National Examinations – May 2019

16-Civ-A6 - Highway Design, Construction and Maintenance

<u>3 Hour Duration</u>

NOTES

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper a clear statement of any assumptions made.

2. Any data, not given but required, can be assumed.

3. This is a **"CLOSED BOOK"** examination. Candidates may use one of two calculators, the Casio or Sharp approved models.

4. The candidates are required to complete 4 out of the 5 questions (Questions 1 though 5). Only the first four solutions will be marked. Please note that for the last question, the candidate will need to write their answers in the table provided in the last page of the exam. It is then important that all the students return the exam copy together with the answers booklet.

5. All questions are of equal value.

6. For non-numerical questions, clarity and organization of the answer are important.

Marking Scheme

1. 20 marks
2. (a) 5 marks; (b) 10 marks; (c) 5 marks
3. (a) 10 marks; (b) 10 marks
4. (a) 8 marks; (b) 8 marks; (c) 4 marks
5. (a to e) 3 marks each; (f) 5 marks

Question 1:

A Plain Jointed Concrete Pavement is to be designed using the AASHTO 93 Pavement Design Method to withstand a traffic of 10,000,000 ESALs. The slabs are connected longitudinally and transversally using dowel and tie bars and asphalt shoulders were used. The geotechnical investigation conducted before the construction indicated that the subgrade is composed of clay with high plasticity (Class CH according to USCS) and that the bedrock layer was found at five feet from the surface of the subgrade.

The following information and design assumptions are available:

- Concrete slab: $E_c = 3.8\times10^6$, $S'_c = 650$ psi
- Unbound Granular Subbase: $D_{SB} = 12$ in., $E_{SB} = 30{,}000$ psi.
- Resilient Modulus of the Subgrade: $M_R = 4{,}000$ psi.
- Initial Serviceability index value: $p_i = 4.5$
- Terminal Serviceability index value: $p_t = 2.5$
- Reliability and Standard Deviation: $R = 90\%$ and $S_0 = 0.3$

The quality of the drainage is good and the pavement is exposed to moisture level approaching saturation for 15% of the time.

Determine the thickness of the concrete slab to withstand the projected traffic. Assume reasonably any missing information and justify your assumptions.

Question 2:

A 4 km curved section of an urban expressway in an urban area is characterized by a radius of 750 m (to the centreline of the road). According to the MTO study on provincial highway volumes in 2018, the estimated AADT of this section is 52,200 vehicles/day. The highway is a four-lane divided highway with a posted speed of 100 km/h and a lane width of 3.75m. This section is mainly in a cut area with a slope of 3:1 in the clear zone.

a) Would you consider that this curve is safe (The radius is sufficient)?
b) What should be the recommended clear zone distance for this section of the highway both in the inner and outer side of the curve?
c) What length of the spiral curve between the tangent sections and the circular curve would you recommend? (For the calculation of the spiral length, consider a relative slope of 0.004 at 120 km/h)

Question 3:

The first year AADT on a six-lane freeway located in an urban area is expected to be 25,000 veh/day. The growth rate for all the categories of vehicles is expected to be 2% per annum throughout the life of the pavement. The design life of the pavement is 20 years and the projected vehicle mix during the first life of operation is:

- Passenger cars = 50%
- Single-unit trucks, two-axle, four-tire = 28%

- Single-unit trucks, two-axle, six-tire = 18%
- Single-unit trucks, six-axle or more = 4%

a) Determine the design ESAL for the service life of the pavement.
b) Design the flexible pavement to carry this design ESAL based on the following information:

- The effective resilient modulus of the subgrade Mr = 10,000 psi.
- The subbase layer is an untreated silty sand with a resilient modulus (E_{SB}) of 18,000 psi.
- The base layer is an untreated granular material with a resilient modulus (E_B) of 30,000 psi.
- The elastic modulus of the asphalt concrete at 20ºC (E_{AC}) is 450,000 psi.
- The pavement structure will be exposed to moisture levels approaching saturation 15% of the time and it would take about 1 day to drain the water.
- The use of modern pavers allows the achieving of high level of serviceability values after the construction (p_0 = 4.5) and the lowest acceptable value of the serviceability index is 2.5.
- The overall standard deviation is 0.5 and the required level of reliability is 90%

Assume reasonably any missing information and justify your assumptions.

Question 4:

A vehicle was traveling on a curved section on a rural highway that has a speed limit of 90km/h. The driver spotted a stalled vehicle on the same section and quickly acted to stop the vehicle. Unfortunately, the driver was unable to stop the vehicle safely and struck the stalled vehicle. An after-accident investigation revealed that this section of road has a vertical curve that connects a 3% uphill grade to the -2% grade, and has a length of 350 m. The vehicle has an eye height of 1.38 m and the stalled vehicle has a height of 1.10 m. A series of test runs showed that the coefficient of friction is 0.32 @ 80 km/h and 0.29 @ 100 km/h. The driver later claimed that his speed was very close to the posted speed but that there was not enough sight distance available at this vertical curve section.

a) Support or refute the claim of the driver and justify your answer.
b) Determine the available (safe) stopping sight distance based on the geometry of the curve and the conditions of the collision.
c) What could be the other potential factors that had contributed to the collision and support one of these potential factors by proper calculation?

Question 5:

A Superpave mix design was conducted for a pavement construction project on a six-lane urban highway with a traffic ESAL of 8,000,000.

The mix design was conducted for three blends. Each blend is composed of five different materials (three coarse materials and two fine materials). The properties of the aggregates and the composition of each of the three blends are given in the Table 1.

The selected asphalt cement is a **PG 58-28** non-modified and its bulk specific gravity is **1.03**

Table 1: Aggregates properties and composition of blends

Material	G_{sb}	G_{sa}	Blend #1	Blend #2	Blend #3
#1 Stone	2.701	2.784	26.00%	31.00%	10.00%
1/2" Chip	2.688	2.772	14.00%	24.00%	16.00%
3/8" Chip	2.723	2.795	21.00%	13.00%	31.00%
Manf. Sand	2.695	2.745	19.00%	18.00%	30.00%
Screen sand	2.677	2.73	20.00%	14.00%	13.00%

After conducting the mix design for the three blends, it was found that the optimum binder content for blends **#1**, **#2** and **#3** are respectively **4.6%**, **4.8%** and **5.4%**. Two asphalt specimens of each blend were then prepared at the optimal binder content and compacted to $\mathbf{N_{max}}$ with the gyratory compactor. The data obtained from the compaction for the six specimens (two at each binder content) and the measurements for the bulk specific gravity values are presented in Table 2.

Table 2: Data of compaction and measurements for specific gravity of specimens

SUPERPAVE GYRATORY COMPACTOR DATA							Gmb MEASUREMENTS	
			grams	mm	mm	mm	grams	grams
Specimens		% AC	Dry mass	Height @ N_{ini}	Height @ N_{des}	Height @ N_{max}	SSD mass	Mass in Water
Blend #1	A	4.6	4902.7	125.8	115.5	110.8	4729.8	2809.0
	B		4897.3	125.3	115.2	110.9	4718.5	2799.1
Blend #2	A	4.8	4765.0	123.1	110.1	109.1	4726.8	2804.1
	B		4761.0	123.5	110.8	109.4	4722.1	2795.1
Blend #3	A	5.4	4825.0	122.7	112.6	110.4	4722.0	2775.1
	B		4821.0	122.9	112.9	110.8	4739.8	2782.8

a) Determine the combined bulk specific gravity (G_{sb}) and combined apparent specific gravity of the aggregates (G_{sa}) for each blend.

b) Calculate estimated values for the effective specific gravity of aggregates (G_{se}) and the percentage of absorbed asphalt (P_{ba}) for each blend.

c) Calculate the values of estimated bulk specific gravity for each specimen (G_{mb}) at N_{ini}, N_{des} and N_{max} then correct these values based on the values based on the measured G_{mb} at N_{max}.

d) Calculate the value of the G_{mm} of each specimen at the binder content for each blend.

e) Determine the %G_{mm} at N_{ini}, N_{des} and N_{max} for each specimen and find the average value for each blend.

f) Calculate the other volumetrics for each blend and determine the selected mix design between the three blends.

Use to following table to show your calculation

SUPERPAVE GYRATORY COMPACTOR DATA							Gmb MEASUREMENTS	
			grams	mm	mm	mm	grams	grams
Specimens		% AC	Dry mass	Height @ N_{ini}	Height @ N_{des}	Height @ N_{max}	SSD mass	Mass in Water
Blend #1	A	4.6	4902.7	125.8	115.5	110.8	4729.8	2809.0
	B		4897.3	125.3	115.2	110.9	4718.5	2799.1
Blend #2	A	4.8	4765.0	123.1	110.1	109.1	4726.8	2804.1
	B		4761.0	123.5	110.8	109.4	4722.1	2795.1
Blend #3	A	5.4	4825.0	122.7	112.6	110.4	4722.0	2775.1
	B		4821.0	122.9	112.9	110.8	4739.8	2782.8

G_{mb} measured @	Correction factor	G_{mb} @ corrected				%Voids @			% G_{mm} @				
N_{max}		N_{ini}	N_{des}	N_{max}	VMA	N_{des}	VFA	G_{mm}	N_{ini}	N_{des}	N_{max}	P_{be}	DP

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APPENDIX

AASTO Flexible Pavement Design

TABLE IV-5 DISTRIBUTION OF TRUCK FACTORS (TF) FOR DIFFERENT CLASSES OF HIGHWAYS AND VEHICLES—UNITED STATES*

Vehicle Type	Truck Factors: Rural Systems						Truck Factors: Urban Systems					
	INTERSTATE	OTHER PRINCIPAL	MINOR ARTERIAL	COLLECTORS MAJOR	COLLECTORS MINOR	RANGE	INTERSTATE	OTHER FREEWAYS	OTHER PRINCIPAL	MINOR ARTERIAL	COLLECTORS	RANGE
Single-unit trucks												
2-axle, 4-tire	0.003	0.003	0.003	0.017	0.003	0.003-0.017***	0.002	0.015	0.002	0.006	...	0.006-0.015***
2-axle, 6-tire	0.21	0.25	0.28	0.41	0.19	0.19-0.41	0.17	0.13	0.24	0.23	0.13	0.13-0.24
3-axle or more	0.61	0.86	1.06	1.26	0.45	0.45-1.26	0.61	0.74	1.02	0.76	0.72	0.61-1.02
All single-units	0.06	0.08	0.08	0.12	0.03	0.03-0.12	0.05	0.06	0.09	0.04	0.16	0.04-0.16***
Tractor semi-trailers												
4-axle or less	0.62	0.92	0.62	0.37	0.91	0.37-0.91	0.98	0.48	0.71	0.46	0.40	0.40-0.98
5-axle**	1.09	1.25	1.05	1.67	1.11	1.05-1.67	1.07	1.17	0.97	0.77	0.63	0.63-1.17
6-axle or more**	1.23	1.54	1.04	2.21	1.35	1.04-2.21	1.05	1.19	0.90	0.64	...	0.64-1.19
All multiple units	1.04	1.21	0.97	1.52	1.08	0.97-1.52	1.05	0.96	0.91	0.67	0.53	0.53-1.05
All trucks	0.52	0.38	0.21	0.30	0.12	0.12-0.52	0.39	0.23	0.21	0.07	0.24	0.07-0.39

**Compiled from data supplied by the Highway Statistics Division, U.S. Federal Highway Administration.*

***Including full-trailer combinations in some states.*

****See Article 4.06 for values to be used when the number of heavy trucks is low.*

$$ESAL = AADT \times HVP \times DF \times TF \times TDY$$

$$Cumulative\ ESALs = Initial\ year\ ESAL_{(Design\ Lane)} \times \frac{[(1+g)^t - 1]}{g}$$

$$\Delta PSI = p_o - p_t \qquad u_f = 1.18 \times 10^8 M_R^{-2.32}$$

$$SN = a_1 D_1 + a_2 D_2 m_2 + a_3 D_3 m_3 + \cdots + a_i D_i m_i$$

$$\log_{10}(W_{18}) = Z_R \times S_o + 9.36 \times \log_{10}(SN+1) - 0.20 + \frac{\log_{10}\left(\frac{\Delta PSI}{4.2-1.5}\right)}{0.40 + \frac{1094}{(SN+1)^{5.19}}} + 2.32 \times \log_{10}(M_R) - 8.07$$

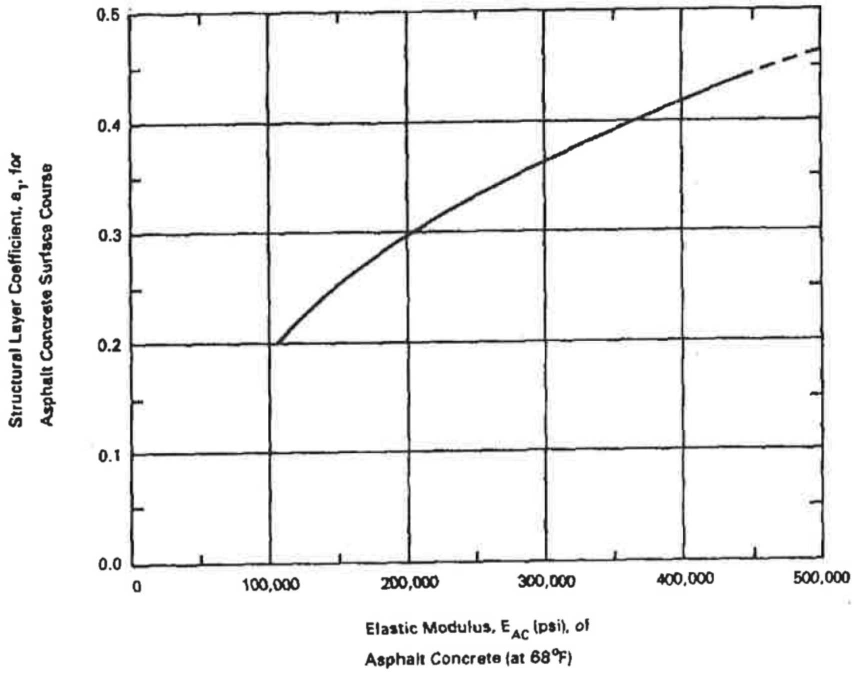
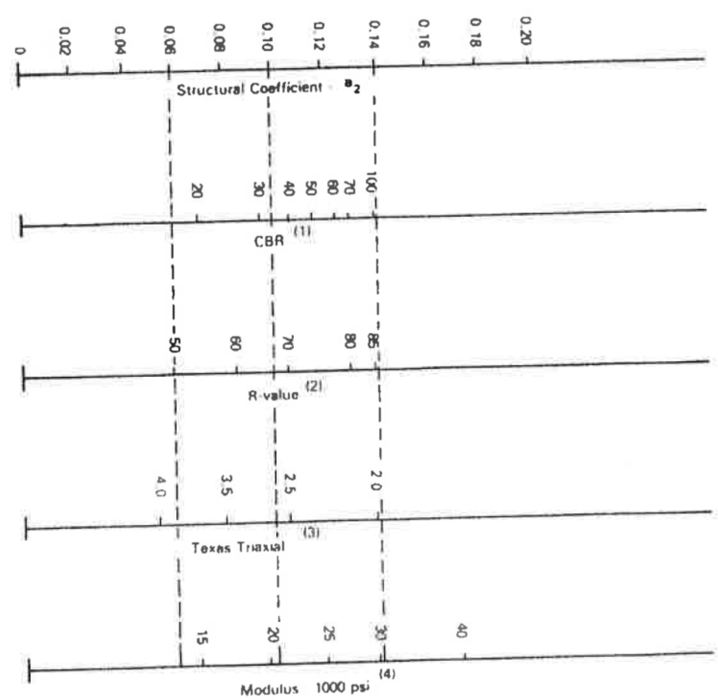
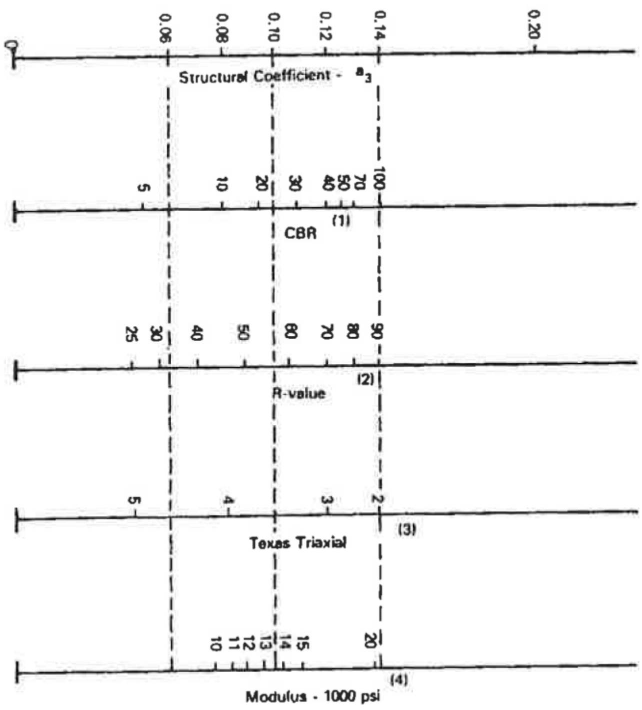


Figure 2.5. Chart for Estimating Structural Layer Coefficient of Dense-Graded Asphalt Concrete Based on the Elastic (Resilient) Modulus (3)



(1) Scale derived by averaging correlations obtained from Illinois.
(2) Scale derived by averaging correlations obtained from California, New Mexico and Wyoming.
(3) Scale derived by averaging correlations obtained from Texas.
(4) Scale derived on NCHRP project (3).

Figure 2.6. Variation in Granular Base Layer Coefficient (a_2) with Various Base Strength Parameters (3)



(1) Scale derived from correlations from Illinois.
(2) Scale derived from correlations obtained from The Asphalt Institute, California, New Mexico and Wyoming.
(3) Scale derived from correlations obtained from Texas.
(4) Scale derived on NCHRP project (3).

Figure 2.7. Variation in Granular Subbase Layer Coefficient (a_3) with Various Subbase Strength Parameters (3)

	Minimum Thickness (inches)	
Traffic, ESAL's	**Asphalt Concrete**	**Aggregate Base**
Less than 50,000	1.0 (or surface treatment)	4
50,001–150,000	2.0	4
150,001–500,000	2.5	4
500,001–2,000,000	3.0	6
2,000,001–7,000,000	3.5	6
Greater than 7,000,000	4.0	6

Table 20.16 Suggested Levels of Reliability for Various Functional Classifications

	Recommended Level of Reliability	
Functional Classification	*Urban*	*Rural*
Interstate and other freeways	85–99.9	80–99.9
Other principal arterials	80–99	75–95
Collectors	80–95	75–95
Local	50–80	50–80

Table 2.4. Recommended m_i Values for Modifying Structural Layer Coefficients of Untreated Base and Subbase Materials in Flexible Pavements

	Percent of Time Pavement Structure is Exposed to Moisture Levels Approaching Saturation			
Quality of Drainage	**Less Than 1%**	**1–5%**	**5–25%**	**Greater Than 25%**
Excellent	1.40–1.35	1.35–1.30	1.30–1.20	1.20
Good	1.35–1.25	1.25–1.15	1.15–1.00	1.00
Fair	1.25–1.15	1.15–1.05	1.00–0.80	0.80
Poor	1.15–1.05	1.05–0.80	0.80–0.60	0.60
Very poor	1.05–0.95	0.95–0.75	0.75–0.40	0.40

NOMOGRAPH SOLVES:

$$\log_{10} W_{18} = Z_R{}^*S_o + 9.36{}^*\log_{10}(SN+1) - 0.20 + \frac{\log_{10}\left[\dfrac{\Delta PSI}{4.2 - 1.5}\right]}{0.40 + \dfrac{1094}{(SN+1)^{5.19}}} + 2.32{}^*\log_{10}M_R - 8.07$$

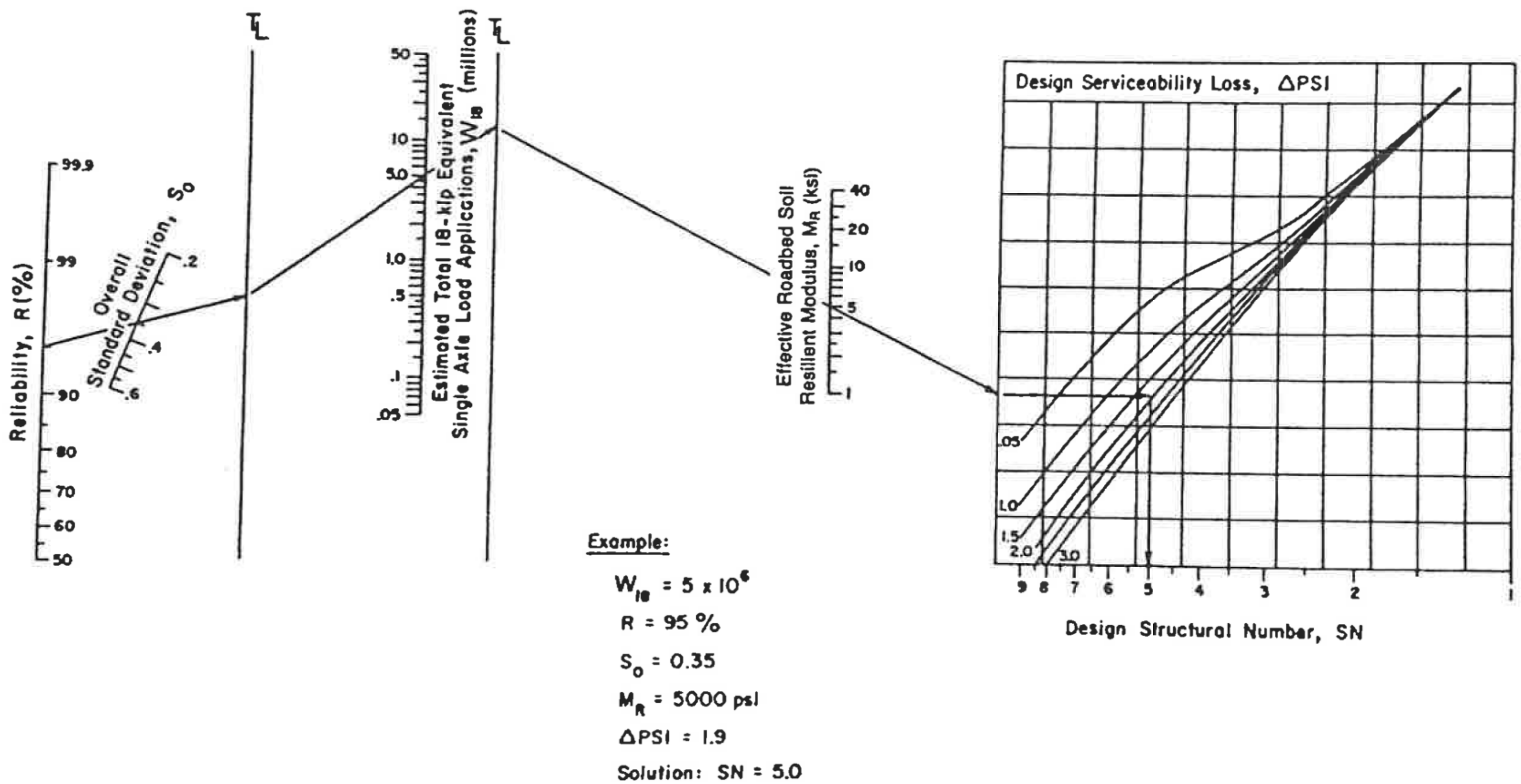


Figure 3.1. Design Chart for Flexible Pavements Based on Using Mean Values for Each Input

AASTO Rigid Pavement Design

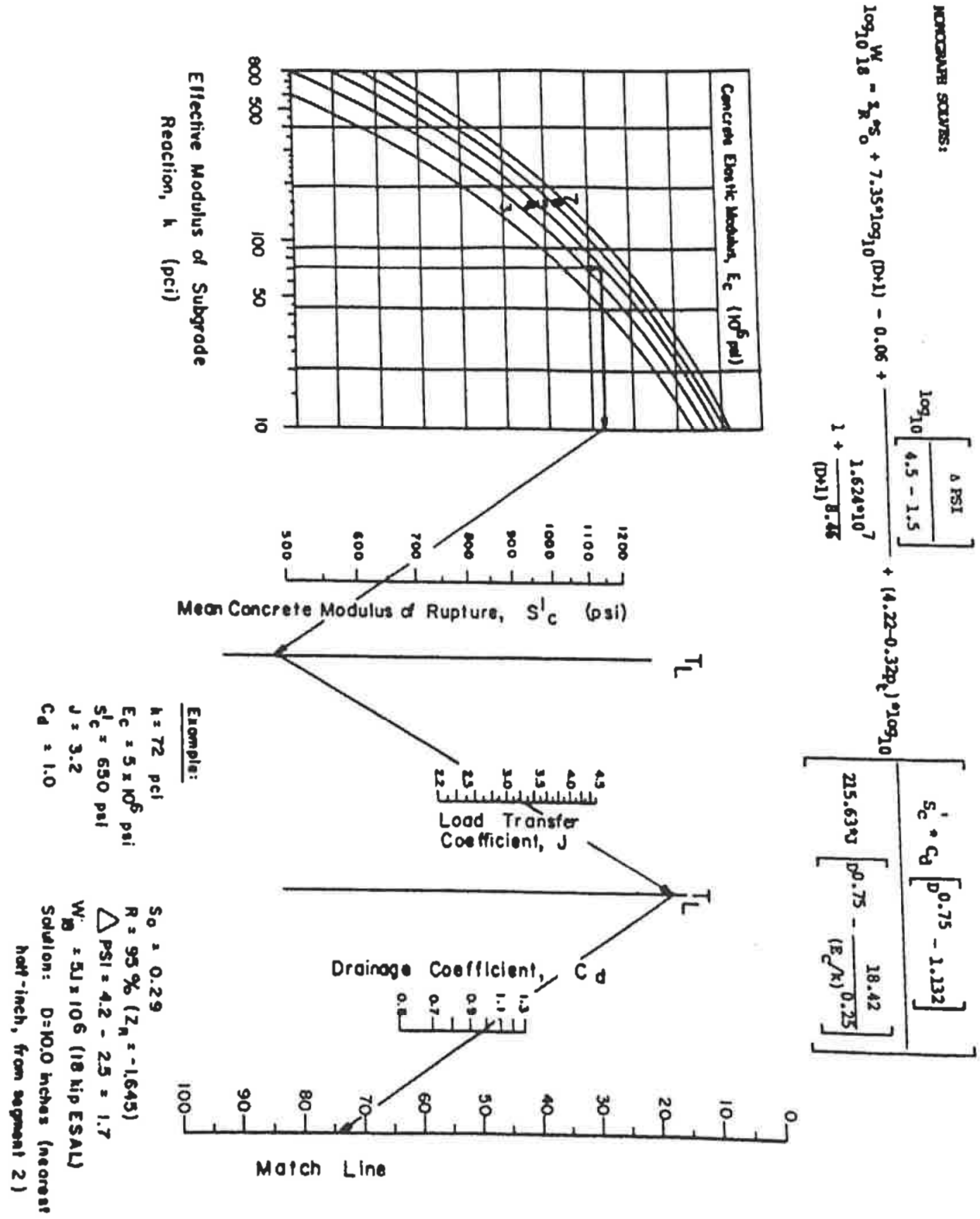


Figure 3.7. Design Chart for Rigid Pavement Based on Using Mean Values for Each Input Variable (Segment 1)

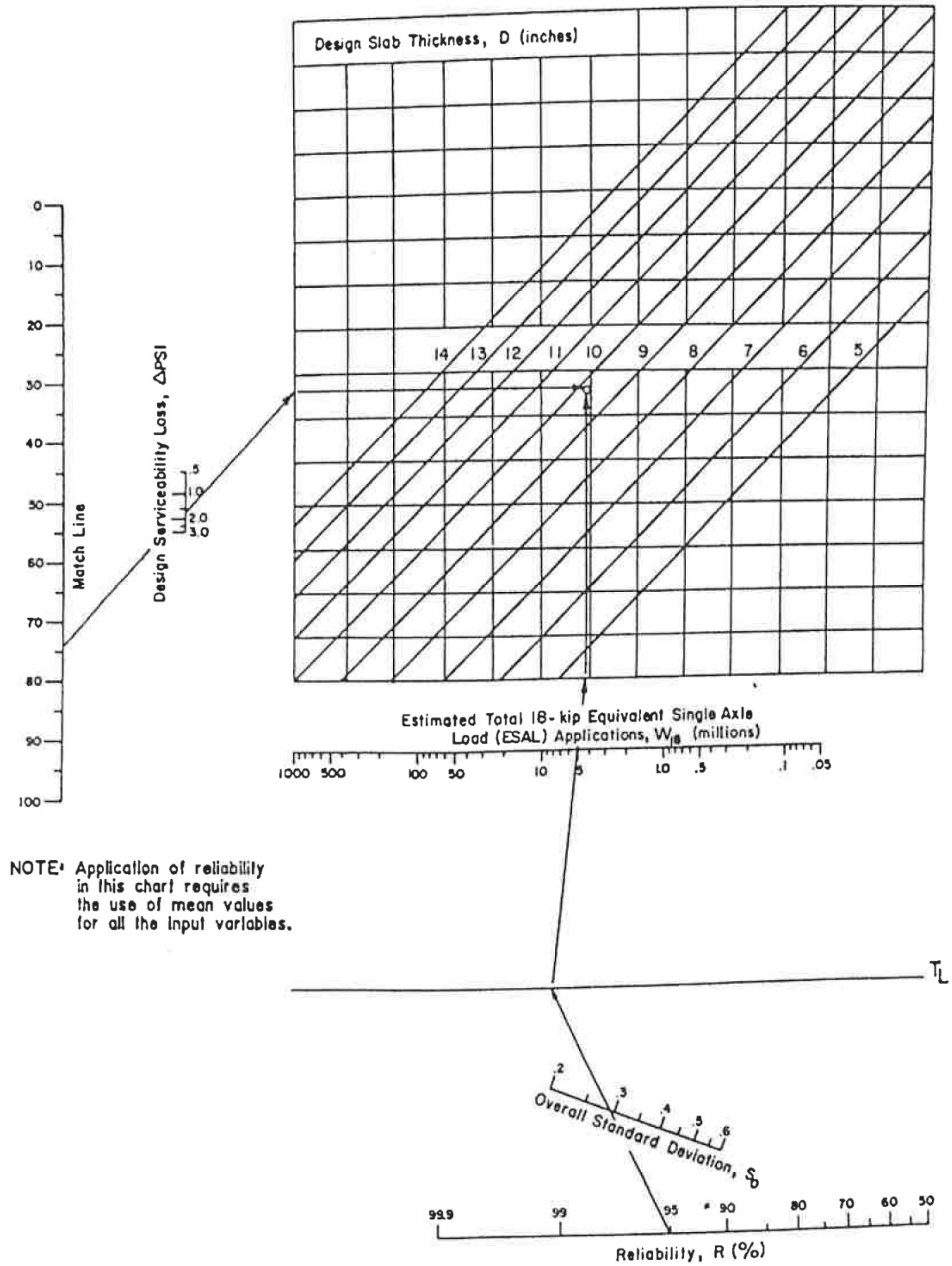


Figure 3.7. Continued—Design Chart for Rigid Pavements Based on Using Mean Values for Each Input Variable (Segment 2)

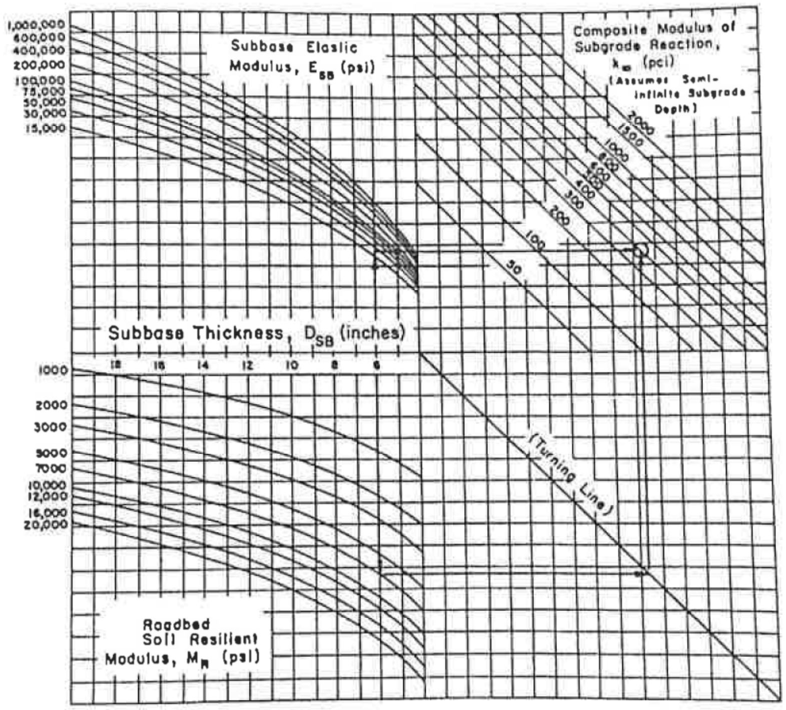


Figure 3.3. Chart for Estimating Composite Modulus of Subgrade Reaction, k_∞, Assuming a Semi-Infinite Subgrade Depth. (For practical purposes, a semi-infinite depth is considered to be greater than 10 feet below the surface of the subgrade.)

Type of Material	Loss of Support (LS)
Cement Treated Granular Base (E = 1,000,000 to 2,000,000 psi)	0.0 to 1.0
Cement Aggregate Mixtures (E = 500,000 to 1,000,000 psi)	0.0 to 1.0
Asphalt Treated Base (E = 350,000 to 1,000,000 psi)	0.0 to 1.0
Bituminous Stabilized Mixtures (E = 40,000 to 300,000 psi)	0.0 to 1.0
Lime Stabilized (E = 20,000 to 70,000 psi)	1.0 to 3.0
Unbound Granular Materials (E = 15,000 to 45,000 psi)	1.0 to 3.0
Fine Grained or Natural Subgrade Materials (E = 3,000 to 40,000 psi)	2.0 to 3.0

NOTE: E in this table refers to the general symbol for elastic or resilient modulus of the material.

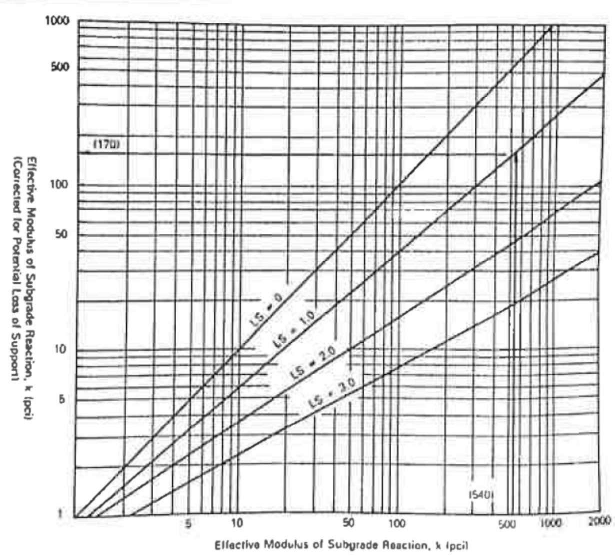


Figure 3.6. Correction of Effective Modulus of Subgrade Reaction for Potential Loss of Subbase Support (6

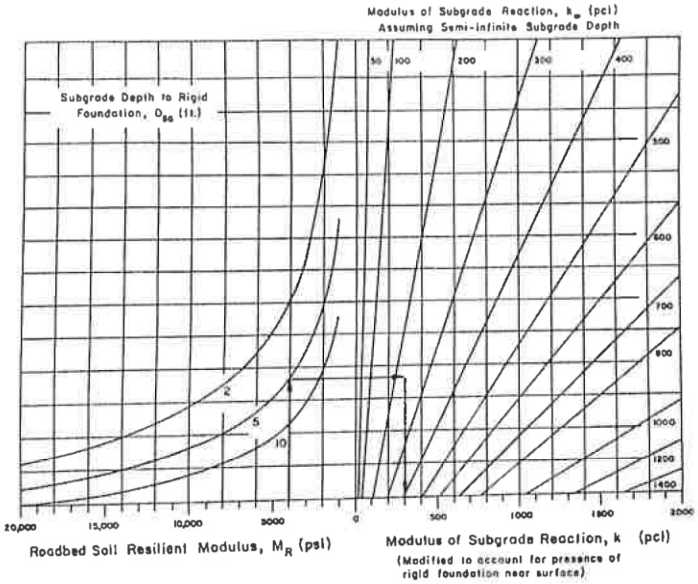


Figure 3.4. Chart to Modify Modulus of Subgrade Reaction to Consider Effects of Rigid Foundation Near Sur

$$k_\infty = M_R / 19.4$$

$$\Delta PSI = p_o - p_t$$

NOMOGRAPH SOLVES:

$$\log_{10} W_{18} = Z_R * S_o + 7.35 * \log_{10}(D+1) - 0.06 + \frac{\log_{10}\left[\frac{\Delta PSI}{4.5 - 1.5}\right]}{1 + \frac{1.624*10^7}{(D+1)^{8.46}}} + (4.22 - 0.32 p_t) * \log_{10}\left[\frac{S_c' * C_d \left[D^{0.75} - 1.132\right]}{215.63 * J \left[D^{0.75} - \frac{18.42}{(E_c/k)^{0.25}}\right]}\right]$$

Table 2.5. Recommended Values of Drainage Coefficient, C_d, for Rigid Pavement Design

Quality of Drainage	Percent of Time Pavement Structure Is Exposed to Moisture Levels Approaching Saturation: Less Than 1%	1–5%	5–25%	Greater Than 25%
Excellent	1.25–1.20	1.20–1.15	1.15–1.10	1.10
Good	1.20–1.15	1.15–1.10	1.10–1.00	1.00
Fair	1.15–1.10	1.10–1.00	1.00–0.90	0.90
Poor	1.10–1.00	1.00–0.90	0.90–0.80	0.80
Very poor	1.00–0.90	0.90–0.80	0.80–0.70	0.70

Table 20.16 Suggested Levels of Reliability for Various Functional Classifications

Functional Classification	Recommended Level of Reliability: Urban	Rural
Interstate and other freeways	85–99.9	80–99.9
Other principal arterials	80–99	75–95
Collectors	80–95	75–95
Local	50–80	50–80

Table 2.6. Recommended Load Transfer Coefficient for Various Pavement Types and Design Conditions

Shoulder	Asphalt		Tied P.C.C.	
Load Transfer Devices	Yes	No	Yes	No
Pavement Type				
1. Plain jointed and jointed reinforced	3.2	3.8–4.4	2.5–3.1	3.6–4.2
2. CRCP	2.9–3.2	N/A	2.3–2.9	N/A

	Standard Deviation, S_o
Flexible pavements	0.40–0.50
Rigid pavements	0.30–0.40

Highway Geometric Design

$d_b = \frac{v^2}{2g(f+G)}$	$K = \frac{L}{\Delta G}$
$L_{crest} = \frac{SSSD^2 \times A}{200(H + h_1 + 2\sqrt{H \times h_1})}$	$K = \frac{SSSD^2}{200(\sqrt{H} + \sqrt{h_1})^2}$
$L_{sag} = \frac{SSSD^2 \times A}{200(H + SSSD\ Tan\beta)}$	$\frac{Gv^2}{gR}cos\gamma = Gsin\gamma + G(cos\gamma)\mu$
$\frac{v^2}{gR} = e + \mu$	$R = \frac{V^2}{127(e+\mu)}$
$A^2 = L_sR = \frac{2RV \times 1000}{3600}$	$A = \sqrt{0.03577V^3}$

$L_s = \frac{we}{2s}$	$A^2 = R \times L$	$A = \sqrt{\frac{RV}{1.8}}$

$S = 2R\theta^\circ\left(\frac{\pi}{180}\right)$	$\theta = \frac{28.65}{R}S$

$$m = R(1 - \cos\frac{28.65}{R}S)$$

Clear zone distances for straight sections (m)

Design Speed (km/h)	Design ADT	6:1 or flatter	Fill Slopes 5:1 to 4:1	3:1	3:1	Cut Slopes 5:1 to 4:1	6:1 or flatter
≤ 60	< 750	2.0 - 3.0	2.0 - 3.0	see note	2.0 - 3.0	2.0 - 3.0	2.0 - 3.0
	750 - 1500	3.0 - 3.5	3.5 - 4.5	see note	3.0 - 3.5	3.0 - 3.5	3.0 - 3.5
	1500 - 6000	3.5 -4.5	4.5 - 5.0	see note	3.5 -4.5	3.5 -4.5	3.5 -4.5
	> 6000	4.5 -5.0	5.0 - 5.5	see note	4.5 -5.0	4.5 -5.0	4.5 -5.0
70 - 80	< 750	3.0 - 3.5	3.5 - 4.5	see note	2.5 - 3.0	2.5 - 3.0	3.0 - 3.5
	750 - 1500	4.5 - 5.0	5.0 - 6.0	see note	3.0 - 3.5	3.5 - 4.5	4.5 - 5.0
	1500 - 6000	5.0 - 5.5	6.0 - 8.0	see note	3.5 - 4.5	4.5 - 5.0	5.0 - 5.5
	> 6000	6.0 - 6.5	7.5 - 8.5	see note	4.5 - 5.0	5.5 - 6.0	6.0 - 6.5
90	< 750	3.5 - 4.5	4.5 - 5.5	see note	2.5 - 3.0	3.0 - 3.5	3.0 - 3.5
	750 - 1500	5.0 - 5.5	6.0 - 7.5	see note	3.0 - 3.5	4.5 - 5.0	5.0 - 5.5
	1500 - 6000	6.0 - 6.5	7.5 - 9.0	see note	4.5 - 5.0	5.0 - 5.5	6.0 - 6.5
	> 6000	6.5 - 7.5	8.0 - 10.0	see note	5.0 - 5.5	6.0 - 6.5	6.5 - 7.5
100	< 750	5.0 - 5.5	6.0 - 7.5	see note	3.0 - 3.5	3.5 - 4.5	4.5 - 5.0
	750 - 1500	6.0 - 7.5	8.0 - 10.0	see note	3.5 - 4.5	5.0 - 5.5	6.0 - 6.5
	1500 - 6000	8.0 - 9.0	10.0 - 12.0	see note	4.5 - 5.5	5.5 - 6.5	7.5 - 8.0
	> 6000	9.0 - 10.0	11.0 - 13.5	see note	6.0 - 6.5	7.5 - 8.0	8.0 - 8.5
≥ 110	< 750	5.5 - 6.0	6.0 - 8.0	see note	3.0 - 3.5	4.5 - 5.0	4.5 - 4.9
	750 - 1500	7.5 - 8.0	8.5 - 11.0	see note	3.5 - 5.0	5.5 - 6.0	6.0 - 6.5
	1500 - 6000	8.5 - 10.0	10.5 - 13.0	see note	5.0 - 6.0	6.5 - 7.5	8.0 - 8.5
	> 6000	9.0 - 10.5	11.5 - 14.0	see note	6.5 - 7.5	8.0 - 9.0	8.5 - 9.0

Clear zone correction factors in curves

Radius (m)	Design Speed 60	70	80	90	100	110+
900	1.1	1.1	1.1	1.2	1.2	1.2
700	1.1	1.1	1.2	1.2	1.2	1.3
600	1.1	1.2	1.2	1.2	1.3	1.4
500	1.1	1.2	1.2	1.3	1.3	1.4
450	1.2	1.2	1.3	1.3	1.4	1.5
400	1.2	1.2	1.3	1.3	1.4	
350	1.2	1.2	1.3	1.4	1.5	
300	1.2	1.3	1.4	1.5	1.5	
250	1.3	1.3	1.4	1.5		
200	1.3	1.4	1.5			
150	1.4	1.5				
100	1.5					

Note: The clear zone horizontal curve adjustment factor is applied to the outside of curves only. Curve flattor than 900 m do not require an adjusted clear zone.

Friction Coefficients to be used when needed

Vehicle Speed (km/h)	≤ 60	70	80	90	100	110	120
Stopping Coefficient of Friction	0.38	0.37	0.35	0.31	0.30	0.28	0.27
Coefficient of Side Friction	0.17	0.16	0.15	0.12	0.11	0.09	0.07

Superpave Mix Design

$$G_{mm} = \frac{100}{\dfrac{P_s}{G_{se}} + \dfrac{P_b}{G_b}}$$

$$V_a = \left(\frac{G_{mm} - G_{mb}}{G_{mm}}\right) x\ 100$$

$$VMA = 100 - \frac{G_{mb}\ x\ P_s}{G_{sb}}$$

$$VFA = \left(\frac{VMA - V_a}{VMA}\right) x\ 100$$

$$G_{se} = \frac{100 - P_b}{\dfrac{100}{G_{mm}} - \dfrac{P_b}{G_b}}$$

$$P_{be} = P_b - \frac{P_{ba}\ x\ P_s}{100}$$

$$P_{ba} = 100\ x \left(\frac{G_{se} - G_{sb}}{G_{se}\ x\ G_{sb}}\right) x\ G_b$$

$$G_{sb}\ (COMBINED) = \frac{100}{\dfrac{P_1\%}{G_{sb1}} + \dfrac{P_2\%}{G_{sb2}} + \dfrac{P_3\%}{G_{sb3}} + \ldots\ldots\ldots + \dfrac{P_n\%}{G_{sb(n)}}}$$