National Exams December 2017

04-BS-1, Mathematics

3 hours Duration

Notes:

1. If doubt exists as to the interpretation of any question, the candidate is urged to include a clear statement of any assumptions made along with their answer.
2. Any APPROVED CALCULATOR is permitted. This is a CLOSED BOOK exam. However, candidates are permitted to bring ONE AID SHEET written on both sides.
3. Any five questions constitute a complete paper. Only the first five questions as they appear in your answer book will be marked.
4. All questions are of equal value.

Marking Scheme:

1. 20 marks
2. 20 marks
3. (a) 8 marks, (b) 12 marks
4. 20 marks
5. 20 marks
6. 20 marks
7. 20 marks
8. 20 marks

1. Solve the initial value problem

$$y'' + 4y = 6\cos(2t), \qquad y(0) = 1,\ y'(0) = 0.$$

Note that $'$ denotes differentiation with respect to t.

2. Find the general solution of the differential equation $x^2y'' - 2xy' + 2y = (1-2x)x^3e^{-2x}$.
Note that $'$ denotes differentiation with respect to x.

3. (a) Find the eigenvalues and the eigenvectors of the matrix

$$\begin{pmatrix} 4 & 2 \\ 3 & -1 \end{pmatrix}$$

(b) Solve the initial value problem

$$\frac{dx}{dt} = 4x + 2y,$$
$$\frac{dy}{dt} = 3x - y,$$

with $x(0) = 0$, $y(0) = 7$.

4. Find the volume of the solid region inside the sphere

$$x^2 + y^2 + z^2 = 4$$

and above the cone

$$\sqrt{3}z = \sqrt{x^2 + y^2}.$$

5. Evaluate the line integral $\oint_C \mathbf{v} \cdot d\mathbf{r}$, where C is the curve formed by the intersection of the cylinder $x^2 + y^2 = 9$ and the plane $x + z = 5$, travelled clockwise as viewed from the positive z-axis, and $\mathbf{v}$ is the vector function $\mathbf{v} = xy\mathbf{i} + 2z\mathbf{j} + 3y\mathbf{k}$.

6. Compute the response of the damped mass-spring system modelled by

$$y'' + 3y' + 2y = r(t), \qquad y(0) = 0, \qquad y'(0) = 0,$$

where r is the square wave

$$r(t) = \begin{cases} 1, & 1 \le t < 2, \\ 0, & \text{otherwise,} \end{cases}$$

and $'$ denotes differentiation with respect to time.

7. Let $f(x, y) = 1 + x\ln(xy - 5)$. Find a formula for the plane tangent to the surface $z = f(x, y)$ at the point $(2, 3)$ and use the tangent plane to approximate $f(2.1, 2.95)$.

8. Let S be the boundary of the region defined by $x^2 + 4y^2 \le 1$, $x \ge 0$, $y \ge 0$ and $0 \le z \le 4$, and let F be the vector function $F(x, y, z) = (y^3, x^3, z^3)$. Evaluate the flux of F across the surface S.