National Exams May 2017

04-BS-6: Mechanics of Materials

3 hours duration

Notes:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper a clear statement of any assumptions made.

2. Candidates may use one of two calculators, the Casio or Sharp approved models.

   This is a Closed Book exam. However candidates are permitted to bring the following into the examination room:

   - ONE aid sheet 8.5" x 11" hand-written on both sides containing notes and formulae. **Example problems and solutions to problems are not allowed!**

3. Any FIVE (5) questions (out of 8 given) constitute a complete paper. Only the first five questions as they appear in your answer book will be marked.

4. All questions are of equal value.

5. Information on geometric properties of wide flange or W shape sections is attached at the end of this exam. There are two pages. Note that this information may not be required.

NOTE: The aid sheet must be handed in with the exam!

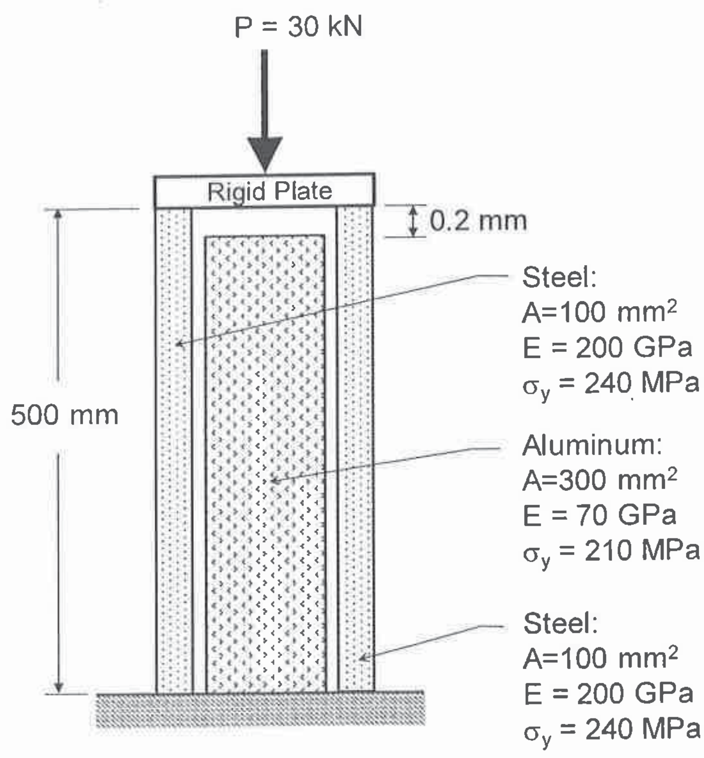
Your exam will not be marked if you do not hand in an aid sheet, unless there is a signed statement by the exam invigilator stating that no aid sheet was used for the exam.

Question 1: **[20 marks]** The assembly below consists of a rigid plate supported by two steel posts and a central aluminum post. Each steel post has an area of 100 mm^2 and the aluminum post has an area of 300 mm^2. The steel has a yield strength of 240 MPa and elastic modulus of 200 GPa, while the aluminum has a yield strength of 210 MPa and elastic modulus of 70 GPa. There is a 0.2 mm gap between the aluminum post and steel posts under no load.

(a) find the forces developed in each post

(b) find the corresponding displacement of the rigid plate

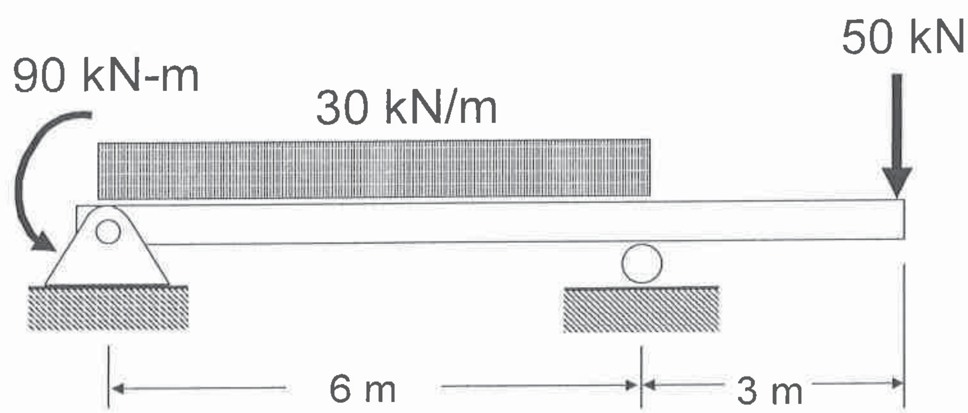
(c) determine the maximum load P that can be supported by the assembly without exceeding the yield strength of either material.



Question 2: A simply supported W 360 x 39 steel beam with an overhang supports a uniformly distributed load acting on the simply supported span, a couple acting at the left support, and a concentrated load at the end of the overhang as shown. The beam is made of steel with a yield strength of 350 MPa and shear stress at yield of 75 MPa. The elastic modulus of the steel is 200 GPa.
[20 marks]

Determine the SHEAR FORCE and BENDING MOMENT along the length of the beam as a function of x. In other words, find V(x) and M(x) for the beam.

Then draw the corresponding shear force and bending moment diagrams for the beam (label all critical points and show your work by indicating exactly how you obtained your answers).

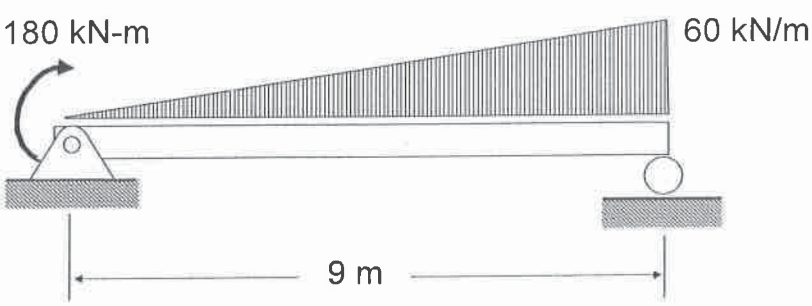


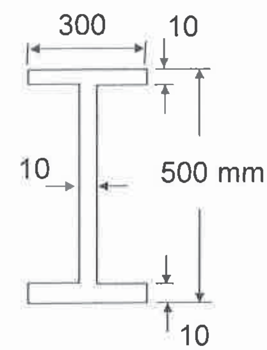
Question 3: A simply supported beam supports a triangularly distributed load and a couple
[20 marks] acting at the left support as shown. The beam has the cross-section given and is made of steel with a yield strength of 350 MPa and shear stress at yield of 75 MPa. The elastic modulus of the steel is 200 GPa.

(a) Determine the maximum deflection using the method of integration.

(b) Determine the slope at the left support using the method of integration.

(c) Sketch the deflected shape of the beam and indicate whether the beam satisfies an allowable deflection limit of L/240 (where L equals the span of the beam).



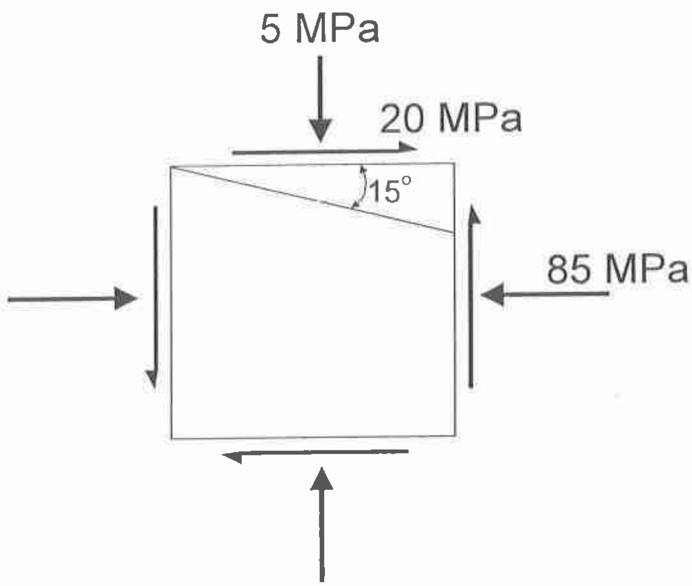


(all dimensions in mm)

Question 4: **[20 marks]** For an element in a state of plane stress subjected to the normal and shear stresses shown below, use the Mohr's circle solution (***not*** the transformation equations) to determine the following:

(a) the stress components acting on the inclined plane (orientated 15° from the horizontal as shown), showing your answer on a properly oriented element.

(b) the maximum in-plane shear stress (and associated normal stresses) and orientation of the corresponding planes. Once again, show your answer on a sketch of a properly oriented element.

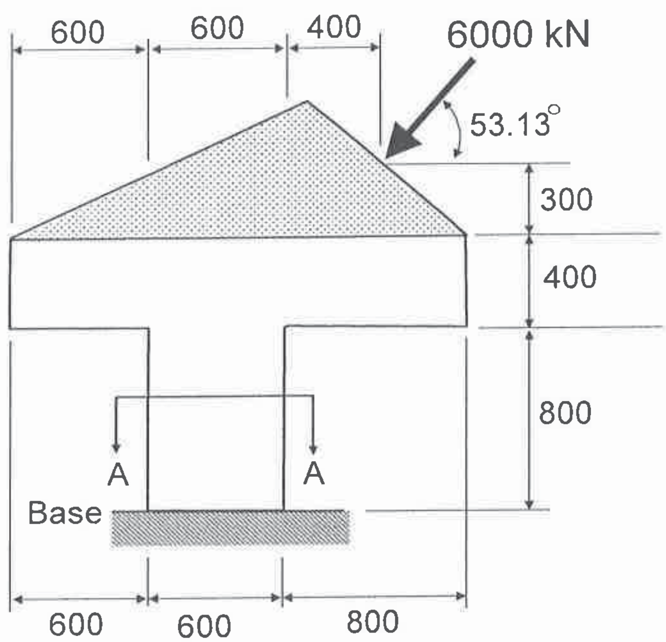


WARNING! Credit will **only** be given for a **solution using Mohr's circle**. Not the stress transformation equations. This means that you need to draw a Mohr's circle based on the stress components given in this problem. Remember to show numbers on your circle. Your calculations must be based on the geometry of your circle. So use your calculator. In other words, you are expected to use trigonometry to construct your Mohr's circle. Do not give a graphical solution that is scaled off!

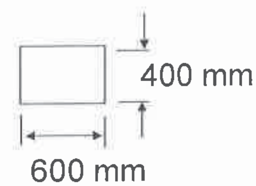
The stress transformation equations can only be used to check your answer.

Question 5: **[20 marks]** The T-shaped element shown below is loaded with an inclined axial load equal to 6000 kN. The element is made of steel with a yield stress of 350 MPa and elastic modulus of 200 GPa. The maximum shear stress allowed equals 60 MPa.

Compute the normal stress distribution and shear stress distribution of the section at the base where the section is fixed. The section has a rectangular cross-section 600 mm x 400 mm at the base. Show your answers on a sketch, and make sure to show maximum and minimum values of stress



(dimensions in mm)



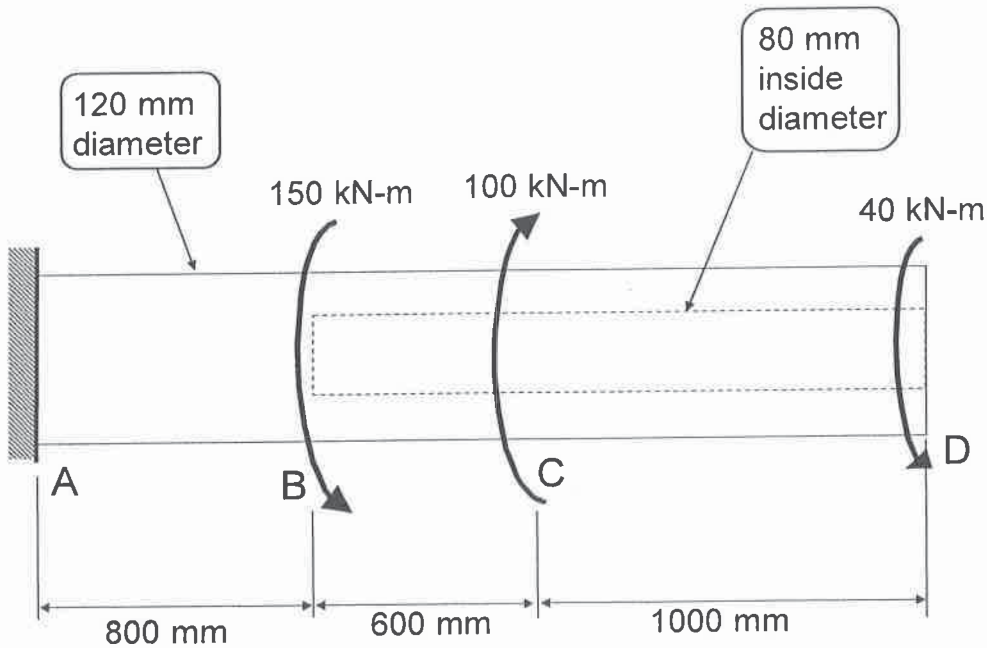
Section A-A

Question 6: A circular steel shaft with G = 80 GPa and τ_y = 280 MPa is subjected to the torques
[20 marks] shown. Part of the shaft (BD) is hollow.

(a) Determine the maximum shear stress in the shaft, and sketch the corresponding variation of shear stress along the shaft radius at this location.

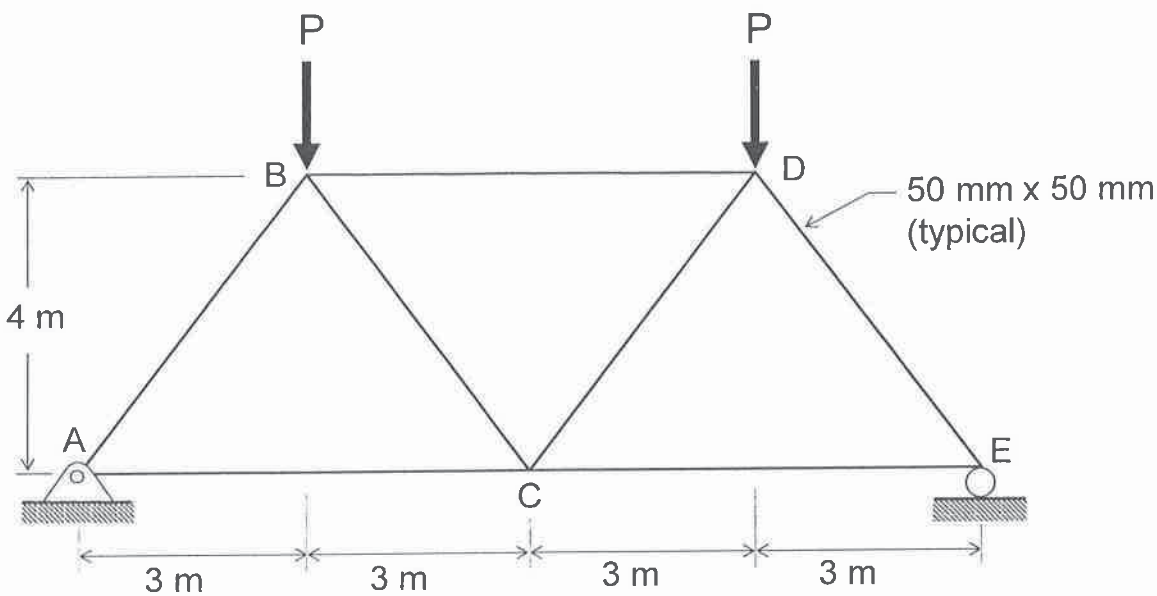
(b) Determine the rotation (in degrees) at the free end of the shaft.

(c) What would happen if the loads on the shaft were doubled?



Question 7 Determine the largest load P that can be applied to the truss structure shown below
[20 marks] given that the members are made of steel with a 50 mm x 50 mm cross section.

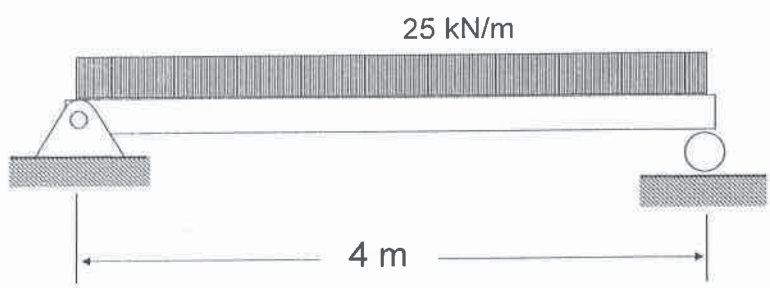
All members are pinned at their ends. Consider in-plane buckling only for the compression members and use a factor of safety of 2 for the Euler buckling load. Do not use a safety factor for yielding of the steel. The steel used in the truss members has an allowable yield strength equal to 240 MPa and elastic modulus of 200 GPa.

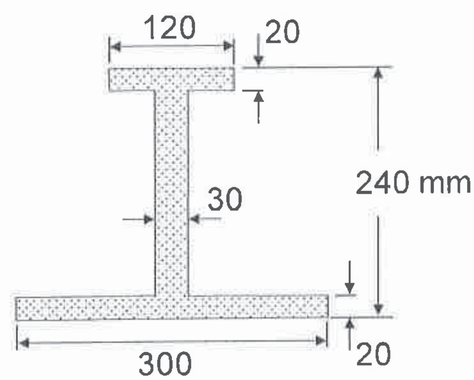


Hint: some of the truss members in this problem may be zero force members

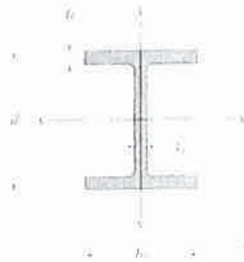
Question 8: A simply supported cast iron beam supports a uniformly distributed load of
[20 marks] 25 kN/m. The beam has a cross-section shape with a wider flange in tension because the cast iron has an allowable tensile strength of 45 MPa that is less than the allowable compressive strength of 100 MPa. The allowable shear stress is 10 MPa and the elastic modulus equals 200 GPa.

Determine whether:
(a) the beam fails in flexure, or
(b) the beam fails in shear, and
(c) how much load can the beam support before it fails.





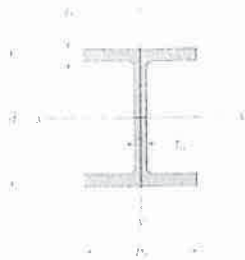
beam cross-section
(all dimensions in mm)



Wide-Flange Sections or W Shapes SI Units

Designation	Area A	Depth d	Web thickness t_w	Flange width b_f	Flange thickness t_f	x-x axis I	x-x axis S	x-x axis r	y-y axis I	y-y axis S	y-y axis r
mm × kg/m	mm^2	mm	mm	mm	mm	10^6 mm^4	10^3 mm^3	mm	10^6 mm^4	10^3 mm^3	mm
W610 × 155	19 800	611	12.70	324.0	19.0	1 290	4 220	255	108	667	73.9
W610 × 140	17 900	617	13.10	230.0	22.2	1 120	3 630	250	45.1	392	50.2
W610 × 125	15 900	612	11.90	229.0	19.6	985	3 220	249	39.3	343	49.7
W610 × 113	14 400	608	11.20	228.0	17.3	875	2 880	247	34.3	301	48.8
W610 × 101	12 900	603	10.50	228.0	14.9	764	2 530	243	29.5	259	47.8
W610 × 92	11 800	603	10.90	179.0	15.0	646	2 140	234	14.4	161	34.9
W610 × 82	10 500	599	10.00	178.0	12.8	560	1 870	231	12.1	136	33.9
W460 × 97	12 300	466	11.40	193.0	19.0	445	1 910	190	22.8	236	43.1
W460 × 89	11 400	463	10.50	192.0	17.7	410	1 770	190	20.9	218	42.8
W460 × 82	10 400	460	9.91	191.0	16.0	370	1 610	189	18.6	195	42.3
W460 × 74	9 460	457	9.02	190.0	14.5	333	1 460	188	16.6	175	41.9
W460 × 68	8 730	459	9.14	154.0	15.4	297	1 290	184	9.41	122	32.8
W460 × 60	7 590	455	8.00	153.0	13.3	255	1 120	183	7.96	104	32.4
W460 × 52	6 640	450	7.62	152.0	10.8	212	942	179	6.34	83.4	30.9
W410 × 85	10 800	417	10.90	181.0	18.2	315	1 510	171	18.0	199	40.8
W410 × 74	9 510	413	9.65	180.0	16.0	275	1 330	170	15.6	173	40.5
W410 × 67	8 560	410	8.76	179.0	14.4	245	1 200	169	13.8	154	40.2
W410 × 53	6 820	403	7.49	177.0	10.9	186	923	165	10.1	114	38.5
W410 × 46	5 890	403	6.99	140.0	11.2	156	774	163	5.14	73.4	29.5
W410 × 39	4 960	399	6.35	140.0	8.8	126	632	159	4.02	57.4	28.5
W360 × 79	10 100	354	9.40	205.0	16.8	227	1 280	150	24.2	236	48.9
W360 × 64	8 150	347	7.75	203.0	13.5	179	1 030	148	18.8	185	48.0
W360 × 57	7 200	358	7.87	172.0	13.1	160	894	149	11.1	129	39.3
W360 × 51	6 450	355	7.24	171.0	11.6	141	794	148	9.68	113	38.7
W360 × 45	5 710	352	6.86	171.0	9.8	121	688	146	8.16	95.4	37.8
W360 × 39	4 960	353	6.48	128.0	10.7	102	578	143	3.75	58.6	27.5
W360 × 33	4 190	349	5.84	127.0	8.5	82.9	475	141	2.91	45.8	26.4

Wide-Flange Sections or W Shapes FPS Units



Wide-Flange Sections or W Shapes SI Units

Designation	Area A	Depth d	Web thickness t_w	Flange width b_f	Flange thickness t_f	x–x axis I	x–x axis S	x–x axis r	y–y axis I	y–y axis S	y–y axis r
mm × kg/m	mm²	mm	mm	mm	mm	10^6 mm⁴	10^3 mm³	mm	10^6 mm⁴	10^3 mm³	mm
W310 × 129	16 500	318	13.10	308.0	20.6	308	1940	137	100	649	77.8
W310 × 74	9 480	310	9.40	205.0	16.3	165	1060	132	23.4	228	49.7
W310 × 67	8 530	306	8.51	204.0	14.6	145	948	130	20.7	203	49.3
W310 × 39	4 930	310	5.84	165.0	9.7	84.8	547	131	7.23	87.6	38.3
W310 × 33	4 180	313	6.60	102.0	10.8	65.0	415	125	1.92	37.6	21.4
W310 × 24	3 040	305	5.59	101.0	6.7	42.8	281	119	1.16	23.0	19.5
W310 × 21	2 680	303	5.08	101.0	5.7	37.0	244	117	0.986	19.5	19.2
W250 × 149	19 000	282	17.30	263.0	28.4	259	1840	117	86.2	656	67.4
W250 × 80	10 200	256	9.40	255.0	15.6	126	984	111	43.1	338	65.0
W250 × 67	8 560	257	8.89	204.0	15.7	104	809	110	22.2	218	50.9
W250 × 58	7 400	252	8.00	203.0	13.5	87.3	693	109	18.8	185	50.4
W250 × 45	5 700	266	7.62	148.0	13.0	71.1	535	112	7.03	95	35.1
W250 × 28	3 620	260	6.35	102.0	10.0	39.9	307	105	1.78	34.9	22.2
W250 × 22	2 850	254	5.84	102.0	6.9	28.8	227	101	1.22	23.9	20.7
W250 × 18	2 280	251	4.83	101.0	5.3	22.5	179	99.3	0.919	18.2	20.1
W200 × 100	12 700	229	14.50	210.0	23.7	113	987	94.3	36.6	349	53.7
W200 × 86	11 000	222	13.00	209.0	20.6	94.7	853	92.8	31.4	300	53.4
W200 × 71	9 100	216	10.20	206.0	17.4	76.6	709	91.7	25.4	247	52.8
W200 × 59	7 580	210	9.14	205.0	14.2	61.2	583	89.9	20.4	199	51.9
W200 × 46	5 890	203	7.24	203.0	11.0	45.5	448	87.9	15.3	151	51.0
W200 × 36	4 570	201	6.22	165.0	10.2	34.4	342	86.8	7.64	92.6	40.9
W200 × 22	2 860	206	6.22	102.0	8.0	20.0	194	83.6	1.42	27.8	22.3
W150 × 37	4 730	162	8.13	154.0	11.6	22.2	274	68.5	7.07	91.8	38.7
W150 × 30	3 790	157	6.60	153.0	9.3	17.1	218	67.2	5.54	72.4	38.2
W150 × 22	2 860	152	5.84	152.0	6.6	12.1	159	65.0	3.87	50.9	36.8
W150 × 24	3 060	160	6.60	102.0	10.3	13.4	168	66.2	1.83	35.9	24.5
W150 × 18	2 290	153	5.84	102.0	7.1	9.19	120	63.3	1.26	24.7	23.5
W150 × 14	1 730	150	4.32	100.0	5.5	6.84	91.2	62.9	0.912	18.2	23.0