National Exams May 2017

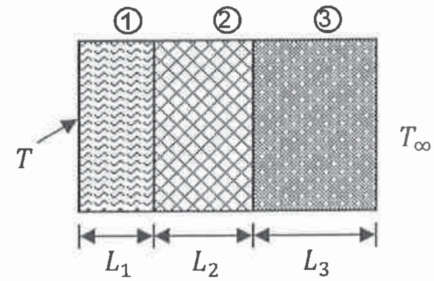
16-Mec-B10, Finite Element Analysis

3 hours duration

NOTES:

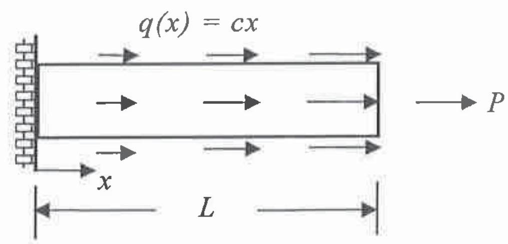
1. If doubt exists as to the interpretation of any question, the candidate is urged to submit, with the answer paper, a clear statement of any assumptions made.
2. This is an OPEN BOOK EXAM.
   Any non-communicating calculator is permitted.
3. FIVE (5) questions constitute a complete exam paper.
   The first five questions as they appear in the answer book will be marked. The questions are to be solved within the context of the finite element method.
4. Each question is of equal value.
5. Some questions require an answer in essay format. Clarity and organization of the answer are important.

Question 1. [20 marks] The composite wall shown in the figure is made of three materials denoted by the numbers 1, 2 and 3. The inside wall temperature is $T = 320\ °\mathrm{C}$, and the outside air temperature is $T_\infty = 65\ °\mathrm{C}$ with a convention coefficient of $h = 20\ \mathrm{W/(m^2\ °C)}$. The thermal conductivities of the materials are $K_1 = 60\ W/(\mathrm{m}\ °\mathrm{C})$, $K_2 = 30\ W/(\mathrm{m}\ °\mathrm{C})$, and $K_3 = 10\ W/(\mathrm{m}\ °\mathrm{C})$. The thickness of each material is $L_1 = 2\ \mathrm{cm}$, $L_2 = 4\ \mathrm{cm}$, and $L_3 = 6\ \mathrm{cm}$, and the cross-sectional area A = 1 cm². Employing only 3 elements, one each across a material



(a) [15 marks] determine the interface temperatures,
(b) [5 marks] determine the heat flux through the third portion.

Question 2. [20 marks] A cantilevered bar is loaded by a constant axial load *P* and a linearly varying distributed load $q(x) = cx$ as shown in the figure - *note that c is a constant*. The cross-sectional area and length of the bar are denoted by A and L, respectively, and it is made of a material with Young's modulus of elasticity E. The system governing equation can be written as



$$EA\frac{d^2u(x)}{dx^2} + cx = 0 \quad 0 < x < L$$

$$\textit{subject to}: \quad u(0) = 0 \quad \textit{and} \quad EA\left.\frac{du(x)}{dx}\right|_{x=L} = P$$

where $u(x)$ represents the displacement in the x-direction. Use the least square method to determine an approximate cubic polynomial solution with evaluation points at $x = \frac{1}{4}L$ and $x = \frac{3}{4}L$.

Question 3. [20 marks] A field variable $f(x, y) = x(x^2 + y)$ is defined over a rectangular domain $\Omega = \{\Re^{2+}: 2 \le x \le 6, 1 \le y \le 7\}$. Given the expression

$$g = \int_1^7 \int_2^6 x\,(x^2 + y)dxdy$$

and assume the following bilinear interpolation shape functions are used to discretize the spatial/geometric variables x and y:

$$N_1 = \frac{1}{4}(1-\xi)(1-\eta),\ N_2 = \frac{1}{4}(1+\xi)(1-\eta),\ N_3 = \frac{1}{4}(1-\xi)(1+\eta),\ N_4 = \frac{1}{4}(1+\xi)(1+\eta)$$

where $-1 \le \xi, \eta \le 1$ for the local coordinates ξ, η.

(a) [15 marks] Use the Gauss quadrature numerical integration method to evaluate *g*. (*The Jacobian must be provided and the evaluation must employ vectors and matrices format.*)

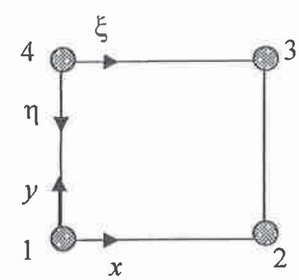
(b) [5 marks] Explain any similarity or difference between your answer and the exact solution g = 2304.

Question 4. [20 marks]

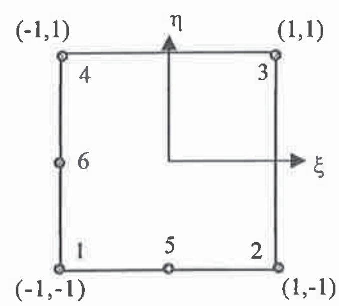
(a) [4 marks] Briefly explain the meaning of geometric isotropy in a sentence or two.
(b) [6 marks] State the **two** properties a polynomial representation of a field variable variation in an element should have to ensure that the element has geometric isotropy?
(c) [10 marks] Consider the square element below for which the field variable u is interpolated in the Cartesian x, y coordinate axes centred at node 1 as

$$u(x,y) = C_1 + C_2 x + C_3 y + C_4 xy$$

Assuming the length of each side of the element is L, use the ξ, η coordinate axes centred at node 4 to show that the element has geometric isotropy.

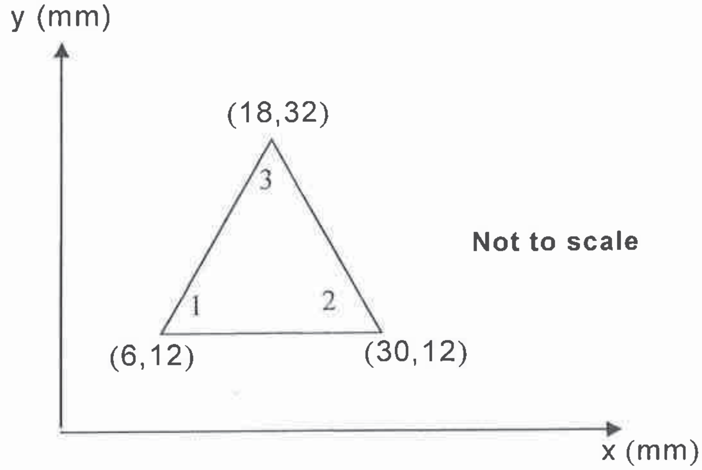


Question 5. [20 marks]



(a) [14 marks] Determine the shape functions (N_i, i = 1 to 6) of the six-node transition element in natural/local coordinates (ξ, η) such that $-1 \leq \xi, \eta \leq 1$.
(b) [4 marks] Evaluate the shape function N_6 at the third and fifth nodes and at the centroid of the element.
(c) [2 marks] Assume the field variables of the problem are displacement components denoted by u and v for the ξ and η directions, respectively. If the nodal displacement components are zero except $v_1 = v_4 = v_6 = -0.025$ mm, determine an expression for the field variables, u and v, in the natural/ local coordinates (ξ, η).

Question 6. [20 marks]



The nodal displacements for the plane strain element shown in the figure above are:

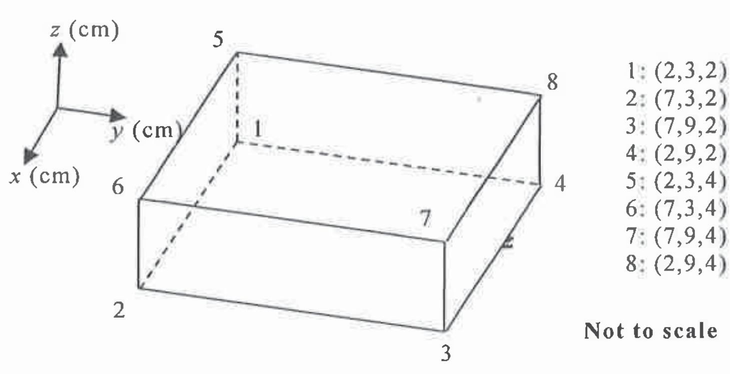
$u_1 = 0.002$ mm and $v_1 = 0.004$ mm; $u_2 = v_2 = 0.0$ mm; $v_3 = 0.006$ mm and $u_3 = 0.0$ mm.

The plate thickness $t = 1$ mm, and it is made of a material with Young's modulus $E = 210$ MPa and Poisson's ratio $\nu = 0.3$.

(a) [15 marks] Determine the element stresses σ_x, σ_y, and τ_{xy}.

(b) [5 marks] Determine the principal stresses σ_1 and σ_2 and principal angle θ_p.

Question 7. [20 marks]



(a) [4 marks] Determine the shape functions (N_i, i = 1 to 8) of an eight-node hexahedron element in natural/local coordinates (ξ, η, ζ) such that $-1 \le \xi, \eta, \zeta \le 1$. The node numbering is identical to that shown in the above representative global element.

(b) [3 marks] Evaluate the shape function N_6 at the second node, the fourth node, and the centroid of the element.

(c) [3 marks] Assume the field variables of the problem are displacement components denoted by $u, v,$ and w in the ξ, η, and ζ directions, respectively. If the nodal displacement components are zero except $v_2 = v_3 = v_6 = v_7 = 0.025$ mm, express the field variables in the natural/ local co-ordinates (ξ, η, ζ).

(d) [7 marks] Determine the Jacobian matrix and evaluate the Jacobian of the above element.

(e) [3 marks] Determine the normal strain $\varepsilon_y = \frac{\partial v}{\partial y}$ and shear strain $\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$ at the centre of the element.