National Exams – December 2016

07–Mec-B6 Advanced Fluid Mechanics

3 hours duration

NOTES:

1. If doubt exists as to the interpretation of any question the candidate is urged to submit with the answer paper a clear statement of the assumptions made.
2. Candidates may use any approved Sharp/Casio calculator.
   The exam is OPEN BOOK.
3. Any FIVE (5) out of the 6 questions constitute a complete exam paper for a total of 100 MARKS.
   The first five questions as they appear in the answer book will be marked.
4. Each question is of equal value (20 marks) and question items are marked as indicated.
5. Clarity and organization of the answer are important.

(20) **Question 1**

Air ($\gamma = 1.4$, $R = 287$ J/(kg K)) flows from a very large reservoir through a convergent-divergent nozzle. The air in the reservoir is kept at a constant temperature of T_0 =300 K. The exit area is A_E = 10 cm^2 and the throat area is A_T = 6.45 cm^2. When the back pressure is P_b = 100 kPa (absolute), the pressure distribution as measured along the centre-line shows a discontinuity at the exit as shown schematically in Figure 1. The flow may be assumed adiabatic for all conditions and isentropic in the absence of shocks.

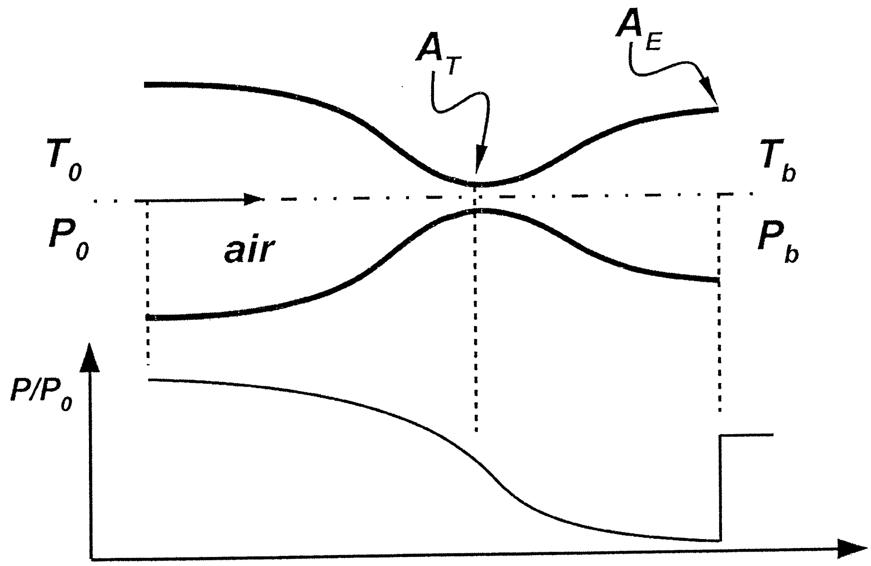

Figure 1: Top: Convergent-divergent nozzle attached to a large air reservoir. Bottom: Pressure distribution along the nozzle centre-line for a back pressure of P_b.

(5) (a) What is the total pressure P_0 in the reservoir.

(5) (b) What is the speed of the flow and the temperature directly downstream of the exit?

(5) (c) What is the mass flow rate?

(5) (d) What is the lowest back pressure for which the flow will be subsonic throughout the channel? What will be the mass flow rate at this back pressure?

(20) **Question 2**

Consider the ideal flow given by the velocity potential function

$$\phi = \frac{-A}{2\pi} \ln r$$

where A is a positive constant.

(5) (a) Determine the stream function ψ.

(5) (b) Sketch the equipotential lines and the stream lines of this flow.

(5) (c) Calculate the radial velocity V_r and identify the flow pattern.

(5) (d) Give the physical meaning of the constant $(2\pi\Gamma)$.

(20) **Question 3**

A large reservoir is connected near its base to a 30 m long (L) annulus (a = 3 cm, b = 5 cm) made of commercial steel (e = 0.046 mm). Consider ρ = 1000 kg/m^3 and $\nu = \mu/\rho = 1.02 \cdot 10^{-6}$ m^2/s for water. Also, assume $D_{eff}/D_h = 0.670$ in the annulus.

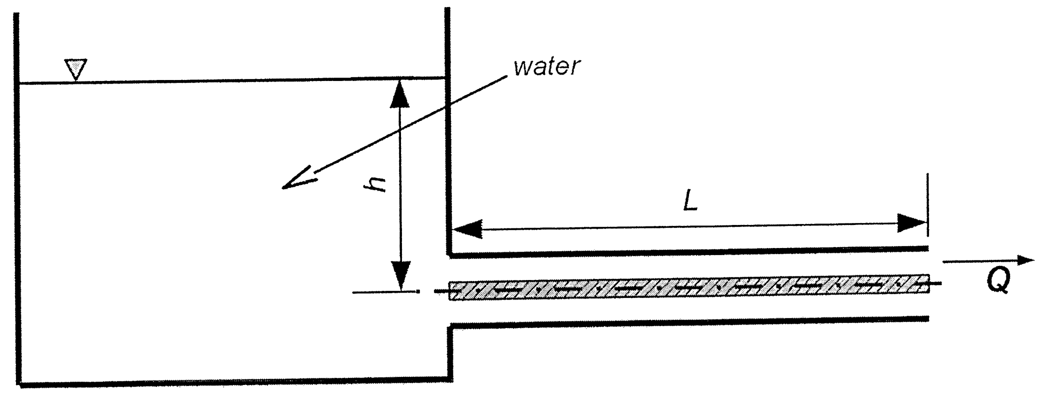

Figure 2: Annular pipe attached to a reservoir.

(15) (a) What should the water level h in the reservoir be to maintain a flow of Q = 0.01 m^3/s? Neglect initially the entrance effects at the annulus.

(5) (b) Estimate the effect in this case of a well-designed entrance, compared with a sharp-edged entrance that can be considered to have a loss coefficient of $K = 0.5$.

(20) **Question 4**

Water rotates as a rigid body about the vertical (z) axis in a spinning cylindrical container. The constant rotation rate is ω rad/s and gravity points in the negative z-direction.

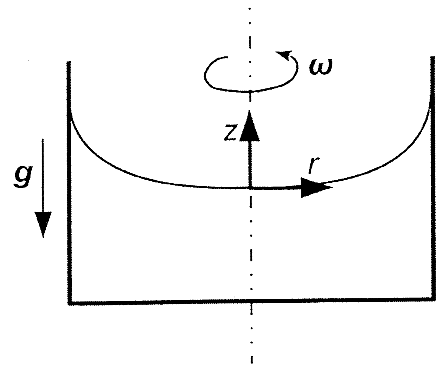

Figure 3: Schematic of spinning cylindrical container.

(6) (a) Simplify the Navier-Stokes equations in r and in z directions. Explain why the viscous stresses can be neglected.

(10) (b) Integrate the simplified inviscid Navier-Stokes equations in r and z directions to derive an expression for the pressure p as a function of r and z everywhere in the fluid.

(4) (c) From your solution to (b), show that the shape of the surface satisfies $z_{\text{surface}} = \omega^2 r^2/2g$, where g is the gravitational acceleration.

(20) **Question 5**

The lift force F on a missile is a function of its length L, velocity V, diameter D, angle of attack α, density ρ, viscosity μ, and speed of sound of the air c.

Find the dimensionless π groups and rewrite the function in terms of known π groups, if there are any.

(20) **Question 6**

A rectangular cross-section suction wind tunnel is designed with a curved upper wall and a straight bottom wall. We are interested in characterising the boundary layer development along the bottom wall. The shape of the upper wall is designed such that the external (irrotational, or main) flow varies along the tunnel according to:

$$U_0 = A\, x^{1/3}$$

where A is a constant.

The flow can be assumed to be two-dimensional and laminar, and the properties of air to be constant (ρ is density, μ is dynamic viscosity, and $\nu = \mu/\rho$ is kinematic viscosity). The boundary layer thickness is represented by δ and it is assumed that at $x = 0$, $\delta = 0$. The velocity distribution in the boundary layer is assumed to be approximated by:

$$\frac{u}{U_0} = 2\frac{y}{\delta} - \left(\frac{y}{\delta}\right)^2 \qquad \text{such that: } \frac{\delta^*}{\delta} = \frac{1}{3} \quad \text{and} \quad \frac{\theta}{\delta} = \frac{2}{15}$$

where δ^* and θ are the momentum and displacement thickness, respectively.

The boundary layer thickness can be assumed to satisfy: $\delta = B\, x^n$.

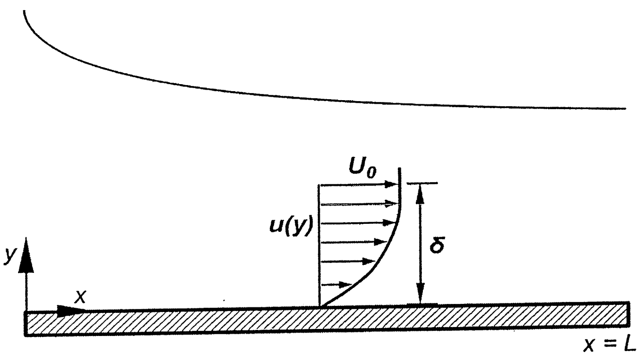

Figure 4: Boundary layer developing over bottom wall of suction wind tunnel.

(5) (a) Find the value of n to satisfy the integral boundary layer equations.

(5) (b) Express δ/x in terms of $\mathrm{Re}_x = \dfrac{U_0\, x}{\nu}$.

(5) (c) What is the local wall shear stress coefficient, $C_{fx} = \dfrac{\tau_w}{\rho U_0^2/2}$, in terms Re_x.

(5) (d) What is the average coefficient $\overline{C_f} = \overline{\tau_w}/(\rho U_0^2/2)$, where $U_0 = A\, L^{1/3}$?