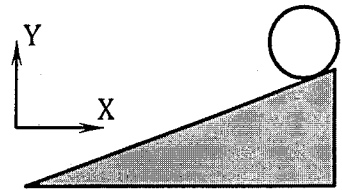## 98-Phys-A1, Classical Mechanics

3 hours duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.

2. This is a CLOSED BOOK EXAM.
   One of two calculators is permitted, Casio or Sharp approved models
   Candidates are allowed one aid sheet 8.5" x 11"

3. FIVE (5) questions constitute a complete exam paper.
   The first five questions as they appear in the answer book will be marked.

4. Each question is of equal value.

5. Most questions require an answer in essay format.
   Clarity and organization of the answer are important.

1. A disk, of 1-m diameter and 2-kg mass, was initially held at the top of a slope of 30-degree; see Figure. After released from rest, the disk rolls down the slope without slipping and maintains its plane-of-motion in the X-Y plane shown in the Figure.
(a) What type of constraint is best suited to describe the motion of the disk as it rolls down the slope?
(b) How many degrees-of-freedom are needed to describe the motion of the disk?
(c) Use the Lagrange's equations to find the force of constraint.

2. Consider the motion of a particle of mass m constrained on a cylindrical surface of radius R; see Figure. The particle is subjected to a force that is proportional to its radial distance and directed towards the origin:
$\vec{F} = -k\vec{r}$. Write down the Hamiltonian of the particle's motion and derive the equations of motion based on the Hamiltonian.
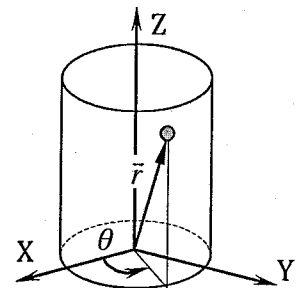
3. The mechanical system shown in the Figure consists of a crank AB, a slider B and a slotted rod CD. At the instant shown, the crank has an angular velocity and acceleration, $\vec{\omega}_{AB} = 3\vec{k}$ rad/s, $\vec{\alpha}_{AB} = -1\vec{k}$ rad/s, respectively. Determine the velocity and acceleration of the slider B <u>relative to the slotted rod</u> at this instant.
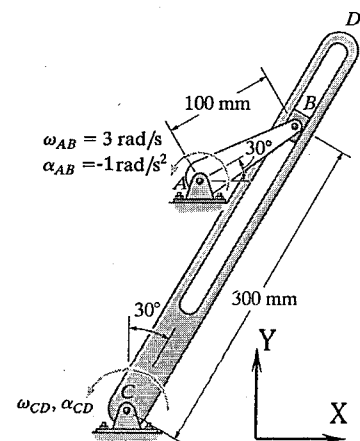
4. A cart of 1 kg, a ball of 0.1 kg, and an elastic cord *connecting the ball and the cart*, are in a weightless environment; see Figure. The cart has a shape of a symmetric trapezoid with its left and right edges at a 60-deg angle from the vertical. It is also attached with four rollers (two on top and two on bottom) so that it can move freely (only) horizontally. Initially at $t = 0$, the system is in static equilibrium with the ball held in the shown position. At this instant, the elastic cord is fully stretched and exerts a force of $\vec{F}_s = -1\vec{i}$ N force on the cart. Then ($t > 0$), the ball is released from rest. The ball hits the cart at $t = 2$ seconds. Note that the elastic cord is located at exactly the same level as the center of mass (C.M.) of the cart. Answer the following questions.
(a) Will the cart move right after the instant the ball is released ($t = 0^+$)?
(b) What are the impulsive forces at the instant the ball hits the cart?
(c) Assume the ball sticks to the cart surface, what is the speed of the cart right after the ball hits the cart?

$$t = 0 \qquad 0 < t < 2 \qquad t = 2$$
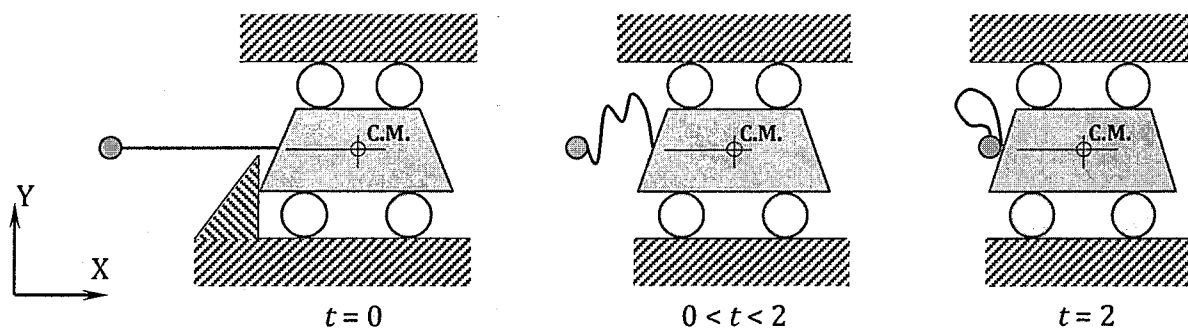
5. In the cable-pulley system shown here, block A is moving upwards at a speed of 5 m/s and block C is moving downwards at a speed of 2.5 m/s. What is the speed of block B?
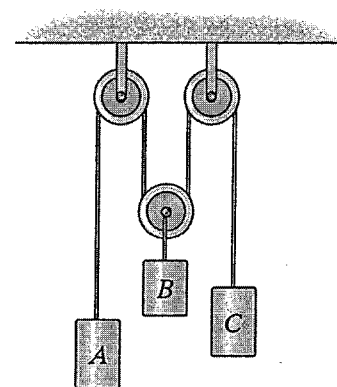
6. At the instant shown, the cable attached to point B snaps. The rod has a mass of 1 kg. Determine the angular acceleration of the rod AB at this moment.