NATIONAL EXAMS December 2015
07-Elec-B2 Advanced Control Systems

3 hours duration

NOTES:

1. If a doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.

3. This is a closed-book examination. Tables of Laplace and z-transforms are attached as page 4 and page 5.

4. Any four questions constitute a complete paper. Only the first four questions as they appear in your answer book will be marked.

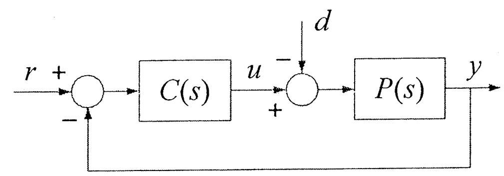
5. All questions are of equal value.

6. One of two calculators permitted Casio or Sharp approved models.

1. Consider the control system below with, $P(s)=\dfrac{10^6}{s+10^6}$, $C(s)=\dfrac{10^4}{s(s+10)}$

(a) Show, by whatever method you choose, that the system is very nearly unstable.

(b) Determine the steady state tracking error, $r-y$ when d is a unit step and $r=0$.



(c) Suppose that $C(s)=K+\dfrac{10^4}{s(s+10)}$. Determine a value for K such that the phase margin is approximately 90°.

(d) Determine the steady state tracking error when $K=0$ and $r(t)=3\sin(100t)$ and $d=0$;

2. For the transfer function, $P(s)=\dfrac{2s+1}{s(s^2+s+4)}$

(a) Determine a state space model for the system.

(b) Consider the feedback control, $u(t)=-Kx(t)+Fr(t)$. Determine gains, K and F, such that the closed loop poles at -1, -2 and -3 and the output, y, tracks a step input, r, in steady state.

3. Input and output measurements from a system are to be used to fit a discrete model of the form, $Y(z)=P(z)U(z)$, where, $P(z)=\dfrac{b_1z+b_0}{a_2z^2+a_1z+a_0}$.

(a) Describe the approach including the mathematical details necessary to arrive at a least squares estimate for $P(z)$.

(b) Under what conditions does the least square estimate converge?

(c) If $u(k)=2$, what is the steady state output as predicted by the identified model?

4. Consider the system,

$$\dot{x}(t) = \begin{pmatrix} -1 & 0 & 0 \\ \alpha - 1 & -2 & 0 \\ 0 & 1 & 0 \end{pmatrix} x(t) + \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} u(t)$$

$$y(t) = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} x(t)$$

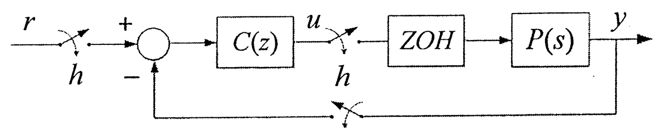
(a) Establish whether the system is controllable and observable. Justify your answer.

(b) Establish whether the system is bounded-input-bounded-output stable. Justify your answer.

(c) Assume $u(t) = 0$ and $\alpha = 3$. Let $x(0) = \begin{pmatrix} 0 & 0 & 9 \end{pmatrix}^T$. Determine the steady state output.

5. Consider the sampled data and digital control system below. The input to the zero order hold, *ZOH*, and the (continuous) output, y, are uniformly sampled with a sample period of $h = 1$ s. $C(z)$ and $P(s)$ *are* given by,

$$C(z) = Kz^{-1}, \quad P(s) = \frac{1}{s + 0.2}$$

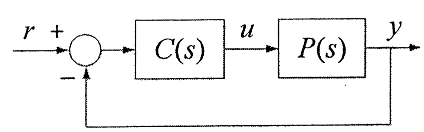


(a) Determine the discrete closed loop transfer function, $T(z)$, that relates $Y(z)$ to $R(z)$.

(b) Determine the range of values of K for stability.

(c) Redesign $C(z)$ such that: the steady state tracking error is zero at the sample instants for a step input at r, (with the closed loop system is stable).

6. Consider the feedback system below with, $C(s) = K, \; P(s) = \dfrac{3e^{-4s}}{s+1}$.



(a) Determine the range of K such that the gain margin is at least 6 dB. Determine the corresponding phase margin.

(b) Assuming stability, determine the steady state tracking error, $e(t) = r(t) - y(t)$, as a function of K when the input is a unit step.

(c) Redesign $C(s)$ such that: i) the steady state tracking error is zero for a step input and ii) the gain margin is at least 6 dB.

Inverse Laplace Transforms	
$F(s)$	$f(t)$
$\dfrac{A}{s+\alpha}$	$Ae^{-\alpha t}$
$\dfrac{C+jD}{s+\alpha+j\beta}+\dfrac{C-jD}{s+\alpha-j\beta}$	$2e^{-\alpha t}\left(C\cos\beta t+D\sin\beta t\right)$
$\dfrac{A}{(s+\alpha)^{n+1}}$	$\dfrac{At^n e^{-\alpha t}}{n!}$
$\dfrac{C+jD}{\left(s+\alpha+j\beta\right)^{n+1}}+\dfrac{C-jD}{\left(s+\alpha-j\beta\right)^{n+1}}$	$\dfrac{2t^n e^{-\alpha t}}{n!}\left(C\cos\beta t+D\sin\beta t\right)$

Inverse z-Transforms	
$F(z)$	$f(nT)$
$\dfrac{Kz}{z-a}$	Ka^n
$\dfrac{\left(C+jD\right)z}{z-re^{j\varphi}}+\dfrac{\left(C-jD\right)z}{z-re^{-j\varphi}}$	$2r^n\left(C\cos n\varphi-D\sin n\varphi\right)$
$\dfrac{Kz}{\left(z-a\right)^r}$, $r=2,3\ldots$	$\dfrac{Kn\left(n-1\right)\ldots\left(n-r+2\right)}{\left(r-1\right)!a^{r-1}}a^n$

Table of Laplace and z-Transforms (h denotes the sample period)		
$f(t)$	$F(s)$	$F(z)$
unit impulse	1	1
unit step	$\frac{1}{s}$	$\frac{z}{z-1}$
$e^{-\alpha t}$	$\frac{1}{s+\alpha}$	$\frac{z}{z-e^{\alpha h}}$
t	$\frac{1}{s^2}$	$\frac{hz}{(z-1)^2}$
$\cos \beta t$	$\frac{s}{s^2+\beta^2}$	$\frac{z(z-\cos\beta h)}{z^2-2z\cos\beta h+1}$
$\sin \beta t$	$\frac{\beta}{s^2+\beta^2}$	$\frac{z\sin\beta h}{z^2-2z\cos\beta h+1}$
$e^{-\alpha t}\cos \beta t$	$\frac{s+\alpha}{(s+\alpha)^2+\beta^2}$	$\frac{z(z-e^{-\alpha h}\cos\beta h)}{z^2-2ze^{-\alpha h}\cos\beta h+e^{-2\alpha h}}$
$e^{-\alpha t}\sin \beta t$	$\frac{\beta}{(s+\alpha)^2+\beta^2}$	$\frac{ze^{-\alpha h}\sin\beta h}{z^2-2ze^{-\alpha h}\cos\beta h+e^{-2\alpha h}}$
$t f(t)$	$-\frac{dF(s)}{ds}$	$-zh\frac{dF(z)}{dz}$
$e^{-\alpha t} f(t)$	$F(s+\alpha)$	$F(ze^{\alpha h})$