assumptions made.

2. Candidates may use one of two calculators, the Casio or Sharp approved models.

This is a CLOSED BOOK exam. Candidates are allowed to bring ONE aid sheet 8.5" X 11" hand-written on both sides containing notes and formulae. Note, example problems and solutions to problems are NOT allowed.

3. Any FIVE (5) questions (out of 8 given) constitute a complete paper. Only the first five questions as they appear in your Answer Book(s) will be marked.

4. All questions are of equal value.

5. Information on geometric properties of wide flange or W shaped sections is provided in Appendices (at the end of this Question Paper). There are two pages. Note that this information may not be required.

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**Note: This Question Paper along with the Aid Sheet (if used) must be handed in with the Answer Book(s).**

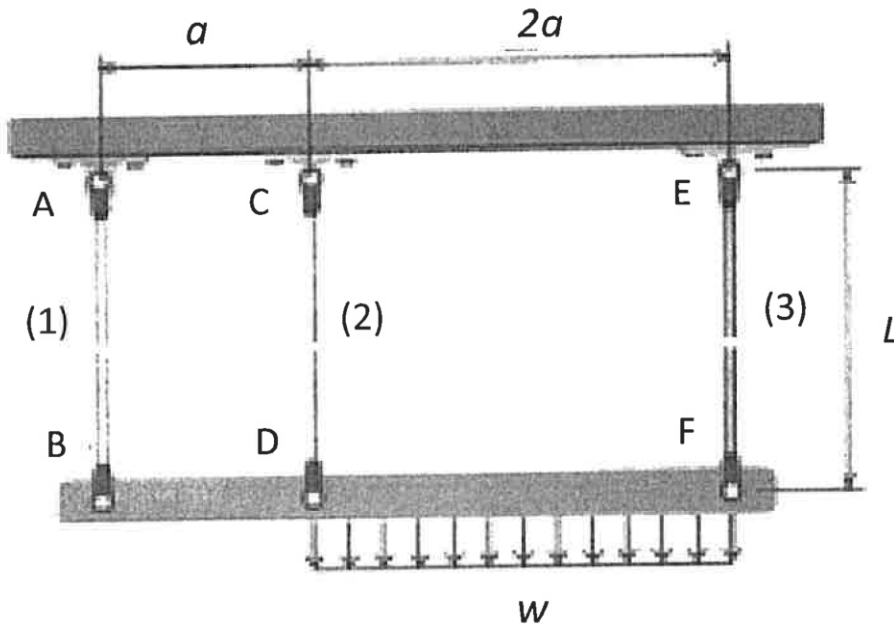
*Your Exam will not be marked if you do not hand in the Aid Sheet, unless there is a signed statement by the exam invigilator stating that no Aid Sheet was used in the exam writing.*

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**Question 1:**

A uniformly distributed load  $w$  is supported by a structure consisting of rigid bar  $BDF$  and three rods, as shown in figure below. Rods (1) and (2) are  $0.75\text{-in.}$  diameter stainless steel rods that have an elastic modulus  $E = 28,000\text{ ksi}$  and a yield strength  $\sigma_Y = 36\text{ ksi}$ . Rod (3) is a  $1.25\text{-in.}$  diameter bronze rod with an elastic modulus  $E = 16,000\text{ ksi}$  and a yield strength  $\sigma_Y = 48\text{ ksi}$ . Use  $a = 4\text{ ft}$  and  $L = 9\text{ ft}$ . If a minimum factor of safety of  $1.8$  is specified for the normal stress in each rod, calculate the maximum magnitude of the distributed load  $w$  that can be supported.

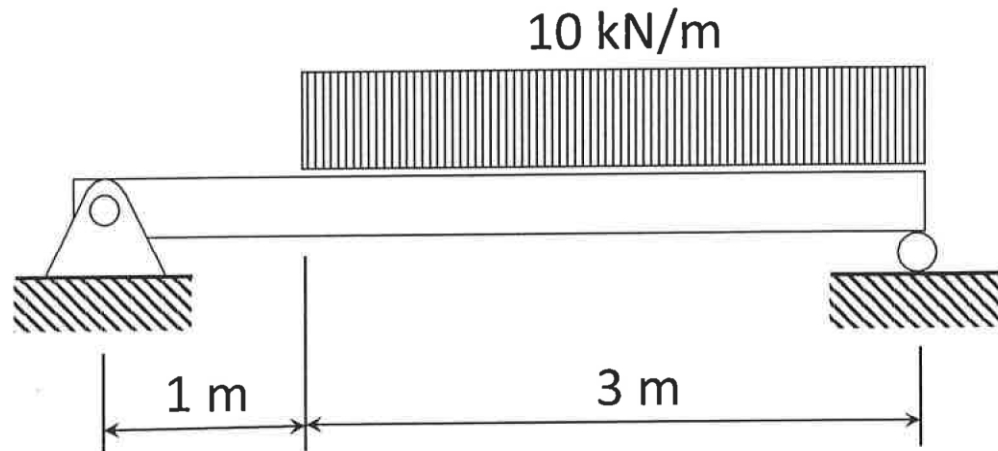
20 marks



**Question 2:**

A simply-supported beam supports a uniformly distributed load over part of the span, as shown in figure below. The beam is a W150x30 section made from steel having an allowable normal stress of  $240\text{ MPa}$  and allowable shear stress of  $60\text{ MPa}$ . The elastic modulus of the steel is  $200\text{ GPa}$ . Determine the maximum deflection of the beam using the Method of Integration.

20 marks



No credit will be given for a solution using the Principle of Superposition, when combination of existing solutions is used to find an answer.

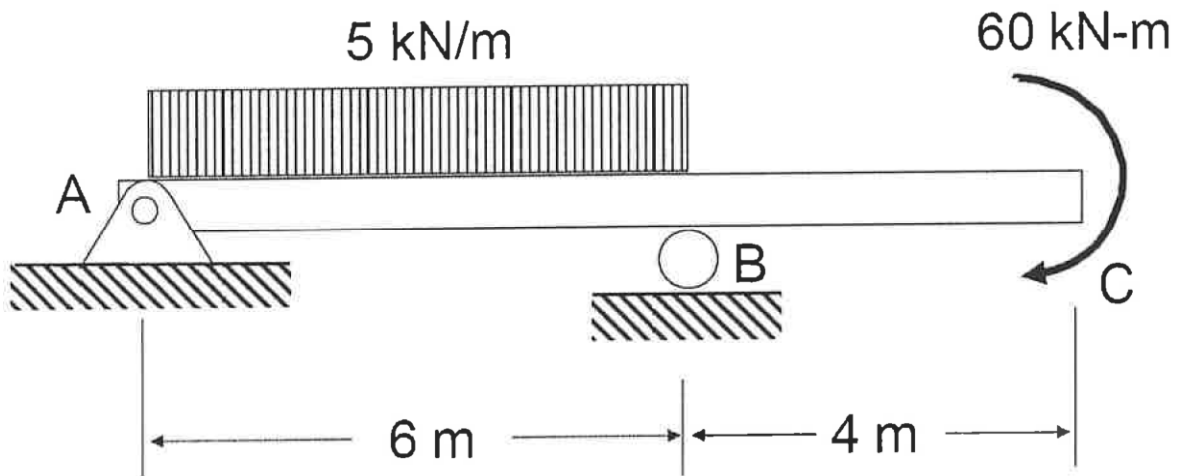
**Question 3:**

A simply-supported beam with a cantilevered overhang is subjected to a uniformly-distributed load over part of the member span and a concentrated moment acting at the free end of the member, as shown in figure below.

Determine the shear and moment throughout the beam as functions of  $x$ . This means that you need to give formulas for  $V(x)$  and  $M(x)$  along the length of the beam.

Next, construct the shear force and bending moment diagrams. Remember to label points of maximum and negative bending moment, as well as any inflection points. Show your work by indicating exactly how you obtained your answer.

20 marks



Remember that it is important you set this problem up correctly by calculating the correct reaction forces at the supports.

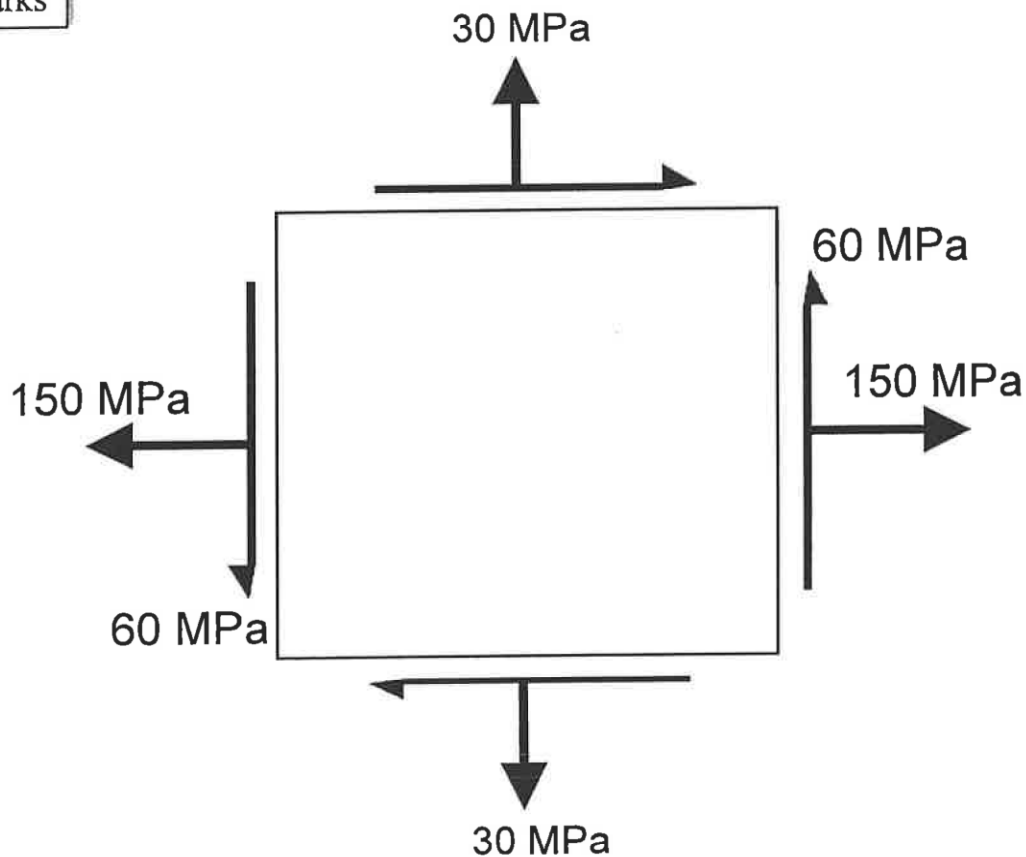
No credit will be given for a solution using the Principle of Superposition, when combinations of existing solutions are used to find an answer.

**Question 4:**

An element in a state of plane stress is subjected to the normal and shear stresses, as shown in figure below. Use the Mohr's circle solution (and not the transformation equations) to determine the following:

- Determine the principal stresses.
- Show, by sketch on a properly oriented element, the planes on which the principal stresses act.
- Determine the maximum shear stress.
- Show, by sketch on a properly oriented element, the plane on which the maximum stress acts.

20 marks



Note that credit will only be **given** for a **solution using Mohr's circle**. This means that you need to draw a Mohr's circle based on the stress components given in this problem. Remember to show numbers on your circle. Your **calculations** must be based on the geometry of your circle. So, use your calculator. In other words, you are expected to use trigonometry to construct your Mohr's circle. Do not give a graphical solution that is scaled off! The stress transformation equations can only be used to check your answer.

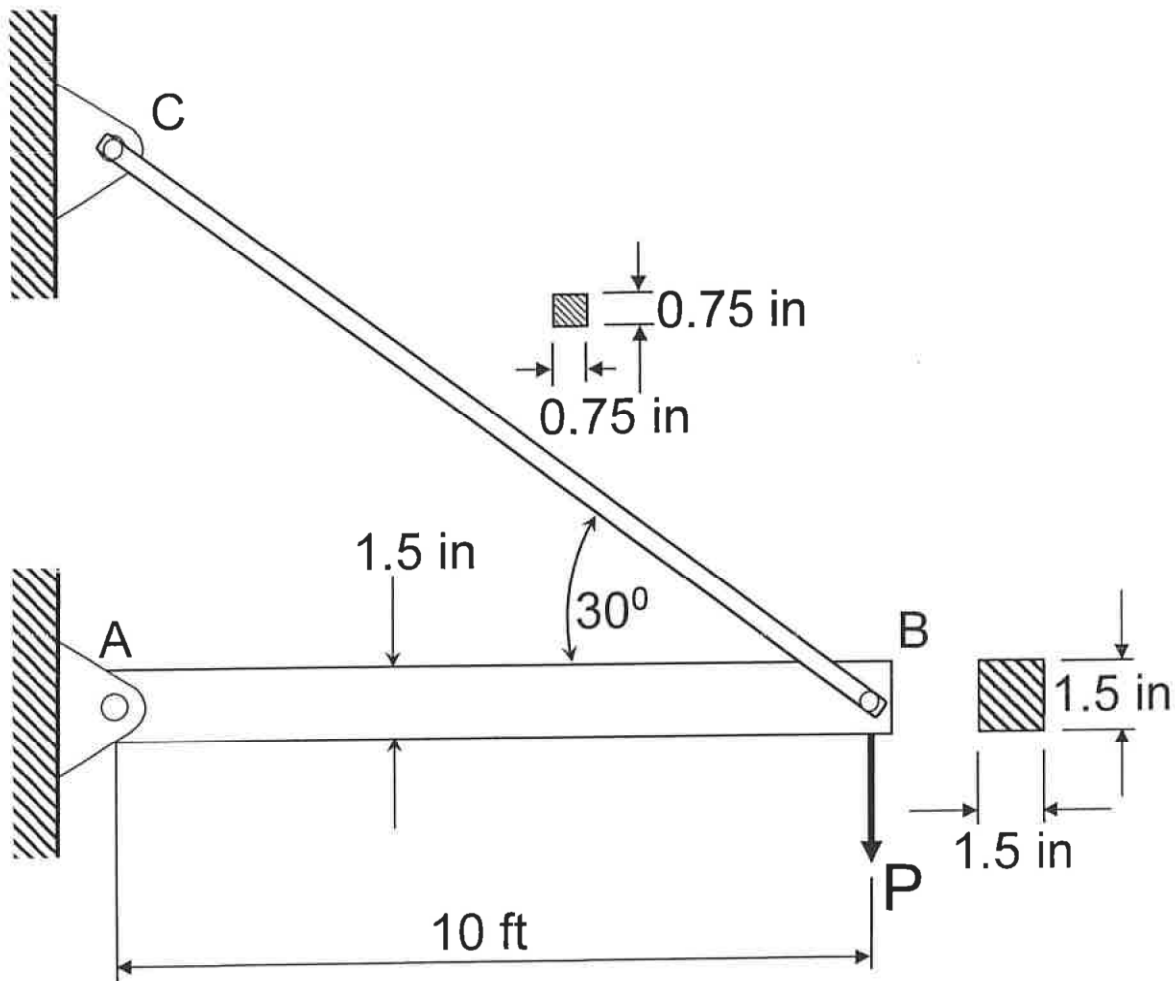
**Question 5:**

The horizontal strut  $AB$  with a cross-sectional area  $1.5 \text{ in.} \times 1.5 \text{ in.}$  is pin-connected at its ends. The member  $BC$  has a cross-sectional area  $0.75 \text{ in.} \times 0.75 \text{ in.}$ , as shown in figure below. Both  $AB$  and  $BC$  are made of steel with elastic modulus  $29 \times 10^3 \text{ ksi}$ , and  $\sigma_Y = 50 \text{ ksi}$ .

Determine the maximum allowable load  $P$  that can be applied to the structure. Use a factor of safety with respect to buckling of 2.

Consider buckling in the plane of the structure only. Do not use a safety factor for yielding of the steel.

20 marks



**Questions 6:**

A circular stepped shaft  $ABCD$  is fixed at  $A$  and subjected to a  $500\text{ N.m}$  torque (counterclockwise) at  $B$ ,  $1,000\text{ N.m}$  torque (clockwise) at  $C$ , and  $200\text{ N.m}$  torque (counterclockwise) at  $D$ .

Part  $AB$  of the shaft is hollow ( $30\text{-mm}$  diameter shaft with a  $20\text{-mm}$  hole drilled in the center), part  $BC$  is a solid shaft with a  $40\text{-mm}$  diameter, and part  $CD$  is a solid shaft but with a smaller  $20\text{-mm}$  diameter. Other shaft dimensions are given in the figure below.

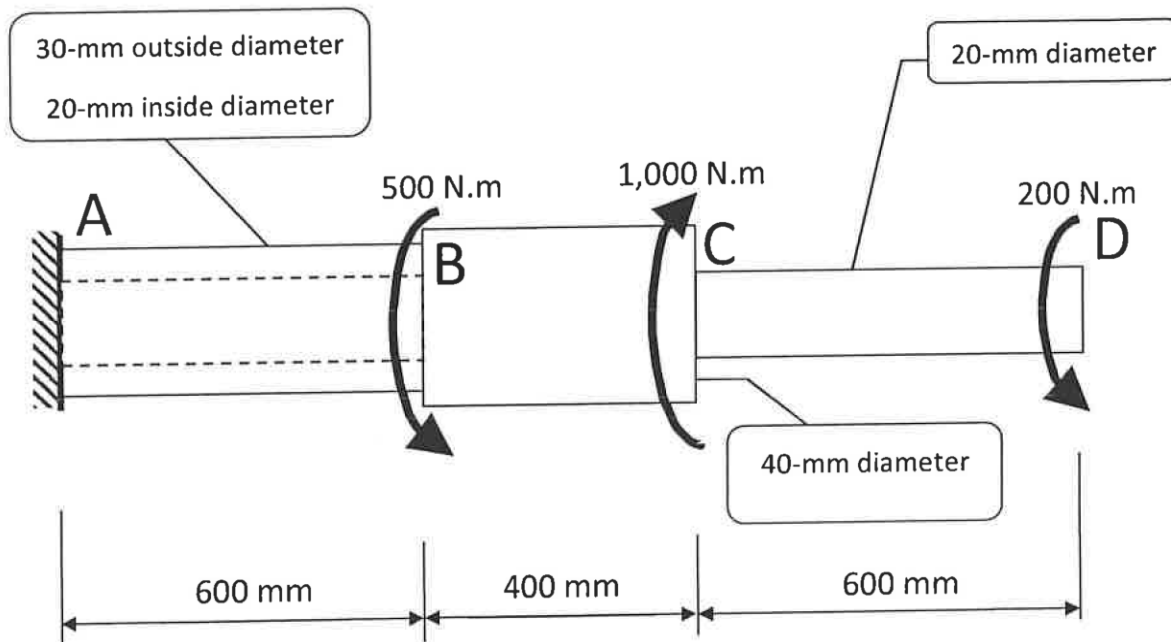
The shaft is made of steel with a shear modulus  $G = 80\text{ GPa}$ , elastic modulus  $E = 200\text{ GPa}$ , and assumed yield stress (in shear) of  $\tau_y = 160\text{ MPa}$ .

12 marks

(a) Determine the maximum shear stress in the stepped shaft and sketch the corresponding variation of shear stress along the shaft radius at this point (where the stress is maximum).

8 marks

(b) Find the angle of twist at the end of the shaft (point  $D$ ) and give your answer in degrees.



**Questions 7:**

The composite member below is made up of a steel section and two wood planks (each with a  $40\text{ mm} \times 280\text{ mm}$  cross-section). The beam supports a maximum positive moment of  $60\text{ kN.m}$  (bending about the horizontal axis). Elastic moduli of the different materials used in the beam are given below.

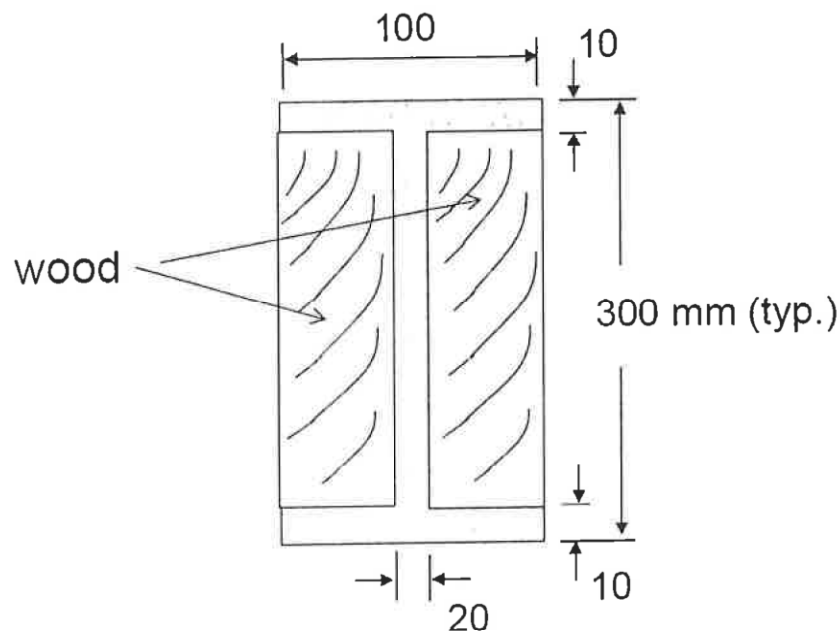
$$E_{\text{wood}} = 7\text{ GPa}$$

$$E_{\text{steel}} = 200\text{ GPa}$$

Determine the following:

- i) Maximum stress in the wood;
- ii) Maximum stress in the steel
- iii) Explain whether the wood planks need to be bonded to the steel section to have composite action.

20 marks



beam cross-section  
(all dimensions in mm)



**Question 8:**

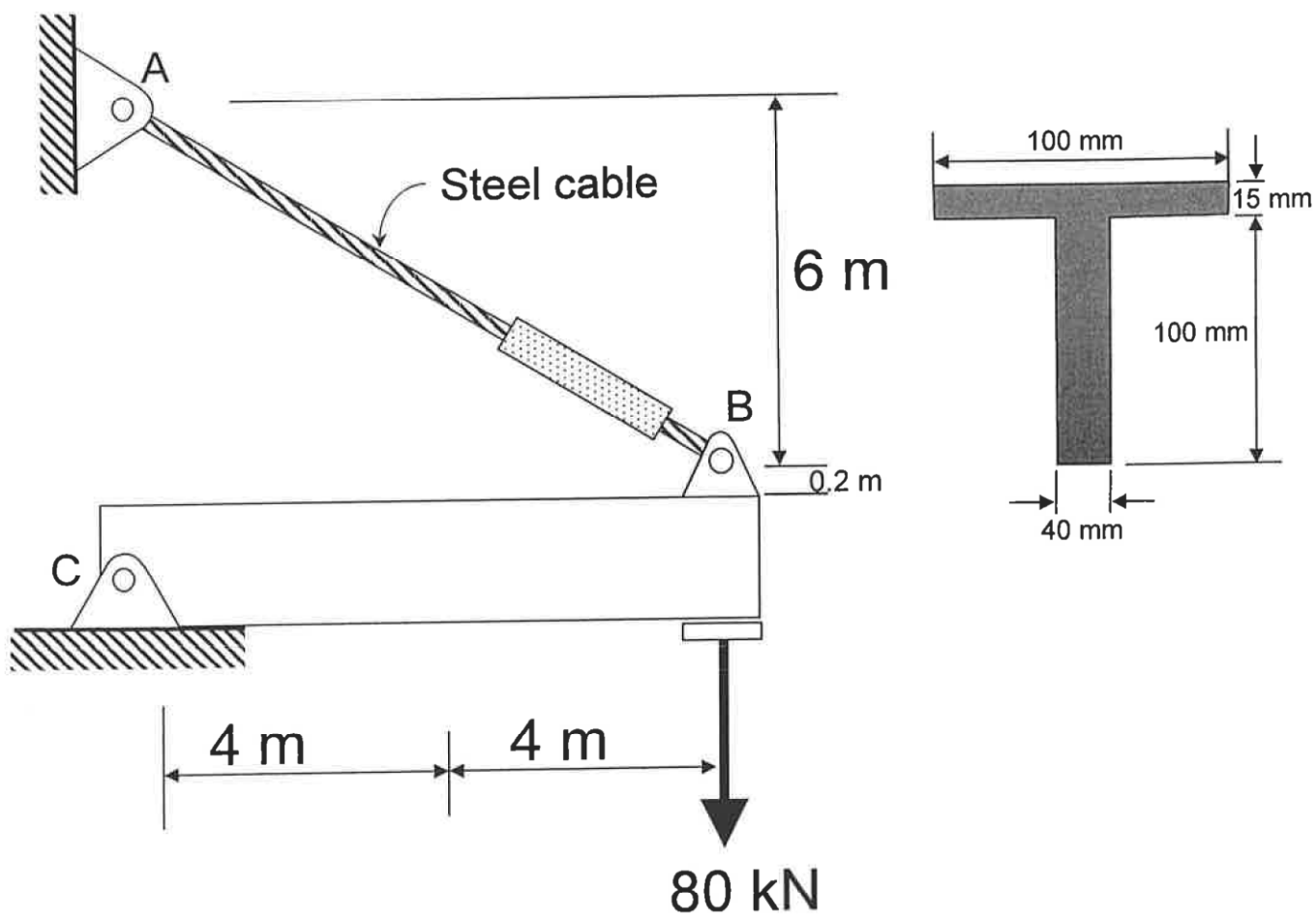
A  $80\text{-kN}$  force is applied at the end of a T-beam ( $CB$ ) supported by a pinned connection at  $C$  and a  $15\text{-mm}$  diameter steel cable ( $AB$ ) at  $B$ . Note that the connection of the cable to the T-beam at point  $B$  is eccentric as shown while the pinned connection at  $C$  is located at the centroid of the member.

16 marks

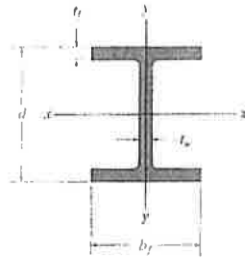
(a) Compute the distribution of normal stress in the T-beam at a section located  $4\text{ m}$  from the support at  $C$  and sketch the normal stress distribution.

4 marks

(b) Compute the maximum shear stress in the T-beam at the same section located  $4\text{ m}$  from the support at  $C$ .

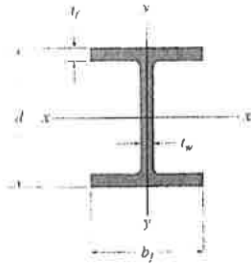


APPENDIX C GEOMETRIC PROPERTIES OF WIDE-FLANGE SECTIONS



Wide-Flange Sections or W Shapes SI Units											
Designation	Area A	Depth d	Web thickness t <sub>w</sub>	Flange		x-x axis			y-y axis		
				width b <sub>f</sub>	thickness t <sub>f</sub>	I	S	r	I	S	r
mm × kg/m	mm <sup>2</sup>	mm	mm	mm	mm	10 <sup>6</sup> mm <sup>4</sup>	10 <sup>3</sup> mm <sup>3</sup>	mm	10 <sup>6</sup> mm <sup>4</sup>	10 <sup>3</sup> mm <sup>3</sup>	mm
W610 × 155	19 800	611	12.70	324.0	19.0	1 290	4 220	255	108	667	73.9
W610 × 140	17 900	617	13.10	230.0	22.2	1 120	3 630	250	45.1	392	50.2
W610 × 125	15 900	612	11.90	229.0	19.6	985	3 220	249	39.3	343	49.7
W610 × 113	14 400	608	11.20	228.0	17.3	875	2 880	247	34.3	301	48.8
W610 × 101	12 900	603	10.50	228.0	14.9	764	2 530	243	29.5	259	47.8
W610 × 92	11 800	603	10.90	179.0	15.0	646	2 140	234	14.4	161	34.9
W610 × 82	10 500	599	10.00	178.0	12.8	560	1 870	231	12.1	136	33.9
W460 × 97	12 300	466	11.40	193.0	19.0	445	1 910	190	22.8	236	43.1
W460 × 89	11 400	463	10.50	192.0	17.7	410	1 770	190	20.9	218	42.8
W460 × 82	10 400	460	9.91	191.0	16.0	370	1 610	189	18.6	195	42.3
W360 × 74	9 460	457	9.02	190.0	14.5	333	1 460	188	16.6	175	41.9
W360 × 68	8 730	459	9.14	154.0	15.4	297	1 290	184	9.41	122	32.8
W460 × 60	7 590	455	8.00	153.0	13.3	255	1 120	183	7.96	104	32.4
W460 × 52	6 640	450	7.62	152.0	10.8	212	942	179	6.34	83.4	30.9
W410 × 85	10 800	417	10.90	181.0	18.2	315	1 510	171	18.0	199	40.8
W410 × 74	9 510	413	9.65	180.0	16.0	275	1 330	170	15.6	173	40.5
W410 × 67	8 560	410	8.76	179.0	14.4	245	1 200	169	13.8	154	40.2
W410 × 53	6 820	403	7.49	177.0	10.9	186	923	165	10.1	114	38.5
W410 × 46	5 890	403	6.99	140.0	11.2	156	774	163	5.14	73.4	29.5
W410 × 39	4 960	399	6.35	140.0	8.8	126	632	159	4.02	57.4	28.5
W360 × 79	10 100	354	9.40	205.0	16.8	227	1 280	150	24.2	236	48.9
W360 × 64	8 150	347	7.75	203.0	13.5	179	1 030	148	18.8	185	48.0
W360 × 57	7 200	358	7.87	172.0	13.1	160	894	149	11.1	129	39.3
W360 × 51	6 450	355	7.24	171.0	11.6	141	794	148	9.68	113	38.7
W360 × 45	5 710	352	6.86	171.0	9.8	121	688	146	8.16	95.4	37.8
W360 × 39	4 960	353	6.48	128.0	10.7	102	578	143	3.75	58.6	27.5
W360 × 33	4 190	349	5.84	127.0	8.5	82.9	475	141	2.91	45.8	26.4

WIDE-FLANGE SECTIONS OR W SHAPES FPS UNITS



Wide-Flange Sections or W Shapes SI Units											
Designation	Area A	Depth d	Web thickness t <sub>w</sub>	Flange		x-x axis			y-y axis		
				width b <sub>f</sub>	thickness t <sub>f</sub>	I	S	r	I	S	r
W310 × 129	16 500	318	13.10	308.0	20.6	308	1940	137	100	649	77.8
W310 × 74	9 480	310	9.40	205.0	16.3	165	1060	132	23.4	228	49.7
W310 × 67	8 530	306	8.51	204.0	14.6	145	948	130	20.7	203	49.3
W310 × 39	4 930	310	5.84	165.0	9.7	84.8	547	131	7.23	87.6	38.3
W310 × 33	4 180	313	6.60	102.0	10.8	65.0	415	125	1.92	37.6	21.4
W310 × 24	3 040	305	5.59	101.0	6.7	42.8	281	119	1.16	23.0	19.5
W310 × 21	2 680	303	5.08	101.0	5.7	37.0	244	117	0.986	19.5	19.2
W250 × 149	19 000	282	17.30	263.0	28.4	259	1840	117	86.2	656	67.4
W250 × 80	10 200	256	9.40	255.0	15.6	126	984	111	43.1	338	65.0
W250 × 67	8 560	257	8.89	204.0	15.7	104	809	110	22.2	218	50.9
W250 × 58	7 400	252	8.00	203.0	13.5	87.3	693	109	18.8	185	50.4
W250 × 45	5 700	266	7.62	148.0	13.0	71.1	535	112	7.03	95	35.1
W250 × 28	3 620	260	6.35	102.0	10.0	39.9	307	105	1.78	34.9	22.2
W250 × 22	2 850	254	5.84	102.0	6.9	28.8	227	101	1.22	23.9	20.7
W250 × 18	2 280	251	4.83	101.0	5.3	22.5	179	99.3	0.919	18.2	20.1
W200 × 100	12 700	229	14.50	210.0	23.7	113	987	94.3	36.6	349	53.7
W200 × 86	11 000	222	13.00	209.0	20.6	94.7	853	92.8	31.4	300	53.4
W200 × 71	9 100	216	10.20	206.0	17.4	76.6	709	91.7	25.4	247	52.8
W200 × 59	7 580	210	9.14	205.0	14.2	61.2	583	89.9	20.4	199	51.9
W200 × 46	5 890	203	7.24	203.0	11.0	45.5	448	87.9	15.3	151	51.0
W200 × 36	4 570	201	6.22	165.0	10.2	34.4	342	86.8	7.64	92.6	40.9
W200 × 22	2 860	206	6.22	102.0	8.0	20.0	194	83.6	1.42	27.8	22.3
W150 × 37	4 730	162	8.13	154.0	11.6	22.2	274	68.5	7.07	91.8	38.7
W150 × 30	3 790	157	6.60	153.0	9.3	17.1	218	67.2	5.54	72.4	38.2
W150 × 22	2 860	152	5.84	152.0	6.6	12.1	159	65.0	3.87	50.9	36.8
W150 × 24	3 060	160	6.60	102.0	10.3	13.4	168	66.2	1.83	35.9	24.5
W150 × 18	2 290	153	5.84	102.0	7.1	9.19	120	63.3	1.26	24.7	23.5
W150 × 14	1 730	150	4.32	100.0	5.5	6.84	91.2	62.9	0.912	18.2	23.0