# National Exams December 2017

## 16-Mec-A3, SYSTEM ANALYSIS AND CONTROL

3 hours duration

**Notes:**

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
2. Candidates may use a Casio **or** Sharp approved calculator. This is a **closed book** exam. No aids other than semi-log graph papers are permitted.
3. Any four (4) questions constitute a complete paper. Only the first four (4) questions as they appear in your answer book will be marked.
4. All questions are of equal value.

1) (a) Obtain the unit-impulse response and the unit-step response of a unity-feedback system whose open loop transfer function is

$$G(s) = \frac{2s+1}{s^2}$$

(b) Consider the differential equation system given by

$$y'' + 3y' + 2y = 0, \qquad y(0) = 0.1, \quad y'(0) = 0.05$$

Obtain the response $y(t)$, subject to the given initial condition.

2) (a) Determine the range of $K$ for stability of a unity-feedback control system whose open-loop transfer function is

$$G(s) = \frac{K}{s(s+1)(s+2)}$$

(b) Consider the unity-feedback control system with the following open-loop transfer function

$$G(s) = \frac{10}{s(s-1)(2s+3)}$$

Is this system stable?

3) (a) Consider a unity-feedback control system with the closed-loop transfer function

$$\frac{C(s)}{R(s)} = \frac{Ks+b}{s^2+as+b}$$

Determine the open-loop transfer function $G(s)$.

Show that the steady-state error in the unit-ramp response is given by

$$e_{ss} = \frac{1}{K_v} = \frac{a - K}{b}$$

Where $e_{ss}$ is the steady-state error, and $K_v$ is the velocity error constant.

(b) Consider a unity-feedback control system whose open-loop transfer function is

$$G(s) = \frac{K}{s(Js + B)}$$

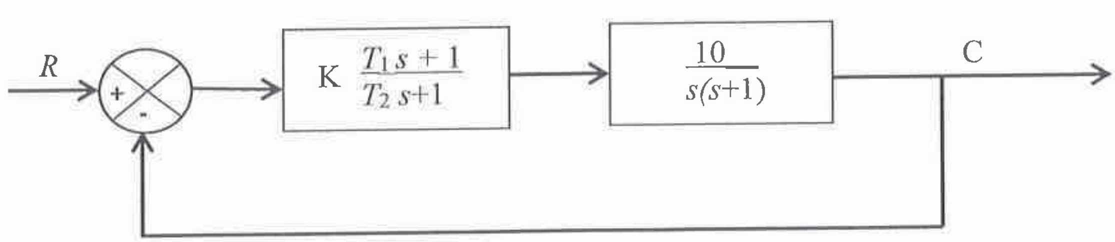
Discuss the effects that varying the values of $K$ and $B$ has on the steady-state error in unit-ramp response. Sketch typical unit-ramp response curves for a small value, medium value, and large value of $K$, assuming that $B$ is constant.

4) Plot the root loci for a system with

$$G(s) = \frac{K}{(s^2 + 2s + 2)(s^2 + 2s + 5)}, \qquad H(s) = 1$$

Determine the points where the root loci cross the $j\omega$ axis.

5) Determine the values of $K$, $T_1$, and $T_2$ of the system shown in the diagram below so that the dominant closed-loop poles have the damping ratio $\zeta = 0.5$ and the undamped natural frequency $\omega_n = 3$ rad/sec



6) (a) Plot the Bode diagram of

$$G(s) = \frac{10\,(s^2 + 0.4\,s + 1)}{s\,(s^2 + 0.8\,s + 9)}$$

(b) Consider a unity-feedback control system with the open-loop transfer function

$$G(s) = \frac{K}{s\,(s^2 + s + 4)}$$

Determine the value of the gain $K$ such that the phase margin is 50°. What is the gain margin with this gain $K$?

# Table of Laplace Transforms

| $f(t)$ | $\mathcal{L}[f(t)] = F(s)$ | |
|---|---|---|
| $1$ | $\frac{1}{s}$ | (1) |
| $e^{at} f(t)$ | $F(s-a)$ | (2) |
| $\mathcal{U}(t-a)$ | $\frac{e^{-as}}{s}$ | (3) |
| $f(t-a)\mathcal{U}(t-a)$ | $e^{-as}F(s)$ | (4) |
| $\delta(t)$ | $1$ | (5) |
| $\delta(t-t_0)$ | $e^{-st_0}$ | (6) |
| $t^n f(t)$ | $(-1)^n \frac{d^n F(s)}{ds^n}$ | (7) |
| $f'(t)$ | $sF(s) - f(0)$ | (8) |
| $f^n(t)$ | $s^n F(s) - s^{(n-1)} f(0) - \cdots - f^{(n-1)}(0)$ | (9) |
| $\int_0^t f(x) g(t-x)\, dx$ | $F(s)G(s)$ | (10) |
| $t^n \ (n = 0, 1, 2, \dots)$ | $\frac{n!}{s^{n+1}}$ | (11) |
| $t^x \ (x \geq -1 \in \mathbb{R})$ | $\frac{\Gamma(x+1)}{s^{x+1}}$ | (12) |
| $\sin kt$ | $\frac{k}{s^2+k^2}$ | (13) |
| $\cos kt$ | $\frac{s}{s^2+k^2}$ | (14) |
| $e^{at}$ | $\frac{1}{s-a}$ | (15) |
| $\sinh kt$ | $\frac{k}{s^2-k^2}$ | (16) |
| $\cosh kt$ | $\frac{s}{s^2-k^2}$ | (17) |
| $\frac{e^{at}-e^{bt}}{a-b}$ | $\frac{1}{(s-a)(s-b)}$ | (18) |
| $\frac{ae^{at}-be^{bt}}{a-b}$ | $\frac{s}{(s-a)(s-b)}$ | (19) |
| $te^{at}$ | $\frac{1}{(s-a)^2}$ | (20) |
| $t^n e^{at}$ | $\frac{n!}{(s-a)^{n+1}}$ | (21) |
| $e^{at} \sin kt$ | $\frac{k}{(s-a)^2+k^2}$ | (22) |
| $e^{at} \cos kt$ | $\frac{s-a}{(s-a)^2+k^2}$ | (23) |
| $e^{at} \sinh kt$ | $\frac{k}{(s-a)^2-k^2}$ | (24) |
| $e^{at} \cosh kt$ | $\frac{s-a}{(s-a)^2-k^2}$ | (25) |
| $t \sin kt$ | $\frac{2ks}{(s^2+k^2)^2}$ | (26) |
| $t \cos kt$ | $\frac{s^2-k^2}{(s^2+k^2)^2}$ | (27) |
| $t \sinh kt$ | $\frac{2ks}{(s^2-k^2)^2}$ | (28) |
| $t \cosh kt$ | $\frac{s^2-k^2}{(s^2-k^2)^2}$ | (29) |
| $\frac{\sin at}{t}$ | $\arctan \frac{a}{s}$ | (30) |
| $\frac{1}{\sqrt{\pi t}} e^{-a^2/4t}$ | $\frac{e^{-a\sqrt{s}}}{\sqrt{s}}$ | (31) |
| $\frac{a}{2\sqrt{\pi t^3}} e^{-a^2/4t}$ | $e^{-a\sqrt{s}}$ | (32) |
| $\operatorname{erfc}\left(\frac{a}{2\sqrt{t}}\right)$ | $\frac{e^{-a\sqrt{s}}}{s}$ | (33) |