National Exams December 2015

07-Elec-A2, Systems & Control

3 hours duration

NOTES:

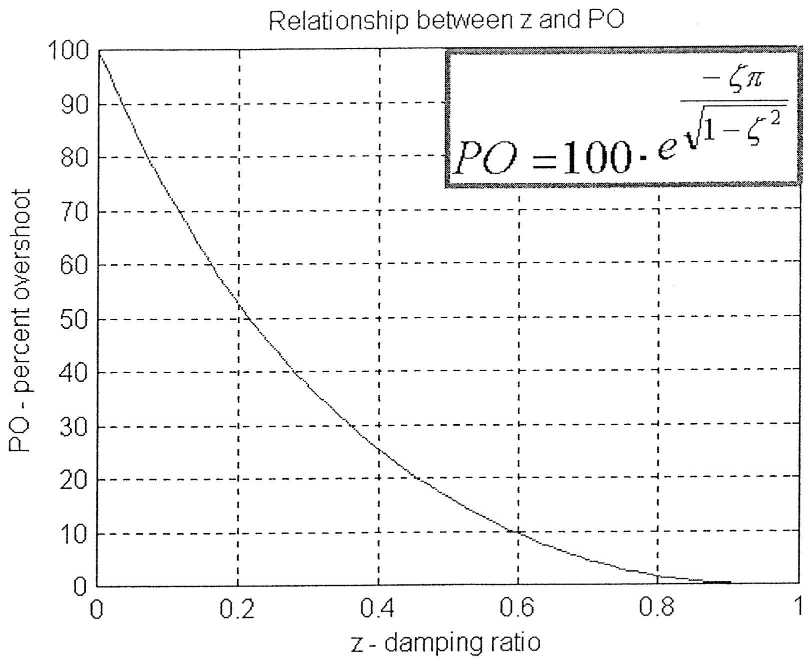
1. This is a CLOSED BOOK EXAM. Only an approved Casio or Sharp calculator is permitted. Candidates are allowed to use a double-sided, **handwritten**, 8.5 by 11" formula and notes sheet. Otherwise, there are no restrictions on the content of the formula sheet. The sheet must be signed and submitted together with the examination paper.
2. If doubt exists as to interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
3. Five (5) questions constitute a complete paper. **YOU ARE REQUIRED TO COMPLETE QUESTION 1 AND QUESTION 2.** Choose three (3) more questions out of the remaining six. Clearly indicate answers to which questions should be marked - otherwise, only the first five answers provided will be marked. Each question is of equal value. Weighting of topics within each question is indicated. Partial marks will be given. Clearly show steps taken in answering each question.
4. **Use exam booklets to answer the questions - clearly indicate which question is being answered.**

YOUR MARKS		
QUESTIONS 1 AND 2 ARE COMPULSORY:		
Question 1	20	
Question 2	20	
CHOOSE THREE OUT OF THE REMAINING SIX QUESTIONS:		
Question 3	20	
Question 4	20	
Question 5	20	
Question 6	20	
Question 7	20	
Question 8	20	
TOTAL:	100	

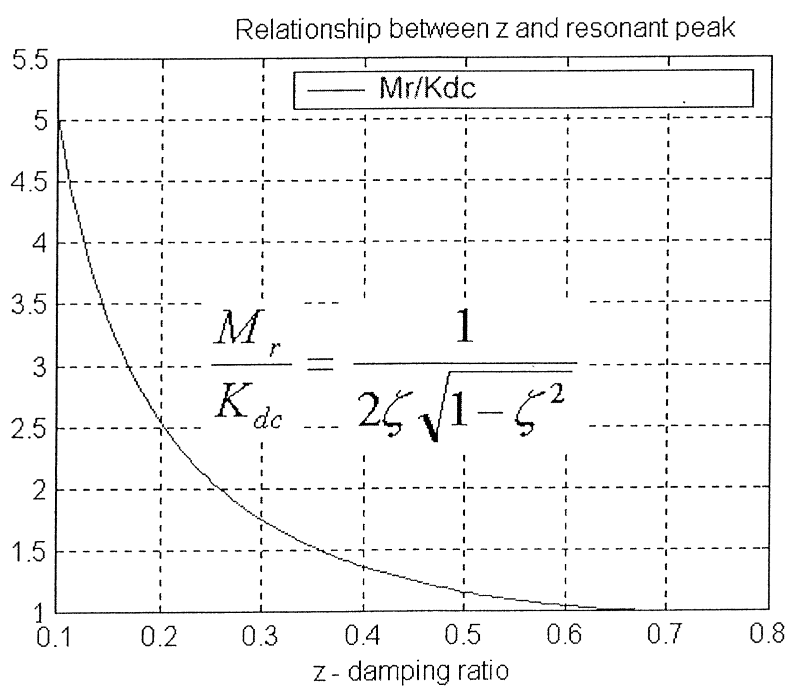
A Short Table of Laplace Transforms

Laplace Transform	Time Function
1	$\sigma(t)$
$\frac{1}{s}$	$1(t)$
$\frac{1}{(s)^2}$	$t \cdot 1(t)$
$\frac{1}{(s)^{k+1}}$	$\frac{t^k}{k!} \cdot 1(t)$
$\frac{a}{s+a}$	$e^{-at} \cdot 1(t)$
$\frac{a}{(s+a)^2}$	$te^{-at} \cdot 1(t)$
$\frac{a}{s(s+a)}$	$(1-e^{-at}) \cdot 1(t)$
$\frac{a}{s^2+a^2}$	$\sin at \cdot 1(t)$
$\frac{s}{s^2+a^2}$	$\cos at \cdot 1(t)$
$\frac{s+a}{(s+a)^2+b^2}$	$e^{-at} \cdot \cos bt \cdot 1(t)$
$\frac{b}{(s+a)^2+b^2}$	$e^{-at} \cdot \sin bt \cdot 1(t)$
$\frac{a^2+b^2}{s[(s+a)^2+b^2]}$	$\left(1 - e^{-at} \cdot \left(\cos bt + \frac{a}{b} \cdot \sin bt\right)\right) \cdot 1(t)$
$\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$	$\frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \cdot \sin\left(\omega_n\sqrt{1-\zeta^2}t\right) \cdot 1(t)$
$\frac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$	$\left(1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \cdot \sin\left(\omega_n\sqrt{1-\zeta^2}t + \cos^{-1}\zeta\right)\right) \cdot 1(t)$
$F(s) \cdot e^{-Ts}$	$f(t-T) \cdot 1(t)$
$F(s+a)$	$f(t) \cdot e^{-at} \cdot 1(t)$
$sF(s) - f(0+)$	$\frac{df(t)}{dt}$
$\frac{1}{s} F(s)$	$\int_{0+}^{+\infty} f(t)dt$

Useful Plots



PO vs. Damping Ratio



Resonant Peak vs. Damping Ratio

Question 1 (Compulsory)

State Space vs. Transfer Function Representations, Controllability and Observability, Steady State Errors to Step Inputs.

Consider a certain closed loop control system shown below:

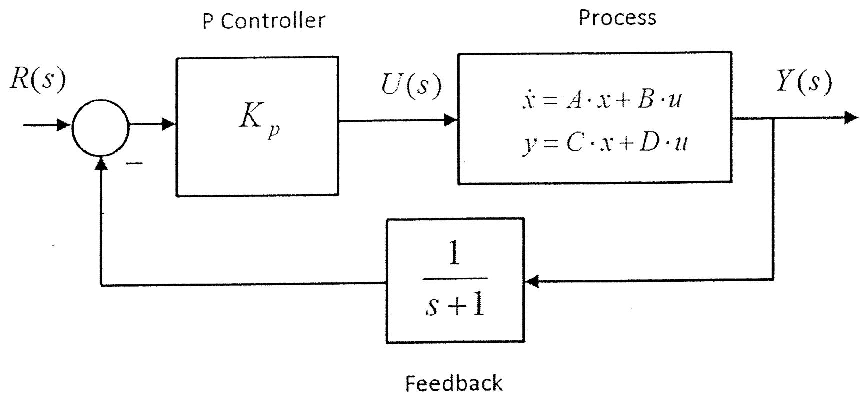


Figure Q1.1 - Closed Loop System in Question 1

1. **(5 marks)** If the state space description of the Process is as follows, find the Open Loop transfer function of the system shown.

$$\begin{aligned} \dot{x_1} &= x_2 \\ \dot{x_2} &= -3x_1 - 2x_2 + u \\ y &= x_1 + x_2 + u \end{aligned}$$

2. **(5 marks)** Determine if the system is controllable and observable.

3. **(10 marks)** Find the range of Proportional Controller gains such that the closed loop system is stable and the closed loop system step response has a Steady State Error, $e_{ss(step)\%}$, of no more than 3%.

Question 2 (Compulsory)

System Stability in the s-Domain and in the Frequency Domain: Bode Plots, Root Locus Plots and Routh-Hurwitz Criterion of Stability.

A unit feedback control system that is unstable in an open loop configuration, is to be stabilized using a Proportional + Integral + Derivative (PID) Controller. The combined process-controller Open Loop transfer function is described as follows:

$$G_{open}(s) = K_p\left(1 + \frac{1}{T_i s}\right)(1 + T_d s) \cdot \frac{2}{s(s+10)(s-2)}$$

The PID controller time constants are as follows: $\boldsymbol{T_i} = \mathbf{1}$ seconds and $\boldsymbol{T_d} = \mathbf{0.5}$ seconds. The frequency response (Bode plot) for the compensated open loop system is shown in Figure Q2.1, and the corresponding Root Locus plot is shown in Figure Q2.2.

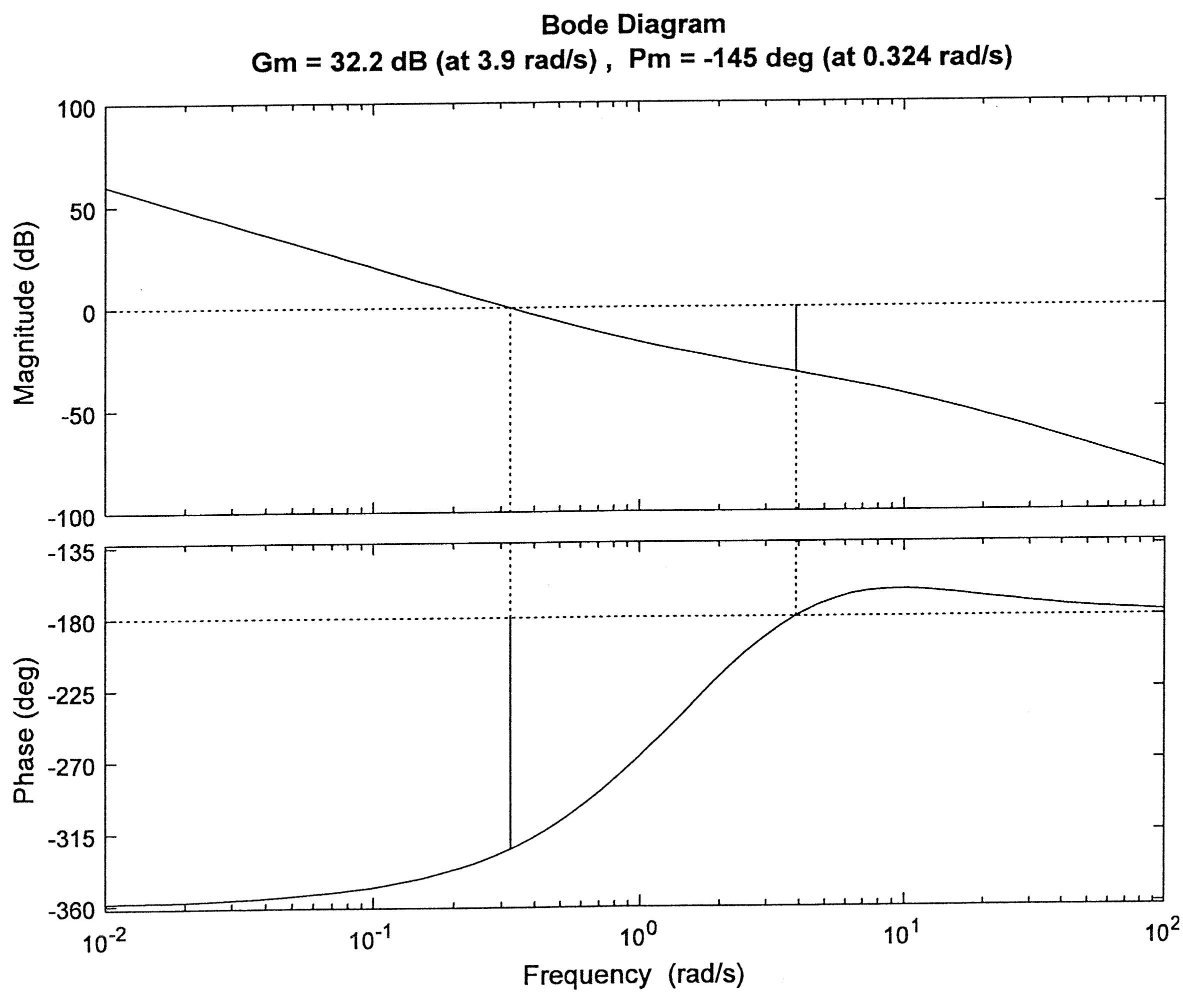


Figure Q2.1 – Open Loop Compensated System Frequency Response

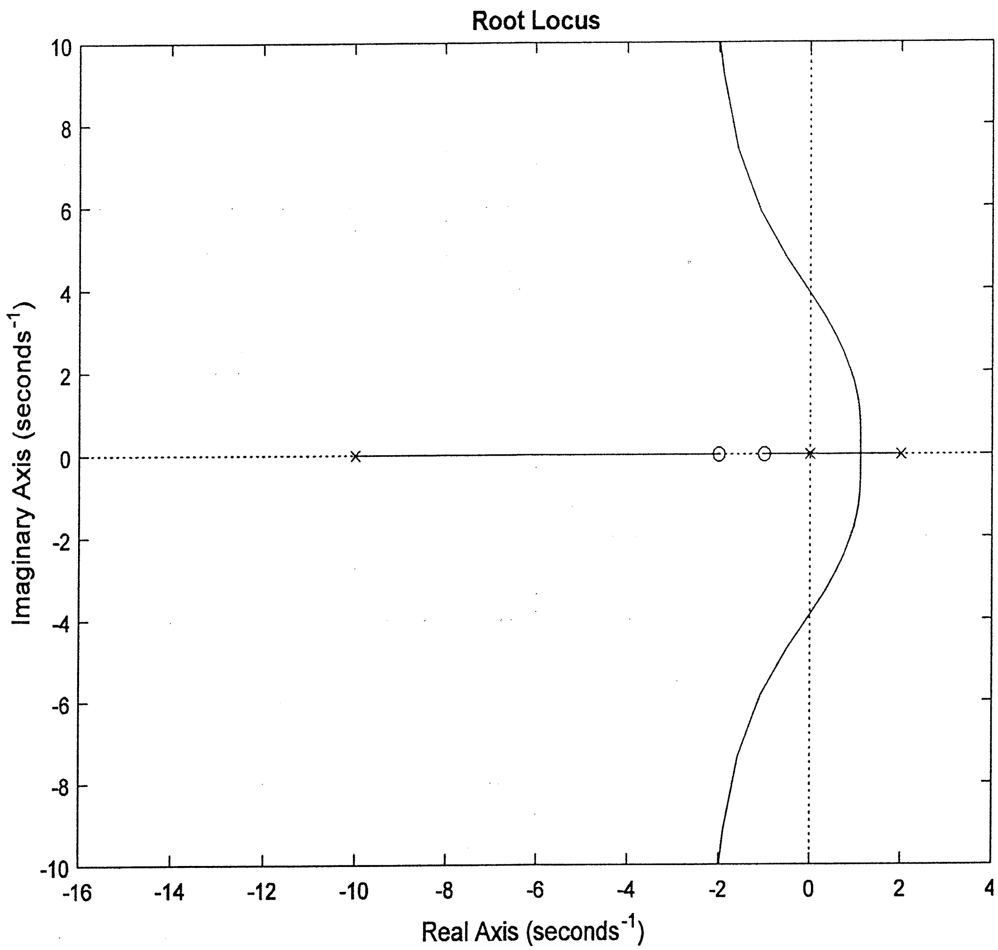


Figure Q2.2 – Root Locus of the PID-Compensated System

1. **(10 marks)** Use the information provided in Figure Q2.1 & Figure Q2.2 to determine the range of Proportional Gain, K_p, for a safe, stable operation of the closed loop system. Next, read off the frequency of oscillations when the system is marginally stable (ω_{osc}).

2. **(10 marks)** Use the Routh-Hurwitz Criterion of Stability to verify both answers as found above, i.e. the range of Proportional Gain, K_p, for the safe, stable operation of the closed loop system, as well as the frequency of marginal oscillations, ω_{osc}.

Question 3

Root Locus Analysis and Gain Selection, Stability, Second Order Model, System Damping.

Consider a closed loop control system working in a unit feedback configuration under Proportional Control ($K_p > 0$), where the process transfer function is described as follows:

$$G(s) = \frac{s+1}{s(s-3)}$$

PART A (10 marks) - *Root Locus Analysis*

Sketch a Root Locus plot for this system and place it in Figure Q3.1 - show all relevant coordinates, such as the crossovers through the Imaginary Axis, the break-away, the centroid and the asymptotic angles.

PART B (10 marks) - *Root Locus Gain Selection*

Next, based on the plot in Part A, determine values of the Proportional Controller gain (K_p) corresponding to the following values of the Closed Loop system damping ratio:

a. $\zeta_1 = 1$ NOTE: the system is critically damped;
b. $\zeta_2 = 0$ NOTE: the system is marginally stable;
c. $\zeta_3 = -1$ NOTE: the system is unstable;

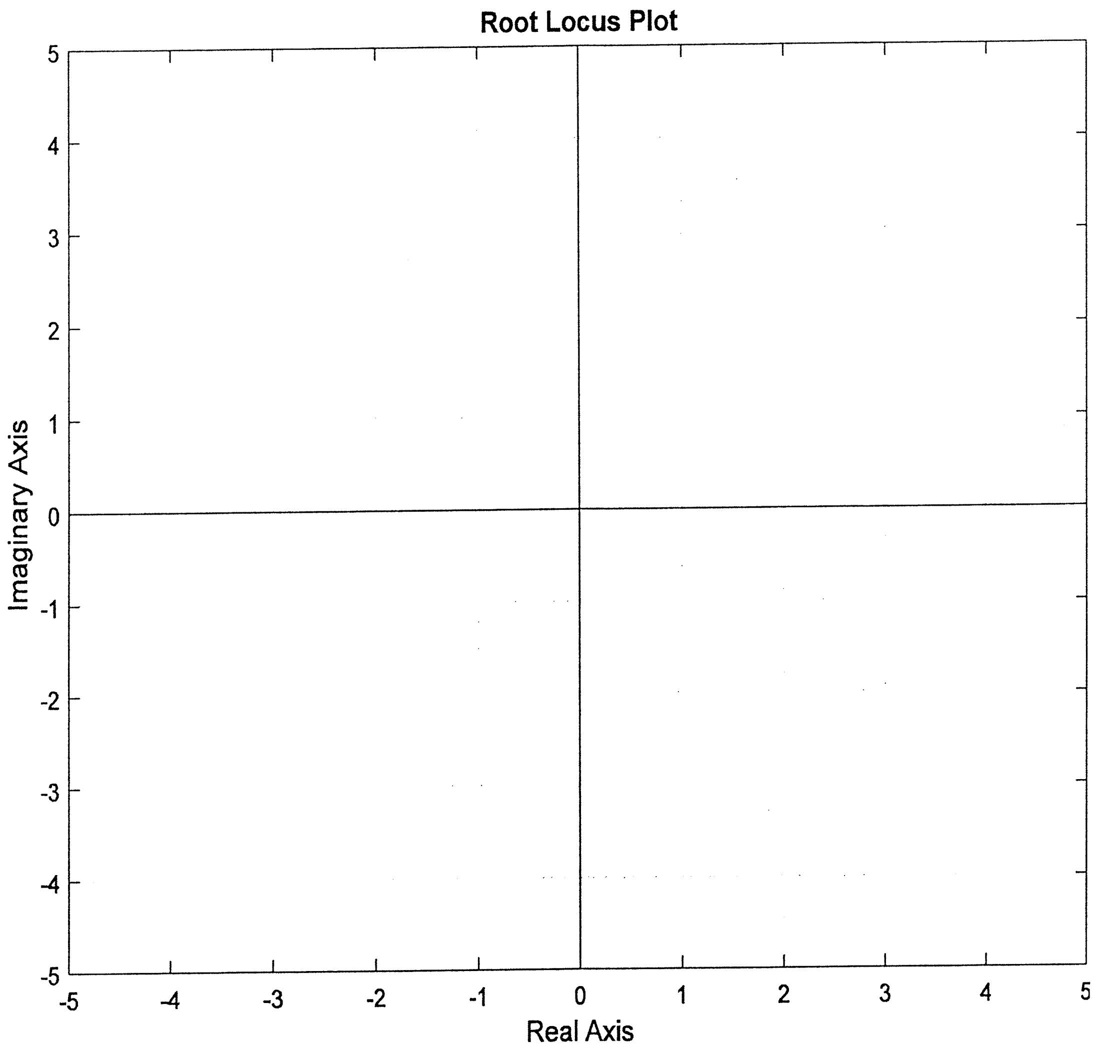


Figure Q3.1 – Place Root Locus Plot for Question 3 Here

Question 4

Second Order Dominant Poles Model, System Type and Error Analysis, Transient Response Specifications, Effect of Additional Poles and Zeros on the Closed Loop Response.

Recall the general form of a factorized transfer function:

$$G(s) = \frac{K \cdot \prod_i^m (s - z_i)}{\prod_i^n (s - p_i)}$$

Next, consider a certain 5th order process that has:

- gain factor, K, of 5,000
- the following singularities:

Poles:	$-1 \pm j4$	-15	$-20 \pm j2$
Zeros:		-20	

1. **(5 marks)** Find an appropriate 2nd order model for this process and write its transfer function in the standard form as shown below. Clearly identify parameters of interest, i.e. damping ratio, frequency of natural oscillations and DC gain.

$$G_m(s) = K_{dc} \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

2. **(5 marks)** For the system in question, determine the System Type. Next, calculate the Steady State Error, $e_{ss(step)\%}$, for a unit step input, and Steady State Error, $e_{ss(ramp)}$, for a unit ramp input.

3. **(5 marks)** For the system in question, estimate the following unit step response specifications: Percent Overshoot, PO, and Settling Time, $T_{settle(\pm 2\%)}$.

4. **(5 marks)** Describe briefly how these estimates would be affected if the system zero were located at -2.

Question 5

Basic System Representation using Transfer Functions; Second Order Model, Dynamic System Response, Response Specifications.

Consider the electric circuit, a two-port, shown in Figure Q5.1, where its components have the following values: $R = 10\ \Omega$, $C = 6.25\ mF$, and $L = 100\ mH$.

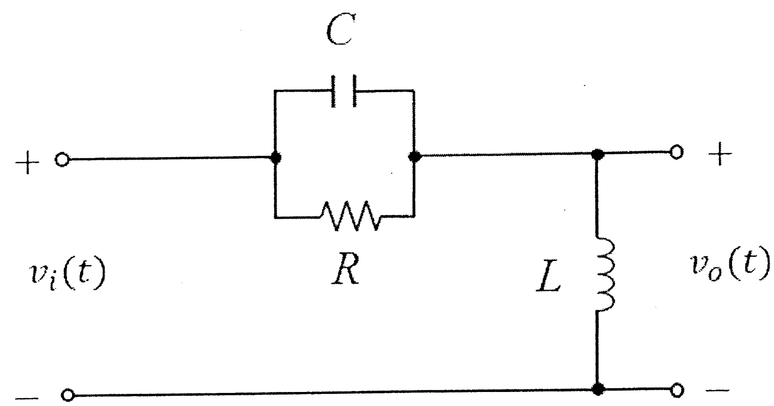


Figure Q5.1 – Two-Port Network

1. **(5 marks)** Find the transfer function of this two-port, $G(s) = \frac{V_o(s)}{V_i(s)}$
2. **(5 marks)** If the input voltage signal $v_i(t)$ is a normalized unit step function, find the analytical function of the network response, $v_o(t)$.
3. **(5 marks)** Evaluate the following parameters of the step response: its steady state value, $v_o(\infty)$, its initial value, $v_o(0)$, its Percent Overshoot, PO, and Settling Time, $T_{settle(\pm 2\%)}$.
4. **(5 marks)** What would be the appropriate value of the resistor R to achieve the Settling Time of less than 0.2 seconds ($T_{settle(\pm 2\%)}$<0.2)?

Question 6

System Stability in the s-domain and in the frequency domain: Nyquist Criterion and Routh-Hurwitz Criterion.

Consider a unit feedback loop system under Proportional Control (gain K). The transfer function of its open loop is described as follows:

$$G_{open}(s) = K \cdot \frac{(1+s)}{s(s-5)}$$

1. **(7 marks)** Sketch a polar plot of the normalized open loop transfer function $\frac{G_{open}(j\omega)}{K}$; calculate all relevant coordinates, including the crossovers with the Imaginary and Real axis, and clearly indicate the direction of increasing frequency on the resulting polar plot.
2. **(8 marks)** Apply the Nyquist criterion of stability to establish the range of values of the Proportional Controller gain, K_p, that will result in a stable closed loop system response. NOTE: clearly show the chosen clockwise (CW) Γ path in the s-plane, and the resulting Nyquist Contour.
3. **(5 marks)** Verify the results using Routh-Hurwitz Criterion of Stability.

Question 7

Controller Design in Frequency Domain – Lead Controller, Step Response Specifications.

Consider a closed loop unit feedback control system with the following uncompensated open loop transfer function:

$$G(s) = \frac{100}{s(s+5)(s+10)}$$

Its frequency response plots are shown in Figure Q7.1. The closed loop system is to be compensated by a **Lead Controller** with the transfer function described below as $G_c(s)$. Note that for $G_c(s)$ to represent the Lead-type Controller transfer function, the zero at $-\frac{a_0}{a_1}$ has to be closer to the Imaginary axis than the pole at $-\frac{1}{b_1}$:

$$G_c(s) = \frac{a_1 s + a_0}{b_1 s + 1}$$

The design requirements are:

- The Steady State Error for the unit ramp input for the compensated closed loop system is equal 0.1;
- The compensated Phase Margin is to be 50 degrees ($\Phi_m = 50^0$) and the Crossover Frequency for the Phase Margin is to be 10 rad/sec ($\omega_{cp} = 10$).

1. **(15 marks)** Calculate the appropriate Lead Controller parameters and the Controller transfer function. Show the general shape of the compensated frequency response by overlaying it on top of the uncompensated plot in Figure Q7.1.
2. **(5 marks)** Estimate the following time response specifications of the compensated closed loop unit step response: Percent Overshoot, Settling Time and Steady State Error - PO, $T_{settle(\pm 2\%)}$ and $e_{ss(step)\%}$, respectively.

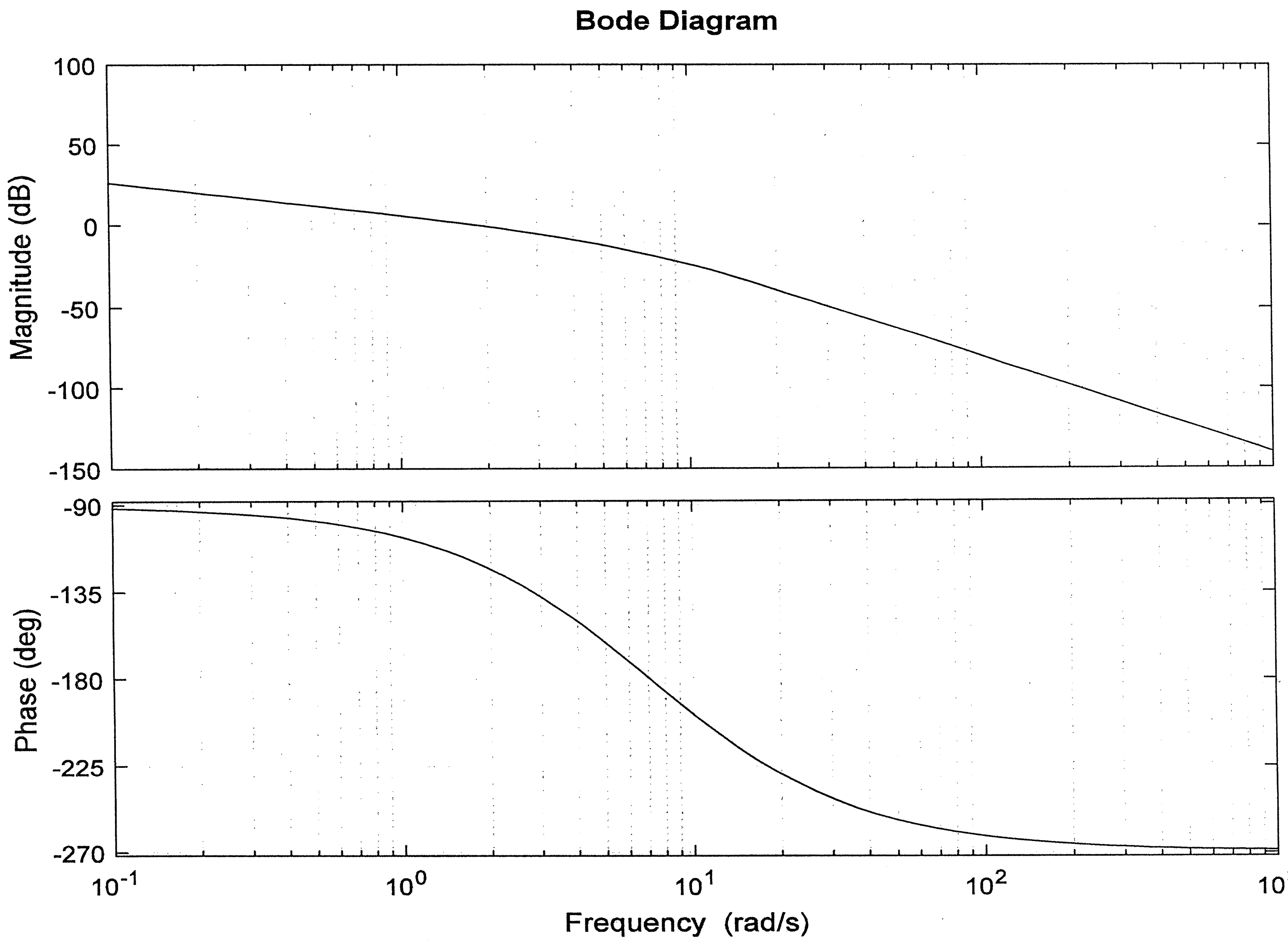


Figure Q7.1 – Open Loop Uncompensated System Frequency Response

Question 8

State Space System Representations and Canonical Forms, Transfer Functions Realizations, Mason's Gain Formula.

PART A (10 marks)

Consider a linear process described by the following transfer function:

$$G(s) = \frac{5s^2 + 3s + 1}{s^3 + 2s^2 + 4s + 1}$$

1. **(5 marks)** Show one possible canonical form of a state variable representation for this system by completing the signal flow graph in Figure Q8.1 – clearly show the required gain values for all its branches.
2. **(5 marks)** Next, derive a set of the corresponding state equations - follow the choice of state variables as indicated in Figure Q8.1. Identify the canonical form in question.

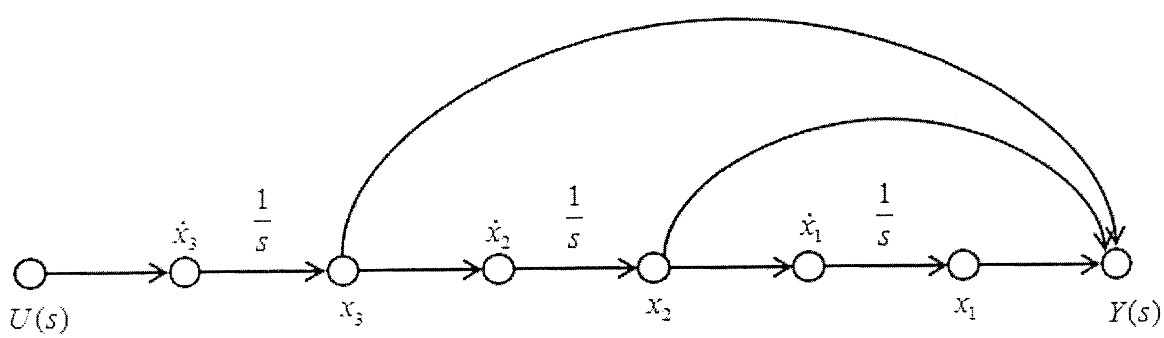


Figure Q8.1 – Complete the Signal Flow Graph Representing G(s)

PART B (10 marks)

A certain system is described by a state space model in an **observable canonical form** as follows:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -4 \\ 1 & 0 & -6 \\ 0 & 1 & -8 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 8 \\ -5 \\ -3 \end{bmatrix} \cdot u$$

$$y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + [1] \cdot u$$

Determine the system transfer function $G(s) = \frac{Y(s)}{U(s)}$. NOTE: Because of the canonical form, the transfer function can be written *by inspection.*