16-CHEM-B1, TRANSPORT PHENOMENA

MAY 2018

3 hours duration

NOTES

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.

2. The examination is an **open book exam.** One textbook of your choice with notations listed on the margins etc., but no loose notes are permitted into the exam.

3. Candidates may use any **non-communicating** calculator.

4. All problems are worth 25 points. **One problem** from **each** of sections A, B, and C must be attempted. A **fourth** problem from **any section** must also be attempted.

5. **Only the first four** questions as they appear in the answer book will be marked.

6. State all assumptions clearly.

SECTION A: Fluid Mechanics

A1. A 42-feet long, 4-inch inside diameter pipe discharges water from a 16-feet diameter tank into a reservoir. The initial height of water surface in the tank is 26 feet from the water surface in the reservoir. Assuming a Darcy friction factor of 0.008 (or Fanning friction factor = 0.002) and accounting for entry and exit losses, calculate the time taken to discharge 1500 ft^3 water from the tank to the reservoir.

A2. The velocity profile for turbulent flow up to a Reynolds number of 1×10^5 in a smooth circular pipe with a radius R varies according the flowing expression:

$$V_r = V_{max}\,[(R - r)/R]^{1/7}$$

where r is the radial distance from center of the pipe and V_{max} is the maximum velocity at the center of the pipe. For an incompressible liquid, obtain an equation relating average velocity (V_{av}) to the maximum velocity (V_{max}).

SECTION B: Heat Transfer

B1. Hydrocarbon oil at 80 °C enters a 4.6 meters long, 1 cm inside diameter pipe. The inside pipe surface temperature is constant at 165 °C. The oil is to be heated to 120 °C in the pipe. Calculated the amount of oil (in kg) that can be heated in an hour.

DATA: Specific heat capacity of oil = 2.0934 kJ/kg.K

Kinematic viscosity of water = 1.43651 x 10^{-4} kW/m.K

Viscosity of oil varies with temperature as follows:

Temperature (in °C)	**Viscosity** (in cP)
65	6.50
95	5.05
120	3.80
150	2.82
175	1.95

B2. Consider heat transfer by conduction through a long hollow cylinder of length L with both ends insulated. The temperature at inner radius r_i is T_i and temperature at outer radius r_o is T_o.

(a) **[7 points]** Write the unsteady-state differential equation for heat transfer.

(b) **[12 points]** Obtain the steady-state temperature profile.

(c) **[6 points]** Derive an expression for steady-state heat flow q.

SECTION C: Mass Transfer

C1. Carbon dioxide from an aqueous solution has to be removed by an air stream flowing at 91 cm/s using a 12.7 cm internal diameter wetted wall column. At one point in the column, CO_2 concentration in air stream is 1 mole percent and CO_2 concentration in water is 0.5 mole percent. The column is operated at 10 atm pressure and 27 °C. Assuming air to be an ideal gas at the operating conditions,

(a) **[22 points]** Determine the gas-phase mass transfer coefficient at this point in the column.

(b) **[3 points]** Determine the mass flux at this point in the column.

DATA:

Viscosity of air at 27 °C = 1.85×10^{-2} cP

Diffusivity of CO_2 in air at 27 °C and 1 atm = 1.36×10^{-5} m^2/s

Henry's Law constant for CO_2 in air at 27 °C = 16.4 atm/mole percent of CO_2 in solution

C2. Gas A diffuses through a stagnant film of gas surrounding a catalyst particle of radius R_1, and undergoes the reaction $3A \rightarrow B$ at the particle surface. Instantaneously, Gas B diffuses back through the stagnant film into the bulk. Assuming isothermal conditions and steady state, obtain an expression for the local reaction rate in terms of the effective gas-film thickness (δ) and the bulk gas stream composition (X_{A0}).

APPENDIX

Summary of the Conservation Equations

Table A.1 The Continuity Equation

$$\frac{\partial \rho}{\partial t} + (\nabla \cdot \rho \vec{u}) = 0$$	(1.1)
Rectangular coordinates (*x*, *y*, *z*) $$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u_x) + \frac{\partial}{\partial y}(\rho u_y) + \frac{\partial}{\partial z}(\rho u_z) = 0$$	(1.1a)
Cylindrical coordinates (*r*, *θ*, *z*) $$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(\rho r u_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho u_\theta) + \frac{\partial}{\partial z}(\rho u_z) = 0$$	(1.1b)
Spherical coordinates (*r*, *θ*, *ϕ*) $$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}(\rho r^2 u_r) + \frac{1}{r \sin\theta}\frac{\partial}{\partial \theta}(\rho u_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}(\rho u_\phi) = 0$$	(1.1c)

Table A.2 The Navier-Stokes equations for Newtonian fluids of constant ρ and μ

	$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla)\vec{u} = -\frac{1}{\rho}\nabla P + \vec{g} + \nu(\nabla^2 \vec{u})$$	(A2)
Rectangular coordinates (*x*, *y*, *z*)		
x-component	$$\frac{\partial u_x}{\partial t} + u_x\frac{\partial u_x}{\partial x} + u_y\frac{\partial u_x}{\partial y} + u_z\frac{\partial u_x}{\partial z} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + g_x + \nu\left(\frac{\partial^2 u_x}{\partial x^2} + \frac{\partial^2 u_x}{\partial y^2} + \frac{\partial^2 u_x}{\partial z^2}\right)$$	(A2a)
y-component	$$\frac{\partial u_y}{\partial t} + u_x\frac{\partial u_y}{\partial x} + u_y\frac{\partial u_y}{\partial y} + u_z\frac{\partial u_y}{\partial z} = -\frac{1}{\rho}\frac{\partial P}{\partial y} + g_y + \nu\left(\frac{\partial^2 u_y}{\partial x^2} + \frac{\partial^2 u_y}{\partial y^2} + \frac{\partial^2 u_y}{\partial z^2}\right)$$	(A2b)
z-component	$$\frac{\partial u_z}{\partial t} + u_x\frac{\partial u_z}{\partial x} + u_y\frac{\partial u_z}{\partial y} + u_z\frac{\partial u_z}{\partial z} = -\frac{1}{\rho}\frac{\partial P}{\partial z} + g_z + \nu\left(\frac{\partial^2 u_z}{\partial x^2} + \frac{\partial^2 u_z}{\partial y^2} + \frac{\partial^2 u_z}{\partial z^2}\right)$$	(A2c)

Cylindrical coordinates (*r*, *θ*, *z*)

r-component

$$\frac{\partial u_r}{\partial t}+u_r\frac{\partial u_r}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta}+u_z\frac{\partial u_r}{\partial z}-\frac{u_\theta^2}{r}$$
$$=-\frac{1}{\rho}\frac{\partial P}{\partial r}+g_r+\nu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial (ru_r)}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u_r}{\partial \theta^2}-\frac{2}{r^2}\frac{\partial u_\theta}{\partial \theta}+\frac{\partial^2 u_r}{\partial z^2}\right] \quad \text{(A2d)}$$

θ-component

$$\frac{\partial u_\theta}{\partial t}+u_r\frac{\partial u_\theta}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta}+u_z\frac{\partial u_\theta}{\partial z}+\frac{u_r u_\theta}{r}$$
$$=-\frac{1}{\rho r}\frac{\partial P}{\partial \theta}+g_\theta+\nu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial (ru_\theta)}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u_\theta}{\partial \theta^2}+\frac{2}{r^2}\frac{\partial u_r}{\partial \theta}+\frac{\partial^2 u_\theta}{\partial z^2}\right] \quad \text{(A2e)}$$

z-component

$$\frac{\partial u_z}{\partial t}+u_r\frac{\partial u_z}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_z}{\partial \theta}+u_z\frac{\partial u_z}{\partial z}$$
$$=-\frac{1}{\rho}\frac{\partial P}{\partial z}+g_z+\nu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u_z}{\partial \theta^2}+\frac{\partial^2 u_z}{\partial z^2}\right] \quad \text{(A2f)}$$

Spherical coordinates (*r*, *θ*, *ϕ*)

r-component

$$\frac{\partial u_r}{\partial t}+u_r\frac{\partial u_r}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta}+\left(\frac{u_\phi}{r\sin\theta}\right)\frac{\partial u_r}{\partial \phi}-\frac{u_\theta^2}{r}-\frac{u_\phi^2}{r}=-\frac{1}{\rho}\frac{\partial P}{\partial r}+g_r$$
$$+\nu\left[\frac{1}{r^2}\frac{\partial^2}{\partial r^2}\left(r^2u_r\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial u_r}{\partial \theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 u_r}{\partial \phi^2}\right] \quad \text{(A2g)}$$

θ-component

$$\frac{\partial u_\theta}{\partial t}+u_r\frac{\partial u_\theta}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta}+\left(\frac{u_\phi}{r\sin\theta}\right)\frac{\partial u_\theta}{\partial \phi}+\frac{u_r u_\theta}{r}-\frac{u_\phi^2}{r}\cot\theta=-\frac{1}{\rho r}\frac{\partial P}{\partial \theta}+g_\theta$$
$$+\nu\left[\begin{array}{l}\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial u_\theta}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial \theta}\left(u_\theta\sin\theta\right)\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 u_\theta}{\partial \phi^2}\\+\frac{2}{r^2}\frac{\partial u_r}{\partial \theta}-\frac{2\cot\theta}{r^2\sin\theta}\frac{\partial u_\phi}{\partial \phi}\end{array}\right] \quad \text{(A2h)}$$

ϕ-component

$$\frac{\partial u_\phi}{\partial t}+u_r\frac{\partial u_\phi}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_\phi}{\partial \theta}+\frac{u_\phi}{r\sin\theta}\frac{\partial u_\phi}{\partial \phi}+\frac{u_r u_\phi}{r}+\frac{u_\theta u_\phi}{r}\cot\theta=-\frac{1}{\rho r\sin\theta}\frac{\partial P}{\partial \phi}$$
$$+g_\varphi+\nu\left[\begin{array}{l}\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial u_\phi}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial \theta}\left(u_\phi\sin\theta\right)\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 u_\phi}{\partial \phi^2}\\+\frac{2}{r^2\sin\theta}\frac{\partial u_r}{\partial \phi}+\frac{2\cot\theta}{r^2\sin\theta}\frac{\partial u_\theta}{\partial \phi}\end{array}\right] \quad \text{(A2i)}$$

Table A.3 The Energy Equation for Incompressible Media

$$\rho c_P\left[\frac{\partial T}{\partial t}+(\vec{u}\cdot\nabla)(T)\right]=\left[\nabla\cdot k\nabla T\right]+\dot{T}_G$$	(A3)
Rectangular coordinates (*x*, *y*, *z*) $$\rho c_P\left[\frac{\partial T}{\partial t}+u_x\frac{\partial T}{\partial x}+u_y\frac{\partial T}{\partial y}+u_z\frac{\partial T}{\partial z}\right]=\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{T}_G$$	(A3a)
Cylindrical coordinates (*r*, *θ*, *z*) $$\rho c_P\left[\frac{\partial T}{\partial t}+u_r\frac{\partial T}{\partial r}+\frac{u_\theta}{r}\frac{\partial T}{\partial \theta}+u_z\frac{\partial T}{\partial z}\right]=\frac{1}{r}\frac{\partial}{\partial r}\left(rk\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \theta}\left(k\frac{\partial T}{\partial \theta}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{T}_G$$	(A3b)
Spherical coordinates (*r*, *θ*, *ϕ*) $$\rho c_P\left[\frac{\partial T}{\partial t}+u_r\frac{\partial T}{\partial r}+\frac{u_\theta}{r}\frac{\partial T}{\partial \theta}+\frac{u_\phi}{r\sin\theta}\frac{\partial T}{\partial \phi}\right]=$$ $$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2k\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(k\sin\theta\frac{\partial T}{\partial \theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right)+\dot{T}_G$$	(A3c)

Table A4: The continuity equation for species *A* in terms of the molar flux

$$\frac{\partial C_A}{\partial t}=-\left(\nabla\cdot\vec{N}_A\right)+\dot{R}_{A,G}$$	(4.)
Rectangular coordinates (*x*, *y*, *z*) $$\frac{\partial C_A}{\partial t}=-\left(\frac{\partial[N_A]_x}{\partial x}+\frac{\partial[N_A]_y}{\partial y}+\frac{\partial[N_A]_z}{\partial z}\right)+\dot{R}_{A,G}$$	(4a)
Cylindrical coordinates (*r*, *θ*, *z*) $$\frac{\partial C_A}{\partial t}=-\left\{\frac{1}{r}\frac{\partial}{\partial r}[rN_A]_r+\frac{1}{r}\frac{\partial}{\partial \theta}[N_A]_\theta+\frac{\partial}{\partial z}[N_A]_z\right\}+\dot{R}_{A,G}$$	(4b)
Spherical coordinates (*r*, *θ*, *ϕ*) $$\frac{\partial C_A}{\partial t}=-\left\{\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2[N_A]_r\right)+\frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}\left([N_A]_\theta\sin\theta\right)+\frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}[N_A]_\phi\right\}+\dot{R}_{A,G}$$	(4c)

Table A.5: The continuity equation for species *A*

$$\frac{\partial C_A}{\partial t} + (\vec{u} \cdot \nabla) C_A = D_A \nabla^2 C_A + \dot{R}_{A,G}$$	(5)
Rectangular coordinates (*x*, *y*, *z*) $$\frac{\partial C_A}{\partial t} + u_x \frac{\partial C_A}{\partial x} + u_y \frac{\partial C_A}{\partial y} + u_z \frac{\partial C_A}{\partial z} = \frac{\partial}{\partial x}\left(D \frac{\partial C_A}{\partial x}\right) + \frac{\partial}{\partial y}\left(D \frac{\partial C_A}{\partial y}\right) + \frac{\partial}{\partial z}\left(D \frac{\partial C_A}{\partial z}\right) + \dot{R}_{A,G}$$	(5a)
Cylindrical coordinates (*r*, *θ*, *z*) $$\frac{\partial C_A}{\partial t} + u_r \frac{\partial C_A}{\partial r} + \frac{u_\theta}{r} \frac{\partial C_A}{\partial \theta} + u_z \frac{\partial C_A}{\partial z} = \frac{1}{r} \frac{\partial}{\partial r}\left(rD \frac{\partial C_A}{\partial r}\right) + \frac{1}{r^2} \frac{\partial}{\partial \theta}\left(D \frac{\partial C_A}{\partial \theta}\right) + \frac{\partial}{\partial z}\left(D \frac{\partial C_A}{\partial z}\right) + \dot{R}_{A,G}$$	(5b)
Spherical coordinates (*r*, *θ*, *ϕ*) $$\frac{\partial C_A}{\partial t} + u_r \frac{\partial C_A}{\partial r} + \frac{u_\theta}{r} \frac{\partial C_A}{\partial \theta} + \frac{u_\phi}{r \sin\theta} \frac{\partial C_A}{\partial \phi} = \frac{1}{r^2} \frac{\partial}{\partial r}\left(r^2 D \frac{\partial C_A}{\partial r}\right)$$ $$+ \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta}\left(D \sin\theta \frac{\partial C_A}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta} \frac{\partial}{\partial \phi}\left(D \frac{\partial C_A}{\partial \phi}\right) + \dot{R}_{A,G}$$	(5c)

The Periodic Table of the Elements

- Alkali metals
- Alkaline earth metals
- Transition metals
- Other metals
- Metalloids (semi-metal)
- Nonmetals
- Halogens
- Noble gases

Element name → Mercury
Atomic # → 80
Symbol → Hg
Avg. Mass → 200.59

1	2		3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Hydrogen 1 H 1.01																		Helium 2 He 4.00
Lithium 3 Li 6.94	Beryllium 4 Be 9.01												Boron 5 B 10.81	Carbon 6 C 12.01	Nitrogen 7 N 14.01	Oxygen 8 O 16.00	Fluorine 9 F 19.00	Neon 10 Ne 20.18
Sodium 11 Na 22.99	Magnesium 12 Mg 24.31												Aluminum 13 Al 26.98	Silicon 14 Si 28.09	Phosphorus 15 P 30.97	Sulfur 16 S 32.07	Chlorine 17 Cl 35.45	Argon 18 Ar 39.95
Potassium 19 K 39.10	Calcium 20 Ca 40.08		Scandium 21 Sc 44.96	Titanium 22 Ti 47.88	Vanadium 23 V 50.94	Chromium 24 Cr 52.00	Manganese 25 Mn 54.94	Iron 26 Fe 55.85	Cobalt 27 Co 58.93	Nickel 28 Ni 58.69	Copper 29 Cu 63.55	Zinc 30 Zn 65.39	Gallium 31 Ga 69.72	Germanium 32 Ge 72.61	Arsenic 33 As 74.92	Selenium 34 Se 78.96	Bromine 35 Br 79.90	Krypton 36 Kr 83.80
Rubidium 37 Rb 85.47	Strontium 38 Sr 87.62		Yttrium 39 Y 88.91	Zirconium 40 Zr 91.22	Niobium 41 Nb 92.91	Molybdenum 42 Mo 95.94	Technetium 43 Tc (98)	Ruthenium 44 Ru 101.07	Rhodium 45 Rh 102.91	Palladium 46 Pd 106.42	Silver 47 Ag 107.87	Cadmium 48 Cd 112.41	Indium 49 In 114.82	Tin 50 Sn 118.71	Antimony 51 Sb 121.76	Tellurium 52 Te 127.60	Iodine 53 I 126.90	Xenon 54 Xe 131.29
Cesium 55 Cs 132.91	Barium 56 Ba 137.33	57-70 *	Lutetium 71 Lu 174.97	Hafnium 72 Hf 178.49	Tantalum 73 Ta 180.95	Tungsten 74 W 183.84	Rhenium 75 Re 186.21	Osmium 76 Os 190.23	Iridium 77 Ir 192.22	Platinum 78 Pt 195.08	Gold 79 Au 196.97	Mercury 80 Hg 200.59	Thallium 81 Tl 204.38	Lead 82 Pb 207.20	Bismuth 83 Bi 208.98	Polonium 84 Po (209)	Astatine 85 At (210)	Radon 86 Rn (222)
Francium 87 Fr (223)	Radium 88 Ra (226)	89-102 **	Lawrencium 103 Lr (262)	Rutherfordium 104 Rf (267)	Dubnium 105 Db (268)	Seaborgium 106 Sg (271)	Bohrium 107 Bh (272)	Hassium 108 Hs (270)	Meitnerium 109 Mt (276)	Darmstadtium 110 Ds (281)	Roentgenium 111 Rg (280)	Copernicium 112 Cn (285)	Ununtrium 113 Uut (284)	Ununquadium 114 Uuq (289)	Ununpentium 115 Uup (288)	Ununhexium 116 Uuh (293)	Ununseptium 117 Uus (294?)	Ununoctium 118 Uuo (294)

*lanthanides	Lanthanum 57 La 138.91	Cerium 58 Ce 140.12	Praseodymium 59 Pr 140.91	Neodymium 60 Nd 144.24	Promethium 61 Pm (145)	Samarium 62 Sm 150.36	Europium 63 Eu 151.97	Gadolinium 64 Gd 157.25	Terbium 65 Tb 158.93	Dysprosium 66 Dy 162.50	Holmium 67 Ho 164.93	Erbium 68 Er 167.26	Thulium 69 Tm 168.93	Ytterbium 70 Yb 173.04
**actinides	Actinium 89 Ac (227)	Thorium 90 Th 232.04	Protactinium 91 Pa 231.04	Uranium 92 U 238.03	Neptunium 93 Np (237)	Plutonium 94 Pu (244)	Americium 95 Am (243)	Curium 96 Cm (247)	Berkelium 97 Bk (247)	Californium 98 Cf (251)	Einsteinium 99 Es (252)	Fermium 100 Fm (257)	Mendelevium 101 Md (258)	Nobelium 102 No (259)