National Exams May 2017

16-Elec-B1, Digital Signal Processing

3 hours duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.

2. This is a Closed Book exam.
   Candidates may use one of two calculators, the Casio or Sharp approved models. They are also entitled to one aid sheet with tables & formulas written both sides. No textbook excerpts or examples solved.

3. FIVE (5) questions constitute a complete exam.
   Clearly indicate your choice of any five of the six questions given otherwise the first five answers found will be considered your pick.

4. All questions are worth 12 points.
   See below for a detailed breakdown of the marking.

Marking Scheme

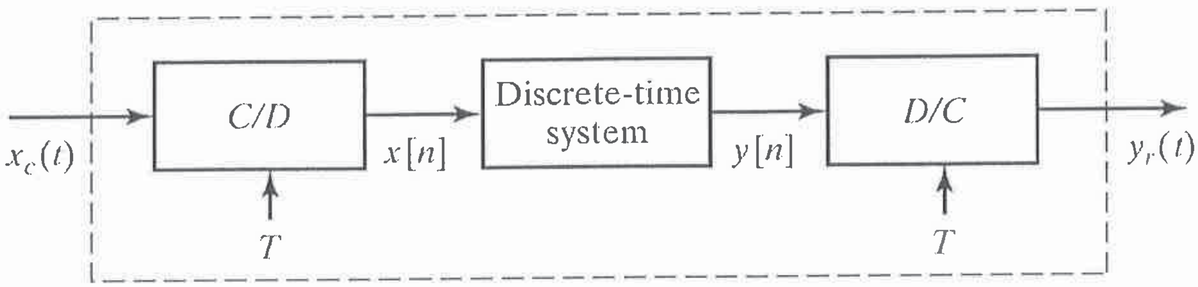
1. (a) 8, (b) 4, total = 12
2. (a) 8, (b) 4, total = 12
3. (a) 6, (b) 2, (c) 2, (d) 2, total = 12
4. (a) 6, (b) 6, total = 12
5. (a) 3, (b) 3, (c) 3, (d) 3, total = 12
6. (a) 6, (b) 6, total = 12

The number beside each part above indicates the points that part is worth

1.- In the system of the figure below assume that the LTI discrete-time system is a differentiator with frequency response

$$H(e^{j\omega}) = j\omega/T, \quad -\pi \le \omega \le \pi,$$

and $T = 1/10\ sec.$



(a) For each of the following inputs $x_c(t)$, find the corresponding output $y_r(t)$ [8 pts]

(i) $x_c(t) = cos(6\pi t)$, (ii) $x_c(t) = cos(14\pi t)$.

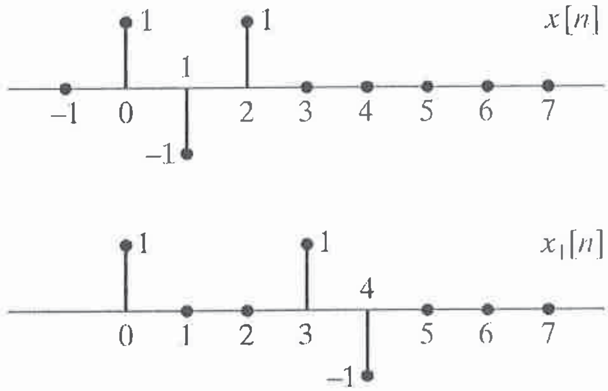
(b) Are the outputs $y_r(t)$ those you would expect from a differentiator? Explain. [4 pts]

Note: Consult tables and formulas provided at the end of this exam as needed

2.- Two finite-length sequences $x[n]$ and $x_1[n]$ are shown in the figure below. The N-point DFTs of these sequences, $X[k]$ and $X_1[k]$, respectively, are related by the equation

$$X_1[k] = X[k]e^{j2\pi k2/N},$$

where N is an unknown constant.



(a) Can you determine a value of N consistent with the figure? [8 pts]

(b) Is your choice of N unique? If so, justify your answer. If not, find another choice of N consistent with the information given. [4 pts]

3.- When the input to an LTI system is

$$x[n] = \left(\frac{1}{3}\right)^n u[n] + 2^n u[-n-1],$$

the corresponding output is

$$y[n] = 5\left(\frac{1}{3}\right)^n u[n] - 5\left(\frac{2}{3}\right)^n u[n].$$

(a) Find the system function $H(z)$. Plot its poles and zeros. Indicate the ROC. [6 pts]

(b) Find the impulse response $h[n]$ of the system. [2 pts]

(c) Write a difference equation that is satisfied by the given input and output. [2 pts]

(d) Is the system stable? Is it causal? Explain. [2 pts]

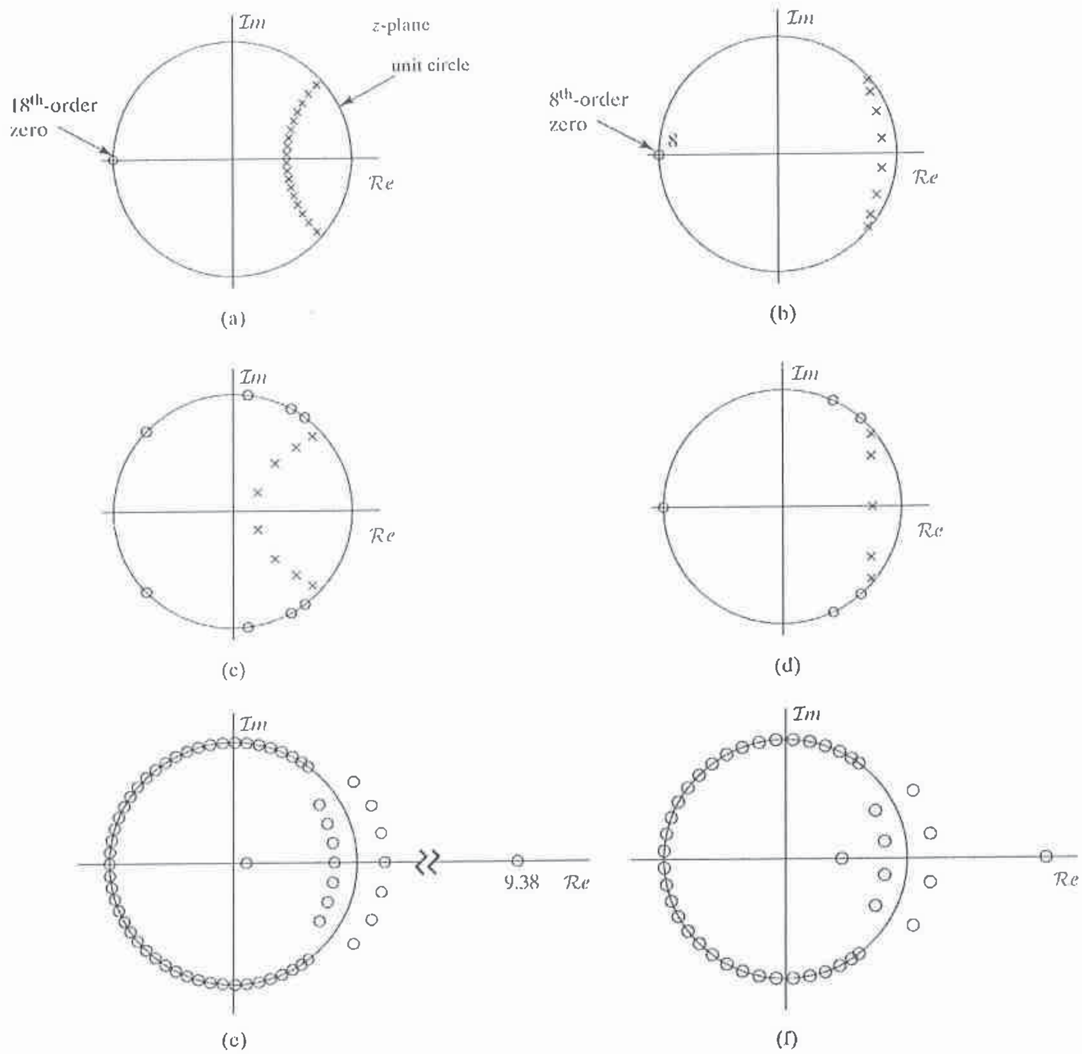
4.- (a) Draw the signal flow graph for the direct form I implementation of the LTI system with system function

$$H(z) = \frac{1 - \frac{1}{2}z^{-2}}{1 - \frac{1}{4}z^{-1} - \frac{1}{8}z^{-2}}.$$

(b) Draw the signal flow graph for the transposed direct form II implementation of the LTI system with system function

$$H(z) = \frac{1 - \frac{7}{6}z^{-1} + \frac{1}{6}z^{-2}}{1 + z^{-1} + \frac{1}{2}z^{-2}}.$$

5.- The pole-zero plots shown below correspond to different filter functions $H(z)$ obtained for the same set of filter specifications.



(a) Which ones are IIR filters? Explain. [3 pts]

(b) Are they low-pass, high-pass, band-pass or band-stop filters? Explain. [3 pts]

(c) Identify Butterworth, Chebyshev I, Chebyshev II and Elliptic filter designs. [3 pts]

(d) Identify which one(s) are linear phase filters. Explain. [3 pts]

6.- The Kaiser window method is used for designing a highpass filter with cutoff frequency $\omega_c = 0.525\pi\ rad/sample$. From the Kaiser formulas seen below values of $\beta = 5.653$ and $M = 96.58 \cong 97$ are found to satisfy the filter specifications except in the neighborhood of π where the error rapidly increases well beyond the specified tolerance.

$$\beta = \begin{cases} 0.1102(A - 8.7), & A > 50, \\ 0.5842(A-21)^{0.4} + 0.07886(A-21), & 21 \le A \le 50, \\ 0.0, & A < 21. \end{cases}$$

$where\ A = -20\log_{10}\delta$, and

$$M = \frac{A-8}{2.285\Delta\omega}$$

$where\ \Delta\omega\ is\ the\ transition\ band\ width\ in\ the\ design\ specifications.$

(a) Provide the filter specifications, including the values of the tolerance δ, the stopband corner frequency ω_s and the passband corner frequency ω_p.

(b) What else is required in order to obtain a final design that satisfies the filter specifications for all values of ω? Explain.

COMPARISON OF COMMONLY USED WINDOWS

Type of Window	Peak Side-Lobe Amplitude (Relative)	Approximate Width of Main Lobe	Peak Approximation Error, $20\log_{10}\delta$ (dB)	Equivalent Kaiser Window, β	Transition Width of Equivalent Kaiser Window
Rectangular	−13	$4\pi/(M+1)$	−21	0	$1.81\pi/M$
Bartlett	−25	$8\pi/M$	−25	1.33	$2.37\pi/M$
Hann	−31	$8\pi/M$	−44	3.86	$5.01\pi/M$
Hamming	−41	$8\pi/M$	−53	4.86	$6.27\pi/M$
Blackman	−57	$12\pi/M$	−74	7.04	$9.19\pi/M$

Additional Information

(Not all of this information is necessarily required today!)

DTFT Synthesis Equation	DTFT Analysis Equation
$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}d\omega$	$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$
Parseval's Theorem	**N-point DFT**
$E = \sum_{n=-\infty}^{\infty}\lvert x[n]\rvert^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi}\lvert X(e^{j\omega})\rvert^2 d\omega$	$X[k] = \sum_{n=0}^{N-1} x[n]\,W_N^{kn}, \quad W_N = e^{-j\frac{2\pi}{N}}$
Z-transform of a sequence x[n]	**Sinusoidal response of LTI systems, real h[n]**
$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$	$y[n] = \lvert H(e^{j\omega_0})\rvert \cos\left(\omega_0 n + \measuredangle H(e^{j\omega_0})\right)$

TABLE 3.2 SOME z-TRANSFORM PROPERTIES

Property Number	Section Reference	Sequence	Transform	ROC
		$x[n]$	$X(z)$	R_x
		$x_1[n]$	$X_1(z)$	R_{x_1}
		$x_2[n]$	$X_2(z)$	R_{x_2}
1	3.4.1	$ax_1[n]+bx_2[n]$	$aX_1(z)+bX_2(z)$	Contains $R_{x_1} \cap R_{x_2}$
2	3.4.2	$x[n-n_0]$	$z^{-n_0}X(z)$	R_x, except for the possible addition or deletion of the origin or ∞
3	3.4.3	$z_0^n x[n]$	$X(z/z_0)$	$\lvert z_0\rvert R_x$
4	3.4.4	$nx[n]$	$-z\frac{dX(z)}{dz}$	R_x
5	3.4.5	$x^*[n]$	$X^*(z^*)$	R_x
6		$\mathcal{Re}\{x[n]\}$	$\frac{1}{2}[X(z)+X^*(z^*)]$	Contains R_x
7		$\mathcal{Im}\{x[n]\}$	$\frac{1}{2j}[X(z)-X^*(z^*)]$	Contains R_x
8	3.4.6	$x^*[-n]$	$X^*(1/z^*)$	$1/R_x$
9	3.4.7	$x_1[n]*x_2[n]$	$X_1(z)X_2(z)$	Contains $R_{x_1} \cap R_{x_2}$

Properties of the Discrete Fourier Transform

	Finite-Length Sequence (Length N)	N-point DFT (Length N)
1.	$x[n]$	$X[k]$
2.	$x_1[n], x_2[n]$	$X_1[k], X_2[k]$
3.	$ax_1[n] + bx_2[n]$	$aX_1[k] + bX_2[k]$
4.	$X[n]$	$Nx[((-k))_N]$
5.	$x[((n-m))_N]$	$W_N^{km}X[k]$
6.	$W_N^{-\ell n}x[n]$	$X[((k-\ell))_N]$
7.	$\sum_{m=0}^{N-1} x_1(m)x_2[((n-m))_N]$	$X_1[k]X_2[k]$
8.	$x_1[n]x_2[n]$	$\frac{1}{N}\sum_{\ell=0}^{N-1} X_1(\ell)X_2[((k-\ell))_N]$
9.	$x^*[n]$	$X^*[((-k))_N]$
10.	$x^*[((-n))_N]$	$X^*[k]$
11.	$\mathcal{R}e\{x[n]\}$	$X_{\text{ep}}[k] = \frac{1}{2}\{X[((k))_N] + X^*[((-k))_N]\}$
12.	$j\mathcal{J}m\{x[n]\}$	$X_{\text{op}}[k] = \frac{1}{2}\{X[((k))_N] - X^*[((-k))_N]\}$
13.	$x_{\text{ep}}[n] = \frac{1}{2}\{x[n] + x^*[((-n))_N]\}$	$\mathcal{R}e\{X[k]\}$
14.	$x_{\text{op}}[n] = \frac{1}{2}\{x[n] - x^*[((-n))_N]\}$	$j\mathcal{J}m\{X[k]\}$
Properties 15–17 apply only when $x[n]$ is real.		
15.	Symmetry properties	$\begin{cases} X[k] = X^*[((-k))_N] \\ \mathcal{R}e\{X[k]\} = \mathcal{R}e\{X[((-k))_N]\} \\ \mathcal{J}m\{X[k]\} = -\mathcal{J}m\{X[((-k))_N]\} \\ \lvert X[k]\rvert = \lvert X[((-k))_N]\rvert \\ \sphericalangle\{X[k]\} = -\sphericalangle\{X[((-k))_N]\} \end{cases}$
16.	$x_{\text{ep}}[n] = \frac{1}{2}\{x[n] + x[((-n))_N]\}$	$\mathcal{R}e\{X[k]\}$
17.	$x_{\text{op}}[n] = \frac{1}{2}\{x[n] - x[((-n))_N]\}$	$j\mathcal{J}m\{X[k]\}$

TABLE 3.1 SOME COMMON z-TRANSFORM PAIRS

Sequence	Transform	ROC
1. $\delta[n]$	1	All z
2. $u[n]$	$\dfrac{1}{1-z^{-1}}$	$\lvert z\rvert > 1$
3. $-u[-n-1]$	$\dfrac{1}{1-z^{-1}}$	$\lvert z\rvert < 1$
4. $\delta[n-m]$	z^{-m}	All z except 0 (if $m>0$) or ∞ (if $m<0$)
5. $a^n u[n]$	$\dfrac{1}{1-az^{-1}}$	$\lvert z\rvert > \lvert a\rvert$
6. $-a^n u[-n-1]$	$\dfrac{1}{1-az^{-1}}$	$\lvert z\rvert < \lvert a\rvert$
7. $na^n u[n]$	$\dfrac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert > \lvert a\rvert$
8. $-na^n u[-n-1]$	$\dfrac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert < \lvert a\rvert$
9. $[\cos\omega_0 n]u[n]$	$\dfrac{1-[\cos\omega_0]z^{-1}}{1-[2\cos\omega_0]z^{-1}+z^{-2}}$	$\lvert z\rvert > 1$
10. $[\sin\omega_0 n]u[n]$	$\dfrac{[\sin\omega_0]z^{-1}}{1-[2\cos\omega_0]z^{-1}+z^{-2}}$	$\lvert z\rvert > 1$
11. $[r^n\cos\omega_0 n]u[n]$	$\dfrac{1-[r\cos\omega_0]z^{-1}}{1-[2r\cos\omega_0]z^{-1}+r^2z^{-2}}$	$\lvert z\rvert > r$
12. $[r^n\sin\omega_0 n]u[n]$	$\dfrac{[r\sin\omega_0]z^{-1}}{1-[2r\cos\omega_0]z^{-1}+r^2z^{-2}}$	$\lvert z\rvert > r$
13. $\begin{cases} a^n, & 0 \le n \le N-1, \\ 0, & \text{otherwise} \end{cases}$	$\dfrac{1-a^N z^{-N}}{1-az^{-1}}$	$\lvert z\rvert > 0$

Initial Value Theorem:

If $x[n]$ is a causal sequence, *i.e.* $x[n]=0,\ \forall n<0$, then

$$x[0] = \lim_{z\to\infty} X(z)$$