

**National Exams December 2015**

**04-Chem-A6, Process Dynamics & Control**

3 hours duration

**NOTES:**

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
2. This is an OPEN BOOK EXAM.  
Any non-communicating calculator is permitted.
3. FIVE (5) questions constitute a complete exam paper.  
The first five questions as they appear in the answer book will be marked.
4. Each question is of equal value.
5. Most questions require an answer in essay format. Clarity and organization of the answer are important.

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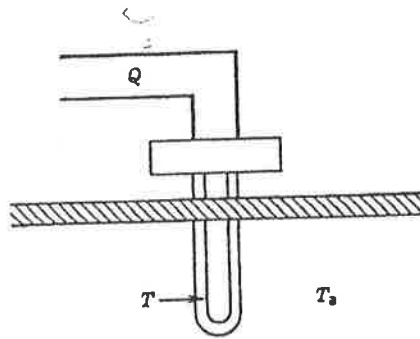
**PROBLEM 1** (20%)

The heating element shown in the drawing transfers heat largely by a radiation mechanism. If the rate of electrical energy input to the heater is  $Q$  and the rod temperature and ambient temperatures are, respectively,  $T$  and  $T_a$ , then an appropriate unsteady-state model for the system is

$$mC \frac{dT}{dt} = Q - k(T^4 - T_a^4)$$

$m$  is the mass of the heater,  $C$  is specific heat and  $k$  is radiation coefficient.

(15%) a) Linearize and then find the transfer functions relating  $\delta T$  to  $\delta Q$  and  $\delta T$  to  $\delta T_a$ . (Be sure they are both in standard form, i.e. show gain and time constant.)



(5%) b) If you were to design a proportional controller to control  $T$  by manipulating  $Q$ , what should be the sign of the controller to guarantee stability? Justify your answer.

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**PROBLEM 2** (20%)

Consider a closed loop system composed of the following elements: a- proportional controller with gain  $K_c$ , b-process with transfer function  $G_p = \frac{1}{(s+1)^3}$  and c-sensor with transfer function  $H$ .

- (5%) a) Find the largest gain  $K_c$  for which the closed loop system is stable for the following two cases: i)  $H=1$  and ii)  $H = e^{-0.7s}$ . Do not use Pade approximation.
- (5%) b) Plot the Bode plots (amplitude ratio normalized and phase) for case ii in item 1 above corresponding to the frequency response of the product  $K_c * G_p * H$ . Indicate clearly asymptotes, corner frequency, value of slopes of asymptotes and extreme values of the phase angle for very small and very large values of frequencies.
- (10%) c) If  $K_c=1$ , calculate the gain and phase margins for case i and ii in item a) above.

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**PROBLEM 3** (20%)

Consider the following system of equations:

$$\begin{aligned}\frac{dx_1}{dt} &= -2.4048x_1 + 7u \\ \frac{dx_2}{dt} &= 0.8333x_1 - 2.2381x_2 - 1.117u \\ y &= x_2\end{aligned}$$

(10%) a) Find the transfer function  $Y(s)/U(s)$

(10%) b) Solve for  $y$  in response to a unit step change in  $u$ .

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**PROBLEM # 4** (20%)

A first order process is given by

$$G_p(s) = \frac{1}{s+5}$$

This process is controlled by a proportional-integral (PI) controller given by:

$$G_c = k_c \left(1 + \frac{1}{s}\right)$$

- (10%) (a) Compute values of  $k_c$  that will result in closed loop stability.
- (10%) (b) Calculate the closed loop response to a unit step change in set point with  $k_c=1$ .

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**Problem #5** (20% total)

A process is described by the following transfer function

$$G_p = \frac{10e^{-5s}}{100s + 1}$$

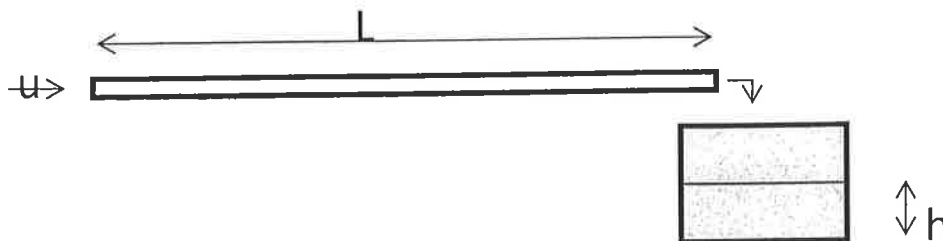
5% (a) Design an IMC (Internal Model Controller) for this process. Show your design with a block diagram. DO NOT USE PADE APPROXIMATION.

10% (b) Compute and plot the closed loop response to a unit step change in the set point using the controller computed in (a). Select the time constant for the IMC filter to be  $\tau_c = 10$ . Assume perfect model (no model error).

5% (c) If the time delay is approximated by using a 1-1 Pade approximation calculate the PID tuning parameters of the feedback controller equivalent to the IMC design.

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**PROBLEM 6** (20%)



A pipe of length  $L=10$  m is feeding liquid into a tank. The speed of the liquid in the pipe is  $u=1$  m/s. The level of liquid in the tank is  $h$ . The cross section area of the tank is  $A=1$  m<sup>2</sup>.

- (5%) a) Calculate the time delay in the pipe between inlet to outlet for the given velocity. Assume that this delay remains constant for the rest of this problem despite changes in flowrate.
- (5%) b) Model the system, i.e. formulate differential equations to calculate  $h(t)$  with respect to inlet velocity  $u$  and find the open loop transfer function between changes in  $h$  to changes  $u$ .
- (10%) c) If the height  $h$  is controlled by manipulating the velocity  $u$  with a proportional controller with gain  $K_c$ , find the closed loop transfer function between the set point changes in  $h$  to changes in  $h$ .

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**PROBLEM 7** (20%)

A process given by:

$$G_p = \frac{20}{s - 3}$$

is to be controlled by a proportional controller with gain  $k_c$ .

- (10%) a) show a qualitative Nyquist plot (show only 2-3 key points along the plot and the general shape of the plot for this problem)  $k_c = 1$ . Is the system stable for this gain?
- (10%) b) Based on the Nyquist criterion, compute a range of  $k_c$  values to obtain closed loop stability.



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**Problem #8** (20% total)

A process given by:

$$G_p = \frac{e^{-0.1s}}{0.5s + 1}$$

is controlled by a proportional controller with gain  $k_c$ .

- (10%)      (a)      Plot qualitatively the Bode Plot for this system (show slope values, corner frequencies and extreme amplitude and phase values).
- (10%)      (b)      Compute the gain  $k_c$  to obtain a gain margin of 1.7.