

National Exams May 2013

04-BS-6: Mechanics of Materials

3 hours duration

Notes:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper a clear statement of any assumptions made.
2. Candidates may use one of two calculators, the Casio or Sharp approved models.

This is a Closed Book exam. However candidates are permitted to bring the following into the examination room:

- ONE aid sheet 8.5" x 11" hand-written on both sides containing notes and formulae.
Example problems and solutions to problems are not allowed!
3. Any five questions (out of 8 given) constitute a complete paper. Only the first five questions as they appear in your answer book will be marked.
 4. All questions are of equal value.

NOTE: The aid sheet must be handed in with the exam!

Your exam will not be marked if you do not hand in an aid sheet, unless there is a signed statement by the exam invigilator stating that no aid sheet was used for the exam.

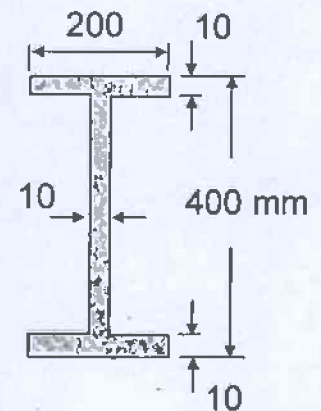
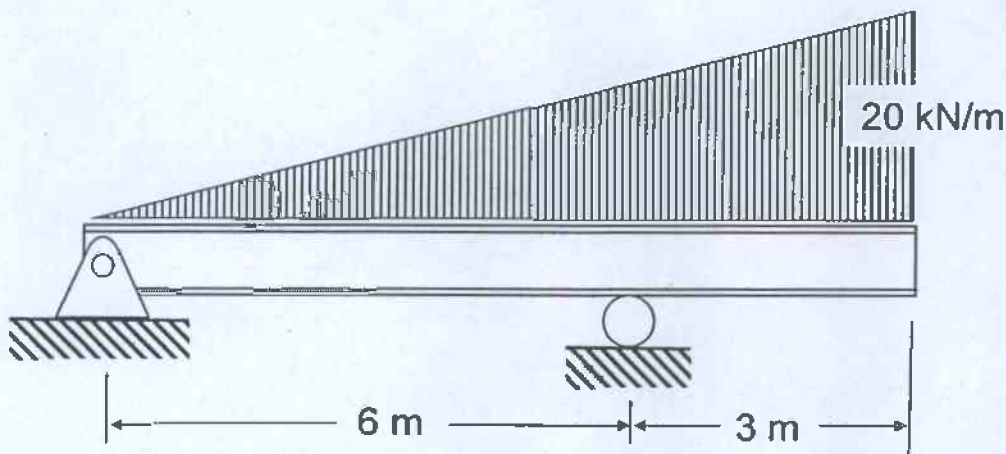
Question 1: A simply supported beam with an overhang supports a triangularly distributed load with a maximum intensity of 20 kN/m. The beam has the cross-section shown below, and is made from steel having an allowable normal stress of 240 MPa and allowable shear stress of 60 MPa. The elastic modulus of the steel equals 200 GPa.

[20 marks]

(a) use the method of integration to determine the deflection and slope of the beam at the end of the overhang.

[bonus]

(b) indicate whether deflection of the beam between the two supports is upwards or downwards (or both) and give reasons for your answer.

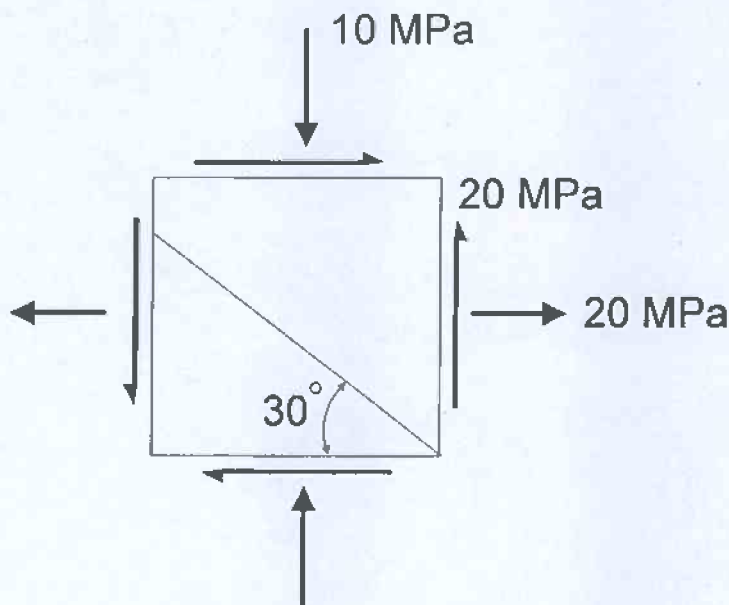


beam cross-section
(all dimensions in mm)

No credit will be given for a solution using the principle of superposition, when combinations of existing solutions are used to find an answer.

Question 2: For an element in a state of plane stress subjected to the normal and shear stresses [20 marks] shown below, use the Mohr's circle solution (*not* the transformation equations) to determine the following:

- (a) the stress components acting on the inclined plane (orientated 30° from the horizontal as shown), showing your answer on a properly oriented element.
- (b) the maximum in-plane shear stress of the element (and associated normal stresses) and orientation of the corresponding planes. Once again, show your answer on a sketch of a properly oriented element.



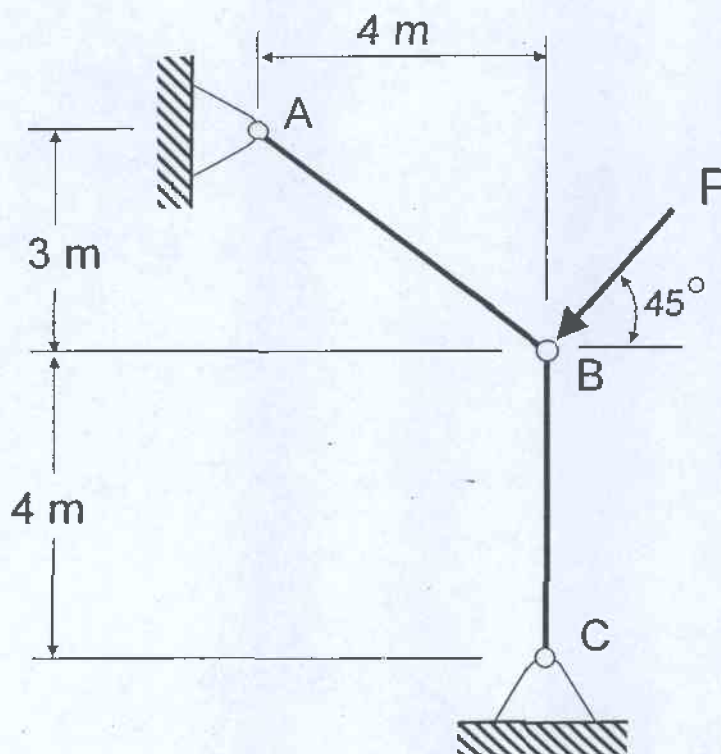
Note that credit will **only** be given for a **solution using Mohr's circle**. This means that you need to draw a Mohr's circle based on the stress components given in this problem. Remember to show numbers on your circle. Your **calculations** must be based on the geometry of your circle. So use your calculator. In other words, you are expected to use trigonometry to construct your Mohr's circle. Do not give a graphical solution that is scaled off! The stress transformation equations can only be used to check your answer.

Question 3: The truss system ABC is constructed from two steel rods and supports an inclined load P applied at B as shown below. The two rods (AB and BC) have a 120 mm diameter and are pinned at both ends. The rods are made of steel with a yield strength of 350 MPa and elastic modulus of 200 GPa.

[20 marks]

Determine the largest load the structure can support.

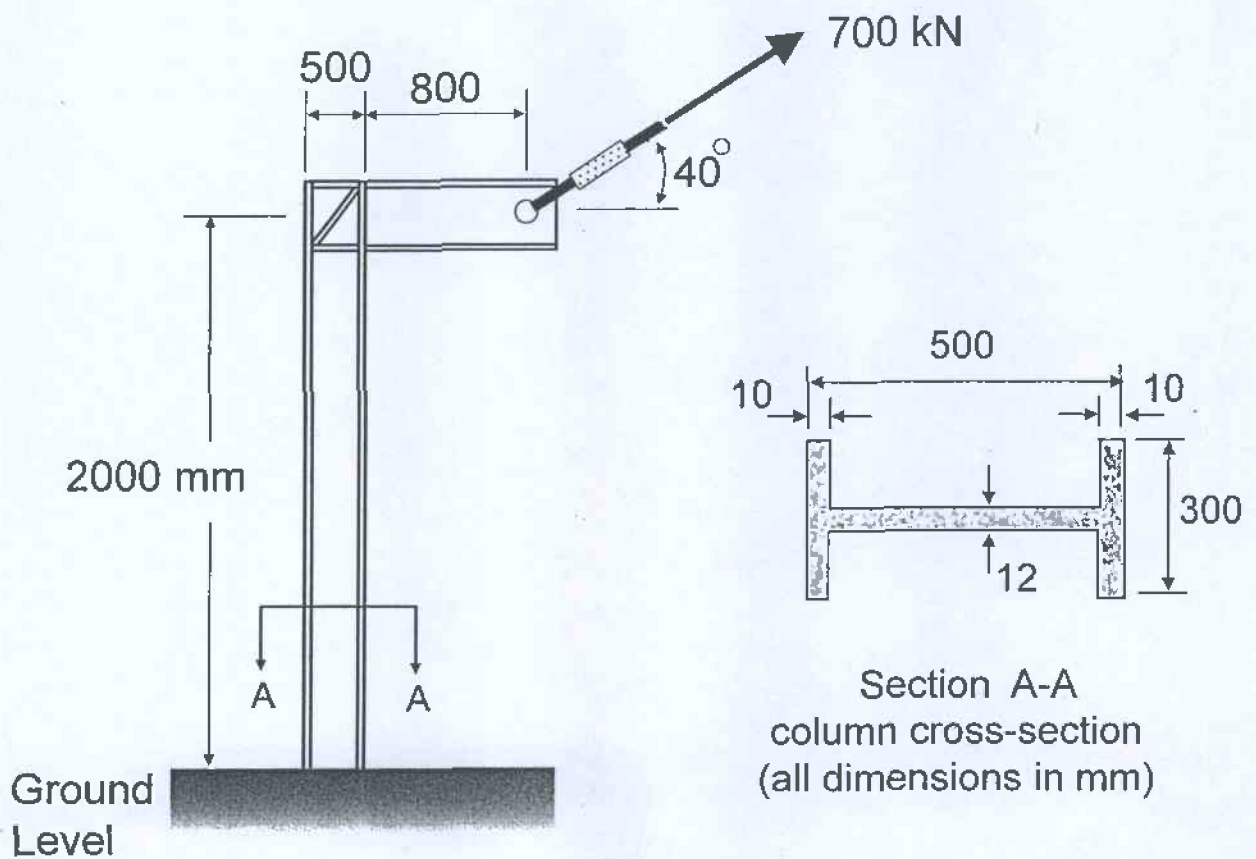
Consider in-plane buckling only and use a safety factor of 2 for Euler buckling. Do not use a safety factor for yielding of the steel.



Note: $A_{\text{circle}} = \pi r^2$ and $I_{\text{circle}} = \pi r^4 / 4$

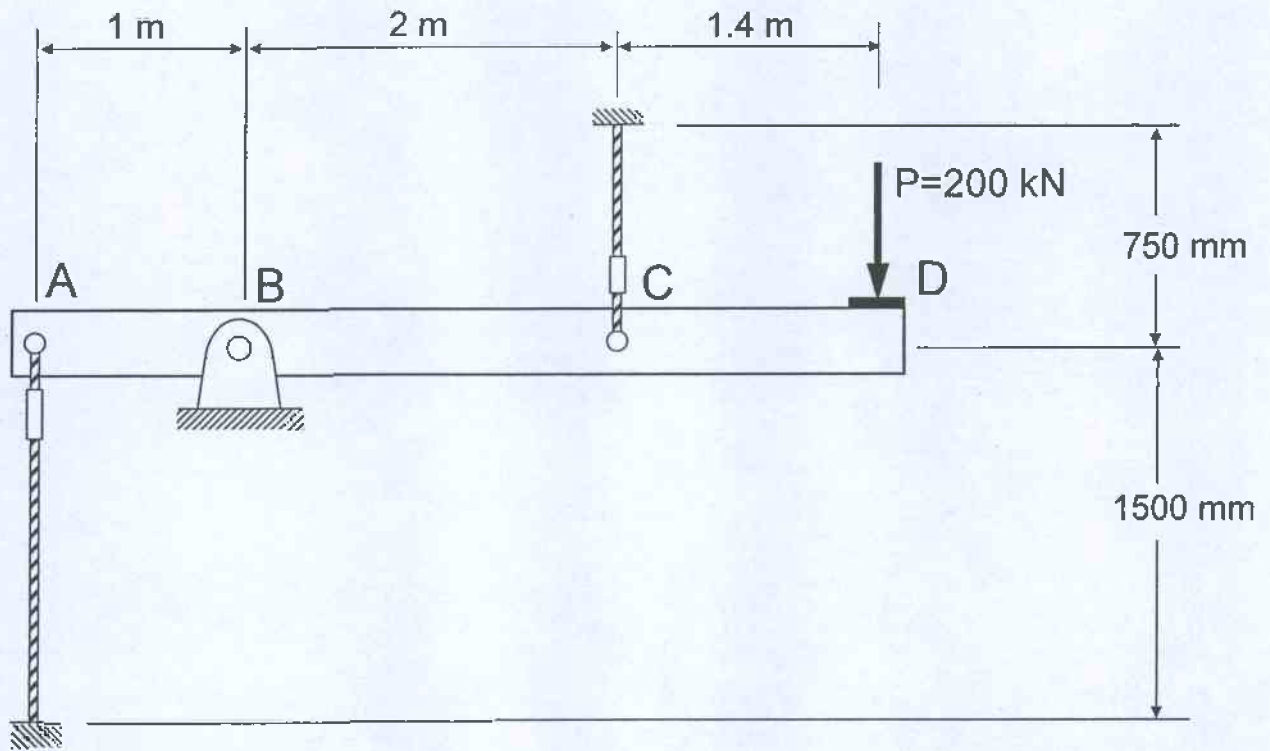
Question 4: An inclined load of 700 kN is applied to a column fixed at the base as shown. The column section has the dimensions given below and is made of steel with a yield stress of 400 MPa and elastic modulus of 200 GPa.
 [20 marks]

- (a) compute the normal stress distribution at the base (ground level) of the column section and show this on a sketch.
- (b) compute the maximum shear stress at the base of the column section.



Question 5: A rigid beam ABCD is supported by a pin at B and two cables located at A and C. The cables have a 30 mm diameter and are made of high strength steel with a yield strength of 800 MPa and elastic modulus of 200 GPa. The rigid beam is designed to resist a vertical load P applied at D equal to 200 kN.

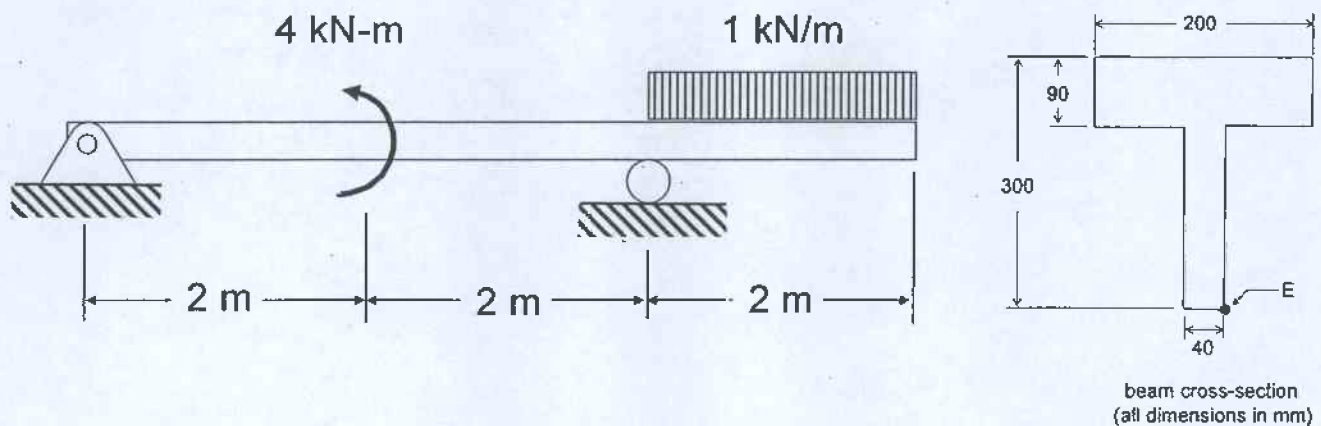
- [12 marks] (a) find the forces developed in each cable
- [4 marks] (b) find the corresponding vertical displacement at point D where the load is applied
- [4marks] (c) find the shear stress in the pin at B given that the pin has a diameter of 50 mm and is loaded in double shear



Question 6: A simply supported beam with an overhang is subjected to a concentrated couple at midspan and a uniformly distributed load acting on the overhang as shown. [20 marks] The beam has the cross-section given and is made of steel with a yield strength of 350 MPa and shear stress at yield of 75 MPa. The elastic modulus of the steel is 200 GPa.

Determine the shear and moment throughout the beam as functions of x (with x starting at the left hand support). This means that you need to give formula(s) for $V(x)$ and $M(x)$ along the length of the beam.

Next construct the shear force and bending moment diagrams. Remember to label points of maximum positive and negative bending moment, as well as any inflection points. Show your work by indicating exactly how you obtained your answer.



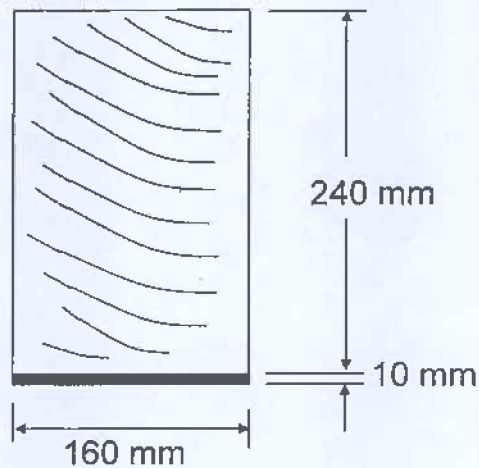
Remember that it is important you set this problem up correctly by calculating the correct reaction forces at the support(s).

No credit will be given for a solution using the principle of superposition, when combinations of existing solutions are used to find an answer.

Question 7:
[20 marks]

A composite beam is made from a rectangular wood section (160 mm wide by 240 mm deep) that is reinforced with a 10 mm thick steel plate as shown. The beam is simply supported with a span of 4 m and supports a uniformly distributed load w equal to 10 kN/m. The wood has an allowable normal stress of 8 MPa and elastic modulus of 10 GPa, while the steel has an allowable normal stress of 240 MPa and elastic modulus of 210 GPa.

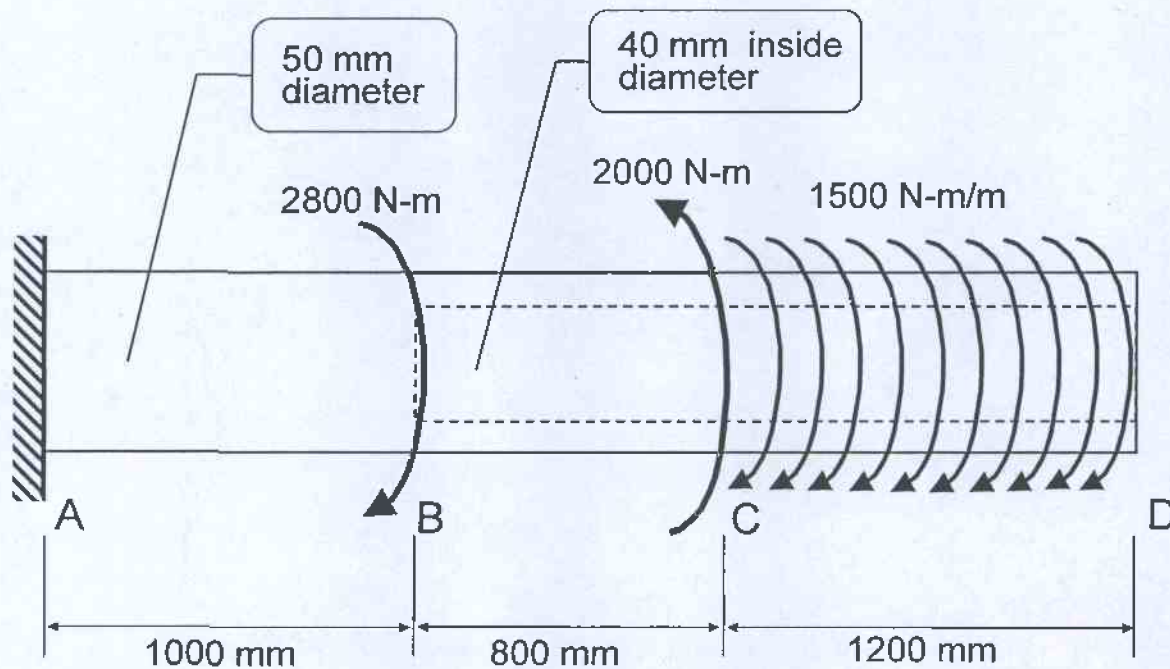
- (a) Determine the maximum normal stress in the wood and steel.
- (b) Give the value of distributed load w that will cause failure of the beam.
- (c) Explain whether the steel plate needs to be bonded to the wood section to have composite behaviour and give reasons for your answer.

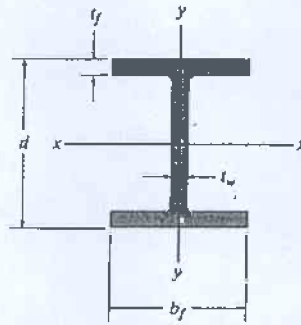


Question 8: A circular shaft ABCD is subjected to two concentrated torques (acting at B and C) and a uniformly distributed torque (acting along CD) as shown. The shaft is fixed at A. Part of the shaft (BCD) is hollow and the entire shaft is made of steel having a shear modulus $G = 80 \text{ GPa}$ and a yield stress of 150 MPa . Dimensions (diameter and length) and magnitude of the torques are given in the diagram.

[12 marks] (a) determine the maximum shear stress in the shaft and sketch the corresponding variation of shear stress along the shaft radius at this point.

[8 marks] (b) find the angle of twist at the end of the shaft (point D) and give your answer in degrees.





Wide-Flange Sections or W Shapes SI Units											
Designation	Area A	Depth d	Web thickness t _w	Flange		x-x axis			y-y axis		
				width b _f	thickness t _f	I	S	r	I	S	r
W310 × 129	16 500	318	13.10	308.0	20.6	308	1940	137	100	649	77.8
W310 × 74	9 480	310	9.40	205.0	16.3	165	1060	132	23.4	228	49.7
W310 × 67	8 530	306	8.51	204.0	14.6	145	948	130	20.7	203	49.3
W310 × 39	4 930	310	5.84	165.0	9.7	84.8	547	131	7.23	87.6	38.3
W310 × 33	4 180	313	6.60	102.0	10.8	65.0	415	125	1.92	37.6	21.4
W310 × 24	3 040	305	5.59	101.0	6.7	42.8	281	119	1.16	23.0	19.5
W310 × 21	2 680	303	5.08	101.0	5.7	37.0	244	117	0.986	19.5	19.2
W250 × 149	19 000	282	17.30	263.0	28.4	259	1840	117	86.2	656	67.4
W250 × 80	10 200	256	9.40	255.0	15.6	126	984	111	43.1	338	65.0
W250 × 67	8 560	257	8.89	204.0	15.7	104	809	110	22.2	218	50.9
W250 × 58	7 400	252	8.00	203.0	13.5	87.3	693	109	18.8	185	50.4
W250 × 45	5 700	266	7.62	148.0	13.0	71.1	535	112	7.03	95	35.1
W250 × 28	3 620	260	6.35	102.0	10.0	39.9	307	105	1.78	34.9	22.2
W250 × 22	2 850	254	5.84	102.0	6.9	28.8	227	101	1.22	23.9	20.7
W250 × 18	2 280	251	4.83	101.0	5.3	22.5	179	99.3	0.919	18.2	20.1
W200 × 100	12 700	229	14.50	210.0	23.7	113	987	94.3	36.6	349	53.7
W200 × 86	11 000	222	13.00	209.0	20.6	94.7	853	92.8	31.4	300	53.4
W200 × 71	9 100	216	10.20	206.0	17.4	76.6	709	91.7	25.4	247	52.8
W200 × 59	7 580	210	9.14	205.0	14.2	61.2	583	89.9	20.4	199	51.9
W200 × 46	5 890	203	7.24	203.0	11.0	45.5	448	87.9	15.3	151	51.0
W200 × 36	4 570	201	6.22	165.0	10.2	34.4	342	86.8	7.64	92.6	40.9
W200 × 22	2 860	206	6.22	102.0	8.0	20.0	194	83.6	1.42	27.8	22.3
W150 × 37	4 730	162	8.13	154.0	11.6	22.2	274	68.5	7.07	91.8	38.7
W150 × 30	3 790	157	6.60	153.0	9.3	17.1	218	67.2	5.54	72.4	38.2
W150 × 22	2 860	152	5.84	152.0	6.6	12.1	159	65.0	3.87	50.9	36.8
W150 × 24	3 060	160	6.60	102.0	10.3	13.4	168	66.2	1.83	35.9	24.5
W150 × 18	2 290	153	5.84	102.0	7.1	9.19	120	63.3	1.26	24.7	23.5
W150 × 14	1 730	150	4.32	100.0	5.5	6.84	91.2	62.9	0.912	18.2	23.0