

National Exams December 2017

98-Ind-B1, Applied Probability & Statistics

3 hours duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
2. This is a closed book exam. Candidates are permitted to use one of the two permitted calculators (Sharp or Casio models).
3. Candidates are permitted to have an aid sheet consisting of one 8.5" x 11.0" sheet of paper. Writing is permitted on both sides of the paper.
4. This exam consists of three sections (A→C). Within each section, candidates will be given a choice of questions to answer. Please read the instructions for each section carefully. A breakdown of questions and marks is as follows:

Section A:	Do 2 of 4 Questions.	Total marks: 30
Section B:	Do 2 of 3 Questions.	Total marks: 30
Section C:	Do 2 of 3 Questions.	Total marks: 40

Exam:	6 Questions.	Total marks: 100
-------	--------------	------------------

4. The value of each question is listed in the exam. Remember to check the instructions for each section. **DO NOT ATTEMPT TO DO ALL QUESTIONS.**
5. Statistical tables are provided.

Section A – Basic Concepts: Complete **two** of the following **four** questions. This section is worth a total of 30 marks. Do not attempt all questions.

1. An oil drilling company drills in various locations, and its success or failure is independent from one location to another. Suppose the probability of a success at any specific location is 0.25. Furthermore, assume that the time required to drill each well is normally distributed with a mean of 72 days and standard deviation of 3 days.
 - a. What is the probability that the company drills at 10 locations and has exactly 1 success?
 - b. The company will go bankrupt if it drills 9 or more times before its first success. What are the firm's prospects for bankruptcy?
 - c. Assume that the time to drill a well is distributed $N(72,3)$ days. What is the probability that a well will require between 75.0 and 80.0 days?
 - d. If a well took 68.3 days to complete, would this be an unusual event and how would you determine whether this was an unusual event or not?
 - e. Assume that the firm's drilling schedule for the next three years calls for 10 wells to be completed. Give a 95% confidence interval for the expected completion date of the drilling program. You may assume that only one well can be drilled at a time, due to equipment capacity constraints. Ignore the potential for going bankrupt during the drilling schedule.

15 Marks

2. Neko Case is an Industrial Engineer analyzing a manufacturing process. Ms. Case has validated her data collection method and is interested in determining the number of data elements she needs to obtain a "good" confidence interval. Neko has conducted a pilot run of 10 samples and obtained the following data, measuring job time in system.

Replication	TIS (Minutes)
1	112.6
2	105.6
3	95.9
4	94.4
5	97.1
6	92.1
7	109.2
8	120.5
9	110.6
10	98.8

- a) If Ms. Case wishes to obtain an estimate of the true process mean that is accurate to within ± 3 minutes, 19 times out of 20.

- b) Ms. Case later completes a total of 57 replications. Assume the sample average from this later sample is 104.0 with a variance of 100. Develop a 95% confidence interval for the second sample mean.
- c) If Ms. Case were to observe, at some time in the future, a simulation result in which the average job time in system was 110 minutes, would this be considered a statistical anomaly?
- d) Is there any evidence that the results from the pilot run are statistically different from those of the later run?

15 Marks

3. The city of Halifax is considering a high-speed ferry for passengers between Bedford, NS and Halifax City. The ferry would have two classes of customers: regular and executive. Executive passengers would have access to priority boarding and nicer accommodations while in transit. Each ferry trip has an operating cost of \$450. Regular passengers will pay \$3, while priority passengers will pay \$8 per trip. Let X denote the number of regular passengers on a trip and Y denote the number of priority passengers. The joint distribution of X and Y is:

y	x		
	100	150	200
0	0.01	0.01	0.03
10	0.03	0.08	0.07
20	0.03	0.06	0.06
30	0.07	0.07	0.13
40	0.12	0.04	0.03
50	0.08	0.06	0.02

- a. Compute the expected profit of a trip.
- b. Find the marginal probability density of X.
- c. Find the marginal probability density of Y.
- d. Assume that priority passengers are required to book 24 hours in advance. If the ferry operator knows that 20 priority passengers have booked for a particular trip, calculate the expected profit for that particular trip.

15 Marks

4. A function $f(x)$ given below:

$$f(x) = \begin{cases} \frac{2(x+2)}{c}, & 1 < x < 2 \\ 0, & \text{elsewhere} \end{cases}$$

- a. What value must c be for $f(x)$ to be a valid probability function? Why?
- b. Find the mean of the function, under the assumption that it is a valid pdf, using the value for c you discovered in part a.

- c. Find the variance of the function using the value for c you discovered in part a.

15 Marks

Section B – Intermediate Concepts: Complete **two** of the following **three** questions. This section is worth a total of 30 marks. Do not attempt all questions.

1. As part of a recent study of patients undergoing a particular medical treatment, researchers recorded the demographics of patients in their study, specifically looking at the relationship between body mass index (BMI) and age. The data is below:

Patient	1	2	3	4	5	6	7	8	9
BMI	19.92	20.59	29.02	20.78	25.97	20.39	23.29	17.27	35.24
Age	45.5	34.6	40.6	32.9	28.2	30.1	52.1	33.3	47.0

- Fit a regression model for age (x) versus BMI (y) of the form $y = ax + b$
- Test for the significance of the regression using $\alpha = 0.05$.
- Determine whether the coefficient for x is significant.
- Give the expected BMI for a 49.0 year-old individual and determine if we would consider a 49.0 year old with a BMI of 32.5 to be a statistical anomaly.

15 Marks

2. The deflection temperature under load for two different types of plastic pipe is being studied. A pilot study involving two random samples of 10 pipes are tested and the temperature of the pipe at the deflection point is measured (in °F):

Type 1: 206, 188, 205, 187, 194, 193, 207, 185, 189, 213

Type 2: 177, 197, 206, 201, 180, 176, 185, 200, 197, 192

- Are the variances of the two samples the same? Using $\alpha = 0.05$ in your calculation.
- Does the data support the supposition that deflection temperature under load for Type 1 exceeds that of Type 2? Use $\alpha = 0.05$ in your calculations. You may assume that deflection temperature is normally distributed and that the variance of deflection temperature is equal in the two types of pipe.
- If the mean deflection temperature for Type 1 exceeds that of type 2 by 2 °F it is important to detect this difference. Can you estimate the sample size needed to be able to detect this difference with a 90% certainty? (You can round your estimates to the nearest 10 observations)

3. In 2008, an engineering professor planted three types of hop plants (*humulus lupulus*), a Willamette plant, a Fuggles plant, and a Centennial plant. Hops are a coniferous plant that produce a flower that is used to bitter beer. Each year, the professor recorded the plant yield (in grams) of the three plants.

W	F	C
118	158	227
118	155	215
135	152	215
126	167	221
130	157	217

- Use Bartlett's test to test the hypothesis at the 0.05 level of significance that the population variances of the three growth records are equal.
- Assuming that growth on a year-by-year basis is normally distributed, determine using ANOVA whether the plant type significantly affects growth. You may assume, for ease of calculations, that SST is 23047.6 and SSE is a number approximately equal to 460 (it is in fact not exactly 460 – please calculate SSE if possible. If you cannot find SSE, use the value 460).
- Using Tukey's test ($\alpha = 0.05$) to determine which, if any, of the plants has a significantly different growth pattern than the other.
- Using single degree of freedom comparison determine if the Centennial plant is different from the Willamette and Fuggle plant as a group.

15 Marks

Section C – Advanced Concepts: Complete **two** of the following **three** questions. This section is worth a total of 40 marks. Do not attempt all questions.

1. After running several marathons, a runner has become convinced that her performance (as measured in time to complete the race) is a distributed function of three elements: x_1 – total number of kilometres she has run in the six weeks prior to the race; x_2 – the longest single run completed during the six weeks prior to the race; and x_3 – the expected temperature on race day. To test this hypothesis, the runner has collected the following dataset:

x_1 Total km (*100)	x_2 Max Dist	x_3 Temp	y Time (Hrs)
4.6	27	11	4.3
4.5	31	10	3.8
4.0	30	25	3.5
4.1	25	22	3.6
4.4	28	14	3.5
4.2	32	17	3.4

The relationship between completion time and (x_1, x_2, x_3) is assumed to be linear in the region of interest.

- a. Build the $(X'X)$ matrix.
- b. A partial table of $(X'X)^{-1}$ is given below, where *, **, and *** remain to be determined:

3186.5127	*	-10.0932	-22.9764
-585.1664	108.4063	1.7075	4.2294
-10.0932	1.7075	0.0562	**
-22.9764	***	0.0685	0.1706

- c. Assume that $(X'X)^{-1}(X'Y)$ is:

-16.29491
4.1060
0.0093
0.1245

Calculate SSE, SSR. You may assume that SST is 0.0548333.

- d. Complete the ANOVA table for this regression model ($\alpha = 0.05$) and determine whether the linear model is a significant predictor of y .
- e. Determine if any of the coefficients in your model is significant at the 95% level.
- f. Based on your analysis, is the model a good predictor of race finishing time?

20 Marks

2. The following data is from an experiment conducted to determine the effect of machine design on photocopier quality. Photocopiers work by essentially gluing ink onto paper using static electricity. Heating and gluing the ink on the paper comprise the final stage of the process. The gluing power during this final process determines the quality of the copy. It is postulated that temperature, surface state of the gluing roller, and hardness of the press roller influence the gluing power of the copier.

Temperature	Surface State	Press (in lb/in ²)					
		20		40		60	
Low	Soft	0.52	0.44	0.54	0.52	0.6	0.55
		0.57	0.53	0.65	0.56	0.78	0.68
	Medium	0.64	0.59	0.79	0.73	0.49	0.48
		0.58	0.64	0.79	0.78	0.74	0.5
	Hard	0.67	0.77	0.58	0.68	0.55	0.65
		0.74	0.65	0.57	0.59	0.57	0.58
Medium	Soft	0.46	0.4	0.31	0.49	0.56	0.42
		0.58	0.37	0.48	0.66	0.49	0.49
	Medium	0.6	0.43	0.66	0.57	0.64	0.54
		0.62	0.61	0.72	0.56	0.74	0.56
	Hard	0.53	0.65	0.53	0.45	0.56	0.66
		0.66	0.56	0.59	0.47	0.71	0.67
High	Soft	0.52	0.44	0.54	0.52	0.65	0.49
		0.57	0.53	0.65	0.56	0.65	0.52
	Medium	0.53	0.65	0.53	0.45	0.49	0.48
		0.66	0.56	0.59	0.47	0.74	0.5
	Hard	0.43	0.43	0.48	0.31	0.55	0.65
		0.47	0.44	0.43	0.27	0.57	0.58

An experiment is run with treatments consisting of a combination of these three factors at three levels and an ANOVA is run. The results, as reported by a particular instance of a commercial statistics package, appear below:

ANOVA: Power versus Temperature, Surface Type, Press

Factor	Type	Levels	Values
Temp	fixed	3	Low, Medium, High
Surface	fixed	3	Soft, Medium, Hard
Press	fixed	3	20, 40, 60

Analysis of Variance for Power

Source	DF	SS	MS	F	P
Temp	X	0.166169	0.083084	SSS.S	0.000
Surface	X	0.078252	0.039126	6.70	0.002
Press	X	0.019469	0.009734	1.67	0.195
Temp*Surface	Y	0.128454	0.032113	5.50	0.001
Temp*Press	Y	0.062804	0.015701	2.69	0.037
Surface*Press	Y	0.126437	0.031609	5.41	0.001
Temp*Surface*Press	Z	0.141224	0.017653	3.02	0.005
Error	81	QQQQQQQQ	R.RRRRRR		
Total	AAA	1.196032			

S = 0.0764348 R-Sq = TT.T% R-Sq(adj) = 47.73%

- Fill in the missing values in the above ANOVA table. These are X, Y, Z, AAA, QQQQQQQQ, R.RRRRRR, SSS.S, and TT.T.
- Are there any significant interactions between factors? If so, what are they?
- Are any of the main factor effects significant? If so which? Are there any for which we are uncertain?
- What do we know about the goodness of fit of this model? Is it a good representation of the underlying data?
- If a larger number is better, what advice can you provide about how to best optimize the design of photocopiers to produce the best possible quality results.

20 Marks

3. The following data is from a 2^2 factorial experiment, replicated three times.

Treatment	Rep 1	Rep 2	Rep 3
(1)	12	19	10
a	15	20	16
b	24	16	17
ab	23	17	27

- Complete the design matrix for this set of experiments
- Compute the contrast and mean effects for the main factors and the interaction.
- Use ANOVA to determine which, if any, of the factors and interactions are significant. For ease of calculations, you may assume SST is 266.

- d. Fit a linear regression model to the data and determine which regression coefficients, if any, are significant.

20 Marks

Areas Under the Normal Curve

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.40	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.30	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.20	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.10	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.00	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.90	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.80	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.70	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.60	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.50	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.40	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.30	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.20	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.10	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.00	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.90	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.80	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.70	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.60	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.50	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.40	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.30	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.20	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.10	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.00	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.90	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.80	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.70	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.60	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.50	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.40	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.30	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.20	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.10	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.00	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Areas Under the Normal Curve

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.00	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.10	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.20	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.30	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.40	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.50	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.60	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.70	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.80	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.90	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.00	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.10	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.20	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.30	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.40	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.50	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.60	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.70	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.80	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.90	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.00	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.10	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.20	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.30	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.40	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.50	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.60	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.70	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.80	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.90	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.00	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.10	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.20	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.30	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.40	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Critical Values of the t-Distribution

v	α						
	0.40	0.30	0.20	0.15	0.10	0.05	0.025
1	0.325	0.727	1.376	1.963	3.078	6.314	12.706
2	0.289	0.617	1.061	1.386	1.886	2.920	4.303
3	0.277	0.584	0.978	1.250	1.638	2.353	3.182
4	0.271	0.569	0.941	1.190	1.533	2.132	2.776
5	0.267	0.559	0.920	1.156	1.476	2.015	2.571
6	0.265	0.553	0.906	1.134	1.440	1.943	2.447
7	0.263	0.549	0.896	1.119	1.415	1.895	2.365
8	0.262	0.546	0.889	1.108	1.397	1.860	2.306
9	0.261	0.543	0.883	1.100	1.383	1.833	2.262
10	0.260	0.542	0.879	1.093	1.372	1.812	2.228
11	0.260	0.540	0.876	1.088	1.363	1.796	2.201
12	0.259	0.539	0.873	1.083	1.356	1.782	2.179
13	0.259	0.538	0.870	1.079	1.350	1.771	2.160
14	0.258	0.537	0.868	1.076	1.345	1.761	2.145
15	0.258	0.536	0.866	1.074	1.341	1.753	2.131
16	0.258	0.535	0.865	1.071	1.337	1.746	2.120
17	0.257	0.534	0.863	1.069	1.333	1.740	2.110
18	0.257	0.534	0.862	1.067	1.330	1.734	2.101
19	0.257	0.533	0.861	1.066	1.328	1.729	2.093
20	0.257	0.533	0.860	1.064	1.325	1.725	2.086
21	0.257	0.532	0.859	1.063	1.323	1.721	2.080
22	0.256	0.532	0.858	1.061	1.321	1.717	2.074
23	0.256	0.532	0.858	1.060	1.319	1.714	2.069
24	0.256	0.531	0.857	1.059	1.318	1.711	2.064
25	0.256	0.531	0.856	1.058	1.316	1.708	2.060
26	0.256	0.531	0.856	1.058	1.315	1.706	2.056
27	0.256	0.531	0.855	1.057	1.314	1.703	2.052
28	0.256	0.530	0.855	1.056	1.313	1.701	2.048
29	0.256	0.530	0.854	1.055	1.311	1.699	2.045
30	0.256	0.530	0.854	1.055	1.310	1.697	2.042
40	0.255	0.529	0.851	1.050	1.303	1.684	2.021
50	0.255	0.528	0.849	1.047	1.299	1.676	2.009
60	0.254	0.527	0.848	1.045	1.296	1.671	2.000
70	0.254	0.527	0.847	1.044	1.294	1.667	1.994
80	0.254	0.526	0.846	1.043	1.292	1.664	1.990
90	0.254	0.526	0.846	1.042	1.291	1.662	1.987
100	0.254	0.526	0.845	1.042	1.290	1.660	1.984
110	0.254	0.526	0.845	1.041	1.289	1.659	1.982
120	0.254	0.526	0.845	1.041	1.289	1.658	1.980
130	0.254	0.526	0.844	1.041	1.288	1.657	1.978
∞	0.253	0.524	0.842	1.036	1.282	1.645	1.960

Critical Values of the t-Distribution

v	α						
	0.02	0.015	0.01	0.0075	0.005	0.0025	0.0005
1	15.895	21.205	31.821	42.433	63.657	127.321	636.619
2	4.849	5.643	6.965	8.073	9.925	14.089	31.599
3	3.482	3.896	4.541	5.047	5.841	7.453	12.924
4	2.999	3.298	3.747	4.088	4.604	5.598	8.610
5	2.757	3.003	3.365	3.634	4.032	4.773	6.869
6	2.612	2.829	3.143	3.372	3.707	4.317	5.959
7	2.517	2.715	2.998	3.203	3.499	4.029	5.408
8	2.449	2.634	2.896	3.085	3.355	3.833	5.041
9	2.398	2.574	2.821	2.998	3.250	3.690	4.781
10	2.359	2.527	2.764	2.932	3.169	3.581	4.587
11	2.328	2.491	2.718	2.879	3.106	3.497	4.437
12	2.303	2.461	2.681	2.836	3.055	3.428	4.318
13	2.282	2.436	2.650	2.801	3.012	3.372	4.221
14	2.264	2.415	2.624	2.771	2.977	3.326	4.140
15	2.249	2.397	2.602	2.746	2.947	3.286	4.073
16	2.235	2.382	2.583	2.724	2.921	3.252	4.015
17	2.224	2.368	2.567	2.706	2.898	3.222	3.965
18	2.214	2.356	2.552	2.689	2.878	3.197	3.922
19	2.205	2.346	2.539	2.674	2.861	3.174	3.883
20	2.197	2.336	2.528	2.661	2.845	3.153	3.850
21	2.189	2.328	2.518	2.649	2.831	3.135	3.819
22	2.183	2.320	2.508	2.639	2.819	3.119	3.792
23	2.177	2.313	2.500	2.629	2.807	3.104	3.768
24	2.172	2.307	2.492	2.620	2.797	3.091	3.745
25	2.167	2.301	2.485	2.612	2.787	3.078	3.725
26	2.162	2.296	2.479	2.605	2.779	3.067	3.707
27	2.158	2.291	2.473	2.598	2.771	3.057	3.690
28	2.154	2.286	2.467	2.592	2.763	3.047	3.674
29	2.150	2.282	2.462	2.586	2.756	3.038	3.659
30	2.147	2.278	2.457	2.581	2.750	3.030	3.646
40	2.123	2.250	2.423	2.542	2.704	2.971	3.551
50	2.109	2.234	2.403	2.519	2.678	2.937	3.496
60	2.099	2.223	2.390	2.504	2.660	2.915	3.460
70	2.093	2.215	2.381	2.494	2.648	2.899	3.435
80	2.088	2.209	2.374	2.486	2.639	2.887	3.416
90	2.084	2.205	2.368	2.480	2.632	2.878	3.402
100	2.081	2.201	2.364	2.475	2.626	2.871	3.390
110	2.078	2.199	2.361	2.471	2.621	2.865	3.381
120	2.076	2.196	2.358	2.468	2.617	2.860	3.373
130	2.075	2.194	2.355	2.465	2.614	2.856	3.367
∞	2.054	2.170	2.327	2.433	2.576	2.808	3.291

Critical Values of the Chi-Squared Distribution

v	α									
	0.995	0.99	0.98	0.975	0.95	0.90	0.80	0.75	0.70	0.50
1	0.000	0.000	0.001	0.001	0.004	0.016	0.064	0.102	0.148	0.455
2	0.010	0.020	0.040	0.051	0.103	0.211	0.446	0.575	0.713	1.386
3	0.072	0.115	0.185	0.216	0.352	0.584	1.005	1.213	1.424	2.366
4	0.207	0.297	0.429	0.484	0.711	1.064	1.649	1.923	2.195	3.357
5	0.412	0.554	0.752	0.831	1.145	1.610	2.343	2.675	3.000	4.351
6	0.676	0.872	1.134	1.237	1.635	2.204	3.070	3.455	3.828	5.348
7	0.989	1.239	1.564	1.690	2.167	2.833	3.822	4.255	4.671	6.346
8	1.344	1.646	2.032	2.180	2.733	3.490	4.594	5.071	5.527	7.344
9	1.735	2.088	2.532	2.700	3.325	4.168	5.380	5.899	6.393	8.343
10	2.156	2.558	3.059	3.247	3.940	4.865	6.179	6.737	7.267	9.342
11	2.603	3.053	3.609	3.816	4.575	5.578	6.989	7.584	8.148	10.341
12	3.074	3.571	4.178	4.404	5.226	6.304	7.807	8.438	9.034	11.340
13	3.565	4.107	4.765	5.009	5.892	7.042	8.634	9.299	9.926	12.340
14	4.075	4.660	5.368	5.629	6.571	7.790	9.467	10.165	10.821	13.339
15	4.601	5.229	5.985	6.262	7.261	8.547	10.307	11.037	11.721	14.339
16	5.142	5.812	6.614	6.908	7.962	9.312	11.152	11.912	12.624	15.338
17	5.697	6.408	7.255	7.564	8.672	10.085	12.002	12.792	13.531	16.338
18	6.265	7.015	7.906	8.231	9.390	10.865	12.857	13.675	14.440	17.338
19	6.844	7.633	8.567	8.907	10.117	11.651	13.716	14.562	15.352	18.338
20	7.434	8.260	9.237	9.591	10.851	12.443	14.578	15.452	16.266	19.337
21	8.034	8.897	9.915	10.283	11.591	13.240	15.445	16.344	17.182	20.337
22	8.643	9.542	10.600	10.982	12.338	14.041	16.314	17.240	18.101	21.337
23	9.260	10.196	11.293	11.689	13.091	14.848	17.187	18.137	19.021	22.337
24	9.886	10.856	11.992	12.401	13.848	15.659	18.062	19.037	19.943	23.337
25	10.520	11.524	12.697	13.120	14.611	16.473	18.940	19.939	20.867	24.337
26	11.160	12.198	13.409	13.844	15.379	17.292	19.820	20.843	21.792	25.336
27	11.808	12.879	14.125	14.573	16.151	18.114	20.703	21.749	22.719	26.336
28	12.461	13.565	14.847	15.308	16.928	18.939	21.588	22.657	23.647	27.336
29	13.121	14.256	15.574	16.047	17.708	19.768	22.475	23.567	24.577	28.336
30	13.787	14.953	16.306	16.791	18.493	20.599	23.364	24.478	25.508	29.336

Critical Values of the Chi-Squared Distribution

v	α									
	0.3	0.25	0.2	0.1	0.05	0.025	0.02	0.01	0.005	0.001
1	1.074	1.323	1.642	2.706	3.841	5.024	5.412	6.635	7.879	10.828
2	2.408	2.773	3.219	4.605	5.991	7.378	7.824	9.210	10.597	13.816
3	3.665	4.108	4.642	6.251	7.815	9.348	9.837	11.345	12.838	16.266
4	4.878	5.385	5.989	7.779	9.488	11.143	11.668	13.277	14.860	18.467
5	6.064	6.626	7.289	9.236	11.070	12.833	13.388	15.086	16.750	20.515
6	7.231	7.841	8.558	10.645	12.592	14.449	15.033	16.812	18.548	22.458
7	8.383	9.037	9.803	12.017	14.067	16.013	16.622	18.475	20.278	24.322
8	9.524	10.219	11.030	13.362	15.507	17.535	18.168	20.090	21.955	26.124
9	10.656	11.389	12.242	14.684	16.919	19.023	19.679	21.666	23.589	27.877
10	11.781	12.549	13.442	15.987	18.307	20.483	21.161	23.209	25.188	29.588
11	12.899	13.701	14.631	17.275	19.675	21.920	22.618	24.725	26.757	31.264
12	14.011	14.845	15.812	18.549	21.026	23.337	24.054	26.217	28.300	32.909
13	15.119	15.984	16.985	19.812	22.362	24.736	25.472	27.688	29.819	34.528
14	16.222	17.117	18.151	21.064	23.685	26.119	26.873	29.141	31.319	36.123
15	17.322	18.245	19.311	22.307	24.996	27.488	28.259	30.578	32.801	37.697
16	18.418	19.369	20.465	23.542	26.296	28.845	29.633	32.000	34.267	39.252
17	19.511	20.489	21.615	24.769	27.587	30.191	30.995	33.409	35.718	40.790
18	20.601	21.605	22.760	25.989	28.869	31.526	32.346	34.805	37.156	42.312
19	21.689	22.718	23.900	27.204	30.144	32.852	33.687	36.191	38.582	43.820
20	22.775	23.828	25.038	28.412	31.410	34.170	35.020	37.566	39.997	45.315
21	23.858	24.935	26.171	29.615	32.671	35.479	36.343	38.932	41.401	46.797
22	24.939	26.039	27.301	30.813	33.924	36.781	37.659	40.289	42.796	48.268
23	26.018	27.141	28.429	32.007	35.172	38.076	38.968	41.638	44.181	49.728
24	27.096	28.241	29.553	33.196	36.415	39.364	40.270	42.980	45.559	51.179
25	28.172	29.339	30.675	34.382	37.652	40.646	41.566	44.314	46.928	52.620
26	29.246	30.435	31.795	35.563	38.885	41.923	42.856	45.642	48.290	54.052
27	30.319	31.528	32.912	36.741	40.113	43.195	44.140	46.963	49.645	55.476
28	31.391	32.620	34.027	37.916	41.337	44.461	45.419	48.278	50.993	56.892
29	32.461	33.711	35.139	39.087	42.557	45.722	46.693	49.588	52.336	58.301
30	33.530	34.800	36.250	40.256	43.773	46.979	47.962	50.892	53.672	59.703

Critical Values of the F Distribution

$f_{0.05(v1,v2)}$

v2	v1									
	1	2	3	4	5	6	7	8	9	10
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54	241.88
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08
50	4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13	2.07	2.03
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83

Critical Values of the F Distribution

$f_{0.05(v_1, v_2)}$

v2	v1									
	10	12	15	20	24	30	40	60	120	∞
1	241.88	243.91	245.95	248.01	249.05	250.10	251.14	252.20	253.25	254.30
2	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50
3	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.37
6	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.41
12	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
26	2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69
27	2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
28	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
29	2.18	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64
30	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
50	2.03	1.95	1.87	1.78	1.74	1.69	1.63	1.58	1.51	1.44
60	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.26
∞	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.01

Critical Values of the F Distribution

$f_{0.01(v1,v2)}$

v2	v1									
	1	2	3	4	5	6	7	8	9	10
1	4052.18	4999.50	5403.35	5624.58	5763.65	5858.99	5928.36	5981.07	6022.47	6055.85
2	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39	99.40
3	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35	27.23
4	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66	14.55
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16	10.05
6	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87
7	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62
8	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81
9	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10
14	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	3.94
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69
17	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68	3.59
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17
25	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22	3.13
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09
27	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15	3.06
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03
29	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09	3.00
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80
50	7.17	5.06	4.20	3.72	3.41	3.19	3.02	2.89	2.78	2.70
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47
∞	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32

Critical Values of the F Distribution

		$f_{0.01}(v_1, v_2)$									
		v1									
v2	10	12	15	20	24	30	40	60	120	∞	
1	6055.85	6106.32	6157.28	6208.73	6234.63	6260.65	6286.78	6313.03	6339.39	6365.55	
2	99.40	99.42	99.43	99.45	99.46	99.47	99.47	99.48	99.49	99.50	
3	27.23	27.05	26.87	26.69	26.60	26.50	26.41	26.32	26.22	26.13	
4	14.55	14.37	14.20	14.02	13.93	13.84	13.75	13.65	13.56	13.46	
5	10.05	9.89	9.72	9.55	9.47	9.38	9.29	9.20	9.11	9.02	
6	7.87	7.72	7.56	7.40	7.31	7.23	7.14	7.06	6.97	6.88	
7	6.62	6.47	6.31	6.16	6.07	5.99	5.91	5.82	5.74	5.65	
8	5.81	5.67	5.52	5.36	5.28	5.20	5.12	5.03	4.95	4.86	
9	5.26	5.11	4.96	4.81	4.73	4.65	4.57	4.48	4.40	4.31	
10	4.85	4.71	4.56	4.41	4.33	4.25	4.17	4.08	4.00	3.91	
11	4.54	4.40	4.25	4.10	4.02	3.94	3.86	3.78	3.69	3.60	
12	4.30	4.16	4.01	3.86	3.78	3.70	3.62	3.54	3.45	3.36	
13	4.10	3.96	3.82	3.66	3.59	3.51	3.43	3.34	3.25	3.17	
14	3.94	3.80	3.66	3.51	3.43	3.35	3.27	3.18	3.09	3.01	
15	3.80	3.67	3.52	3.37	3.29	3.21	3.13	3.05	2.96	2.87	
16	3.69	3.55	3.41	3.26	3.18	3.10	3.02	2.93	2.84	2.75	
17	3.59	3.46	3.31	3.16	3.08	3.00	2.92	2.83	2.75	2.65	
18	3.51	3.37	3.23	3.08	3.00	2.92	2.84	2.75	2.66	2.57	
19	3.43	3.30	3.15	3.00	2.92	2.84	2.76	2.67	2.58	2.49	
20	3.37	3.23	3.09	2.94	2.86	2.78	2.69	2.61	2.52	2.42	
21	3.31	3.17	3.03	2.88	2.80	2.72	2.64	2.55	2.46	2.36	
22	3.26	3.12	2.98	2.83	2.75	2.67	2.58	2.50	2.40	2.31	
23	3.21	3.07	2.93	2.78	2.70	2.62	2.54	2.45	2.35	2.26	
24	3.17	3.03	2.89	2.74	2.66	2.58	2.49	2.40	2.31	2.21	
25	3.13	2.99	2.85	2.70	2.62	2.54	2.45	2.36	2.27	2.17	
26	3.09	2.96	2.81	2.66	2.58	2.50	2.42	2.33	2.23	2.13	
27	3.06	2.93	2.78	2.63	2.55	2.47	2.38	2.29	2.20	2.10	
28	3.03	2.90	2.75	2.60	2.52	2.44	2.35	2.26	2.17	2.07	
29	3.00	2.87	2.73	2.57	2.49	2.41	2.33	2.23	2.14	2.04	
30	2.98	2.84	2.70	2.55	2.47	2.39	2.30	2.21	2.11	2.01	
40	2.80	2.66	2.52	2.37	2.29	2.20	2.11	2.02	1.92	1.81	
50	2.70	2.56	2.42	2.27	2.18	2.10	2.01	1.91	1.80	1.68	
60	2.63	2.50	2.35	2.20	2.12	2.03	1.94	1.84	1.73	1.60	
120	2.47	2.34	2.19	2.03	1.95	1.86	1.76	1.66	1.53	1.38	
∞	2.32	2.18	2.04	1.88	1.79	1.70	1.59	1.47	1.32	1.01	

Critical Values for Bartlett's Test

$b_k(0.01;n)$									
Number of populations, k									
n	2	3	4	5	6	7	8	9	10
3	0.1411	0.1672							
4	0.2843	0.3165	0.3475	0.3729	0.3937	0.4110			
5	0.3984	0.4304	0.4607	0.4850	0.5046	0.5207	0.5343	0.5458	0.5558
6	0.4850	0.5149	0.5430	0.5653	0.5832	0.5975	0.6100	0.6204	0.6293
7	0.5512	0.5787	0.6045	0.6248	0.6410	0.6542	0.6652	0.6744	0.6824
8	0.6031	0.6282	0.6518	0.6704	0.6851	0.6970	0.7069	0.7153	0.7225
9	0.6445	0.6676	0.6892	0.7062	0.7197	0.7305	0.7395	0.7471	0.7536
10	0.6783	0.6996	0.7195	0.7352	0.7475	0.7575	0.7657	0.7726	0.7786
11	0.7063	0.7260	0.7445	0.7590	0.7703	0.7795	0.7871	0.7935	0.7990
12	0.7299	0.7483	0.7654	0.7789	0.7894	0.7980	0.8050	0.8109	0.8160
13	0.7501	0.7672	0.7832	0.7958	0.8056	0.8135	0.8201	0.8256	0.8303
14	0.7674	0.7835	0.7985	0.8103	0.8195	0.8269	0.8330	0.8382	0.8426
15	0.7825	0.7977	0.8118	0.8229	0.8315	0.8385	0.8443	0.8491	0.8532
16	0.7958	0.8101	0.8235	0.8339	0.8421	0.8486	0.8541	0.8586	0.8625
17	0.8076	0.8211	0.8338	0.8436	0.8514	0.8576	0.8627	0.8670	0.8707
18	0.8181	0.8309	0.8429	0.8523	0.8596	0.8655	0.8704	0.8745	0.8780
19	0.8275	0.8397	0.8512	0.8601	0.8670	0.8727	0.8773	0.8811	0.8845
20	0.8360	0.8476	0.8586	0.8671	0.8737	0.8791	0.8835	0.8871	0.8903

Values for Tukey's Test

Upper Percentage Points of the Studentized Range Distribution: Values of $q(0.05, k, \nu)$

Degrees of Freedom ν	Number of Treatments k									
	2	3	4	5	6	7	8	9	10	
1	18.00	27.00	32.80	37.20	40.50	43.10	45.10	47.10	49.10	
2	6.09	5.33	9.80	10.89	11.73	12.43	13.03	13.54	13.99	
3	4.50	5.91	6.83	7.51	8.04	8.47	8.85	9.18	9.46	
4	3.93	5.04	5.76	6.29	6.71	7.06	7.35	7.60	7.83	
5	3.64	4.60	5.22	5.67	6.09	6.80	6.58	6.33	6.03	
6	3.46	4.34	4.90	5.30	6.49	6.32	6.12	5.90	5.63	
7	3.34	4.16	4.68	5.06	6.16	6.00	5.82	5.61	5.36	
8	3.26	4.04	4.53	4.89	5.92	5.77	5.60	5.40	5.17	
9	3.20	3.95	4.41	4.76	5.74	5.59	5.43	5.24	5.02	
10	3.15	3.88	4.33	4.65	5.60	5.46	5.30	5.12	4.91	
11	3.11	3.82	4.26	4.57	5.49	5.35	5.20	5.03	4.82	
12	3.08	3.77	4.20	4.51	5.39	5.27	5.12	4.95	4.75	
13	3.06	3.73	4.15	4.45	5.32	5.19	5.05	4.88	4.69	
14	3.03	3.70	4.11	4.41	5.25	5.13	4.99	4.83	4.64	
15	3.01	3.67	4.08	4.37	5.20	5.08	4.94	4.78	4.59	
16	3.00	3.65	4.05	4.33	5.15	5.03	4.90	4.74	4.56	
17	2.98	3.63	4.02	4.30	5.11	4.99	4.86	4.70	4.52	
18	2.97	3.61	4.00	4.28	5.07	4.96	4.82	4.67	4.49	
19	2.96	3.59	3.98	4.25	5.04	4.92	4.79	4.65	4.47	
20	2.95	3.58	3.96	4.23	5.01	4.90	4.77	4.62	4.45	
24	2.92	3.53	3.90	4.17	4.92	4.81	4.68	4.54	4.37	
30	2.89	3.49	3.85	4.10	4.82	4.72	4.60	4.46	4.30	
40	2.86	3.44	3.79	4.04	4.73	4.63	4.52	4.39	4.23	
60	2.83	3.40	3.74	3.98	4.65	4.55	4.44	4.31	4.16	
120	2.80	3.36	3.68	3.92	4.56	4.47	4.36	4.24	4.10	
∞	2.77	3.31	3.63	3.86	4.47	4.39	4.29	4.17	4.03	

