National Exams December 2014
04-BS-1, Mathematics
3 hours Duration

Notes:

1. If doubt exists as to the interpretation of any question, the candidate is urged to include a clear statement of any assumptions made along with their answer.

2. Any APPROVED CALCULATOR is permitted. This is a CLOSED BOOK exam. However, candidates are permitted to bring ONE AID SHEET written on both sides.

3. Any five questions constitute a complete paper. Only the first five questions as they appear in your answer book will be marked.

4. All questions are of equal value.

Marking Scheme:

1. (a) 10 marks, (b) 10 marks

2. 20 marks

3. (a) 7 marks, (b) 7 marks, (c) 6 marks

4. 20 marks

5. 20 marks

6. 20 marks

7. (a) 6 marks, (b) 6 marks, (c) 8 marks

8. 20 marks

1. For each of the following differential equations, find the general solution, $y(x)$.

    (a) $y'' + 4y = \sec 2x$

    (b) $y'' + y' - 6y = 3x^2 + e^{2x}$

    Note that $'$ denotes differentiation with respect to x.

2. Find the general solution, $y(x)$, of the differential equation

$$2x^2 y'' + xy' - 3y = \frac{4}{x}.$$

    Note that $'$ denotes differentiation with respect to x.

3. Consider the two lines defined as follows:
$$x = 2 - t, \quad y = 3t, \quad\quad z = 1 + t, \text{ (parameter } t\text{)};$$
$$x = 1 + s, \quad y = 3 - 2s, \quad z = 2 + 4s, \text{ (parameter } s\text{)}.$$

    (a) Determine whether or not the two lines intersect, and if so, find the point of intersection.

    (b) Find a third line orthogonal to both lines.

    (c) Is there a plane containing both lines? If so, find an equation for that plane.

4. Evaluate the surface integral $\iint\limits_{S} \mathbf{F} \cdot dS$, where

$$\mathbf{F}(x, y, z) = 4x\mathbf{i} + 2x^2\mathbf{j} - 3\mathbf{k},$$

    S is the surface of the region bounded by the cone $z = 4 - \sqrt{x^2 + y^2}$ and the plane $z = 0$.

5. Let C be the curve formed by the intersection of the cylinder $x^2 + y^2 = 1$ and the plane $z = 1 + y$, and let $\mathbf{v}$ be the vector function $\mathbf{v} = 4z\mathbf{i} - 2x\mathbf{j} + 2x\mathbf{k}$. Evaluate the line integral $\oint_C \mathbf{v} \cdot d\mathbf{r}$. Assume a clockwise orientation for the curve when viewed from above.

6. Find the volume of the solid region outside the cylinder $x^2 + y^2 = 1$, but inside the elipsoid $x^2 + y^2 + 4z^2 = 4$.

7. Consider the matrix
$$A = \begin{pmatrix} 3 & 2 & 0 \\ 0 & 1 & 0 \\ -10 & -4 & -2 \end{pmatrix}$$

    (a) Show that $\begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}$ is an eigenvector of A and find the associated eigenvalue.

    (b) Show that 3 is an eigenvalue of A and find and associated eigenvector.

    (c) Solve the linear system $\mathbf{x}' = \mathbf{A}\mathbf{x}$ for the function $\mathbf{x}(t)$.

8. Let $f(x, y) = 1 + x\ln(xy - 5)$. Find a formula for the plane tangent to the surface $z = f(x, y)$ at the point $(2, 3)$ and use the tangent plane to approximate $f(2.1, 2.95)$.