National Exams **May 2016**

07-Elec-B1, Digital Signal Processing

3 hours duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.

2. This is a Closed Book exam.
   Candidates may use one of two calculators, the Casio or Sharp approved models. They are also entitled to one aid sheet with tables & formulas written both sides. No textbook excerpts or examples solved.

3. FIVE (5) questions constitute a complete exam.
   Clearly indicate your choice of any five of the six questions given otherwise the first five answers found will be considered your pick.

4. All questions are worth 12 points.
   See below for a detailed breakdown of the marking.

Marking Scheme

1. total = 12
2. (a) 4, (b) 4, (c) 4, total = 12
3. (a) 3, (b) 3, (c) 3, (d) 3, total = 12
4. (a) 5, (b) 3, (c) 4, total = 12
5. total = 12
6. (a) 6, (b) 6, total = 12

The number beside each part above indicates the points that part is worth

1.- Let $x[n]$ be a purely real sequence. You are given the following information about $x[n]$ and must determine what it is. Even if you are unable to specify $x[n]$ fully, you may receive partial credit by describing which features of $x[n]$ are determined by each clue.

(a) $x[-n]$ is a causal sequence.

(b) Let $v[n] = x[n-3]$. The discrete-time Fourier transform $V(e^{j\omega})$ is purely imaginary.

(c) $\frac{1}{2\pi}\int_{-\pi}^{\pi}\left|X(e^{j\omega})\right|^2 d\omega = 28$.

(d) $\lim_{z\to 0} X(z) = -1$.

(e) $\frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{-j\omega}d\omega = 2$.

(f) $x[-2] > 0$.

Note: Consult tables and formulas provided at the end of this exam as needed

2.- The figure shows a continuous-time filter that is implemented using an LTI discrete-time ideal lowpass filter with frequency response over $-\pi \leq \omega \leq \pi$ given by

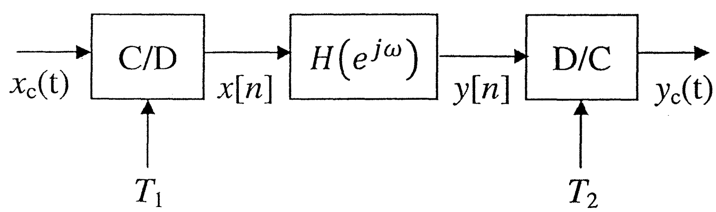
$$H(e^{j\omega}) = \begin{cases} 1, & |\omega| < \omega_c \\ 0, & \omega_c < |\omega| \leq \pi. \end{cases}$$

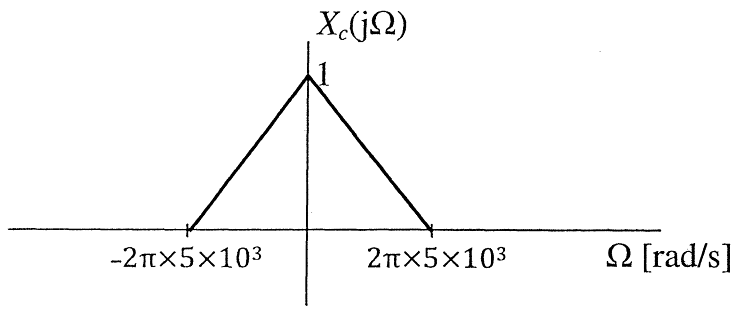
If the continuous-time Fourier transform of $x_c(t)$, namely $X_c(j\Omega)$, is as shown in the figure and $\omega_c = \pi/5$, sketch and label the Fourier transforms of $x[n]$, $y[n]$ and $y_c(t)$, this is $X(e^{j\omega})$, $Y(e^{j\omega})$ and $Y_c(j\Omega)$, for each of the following cases:

(a) $1/T_1 = 1/T_2 = 2 \times 10^4\ s^{-1}$

(b) $1/T_1 = 4 \times 10^4\ s^{-1}$, $1/T_2 = 10^4\ s^{-1}$

(c) $1/T_1 = 10^4\ s^{-1}$, $1/T_2 = 3 \times 10^4\ s^{-1}$





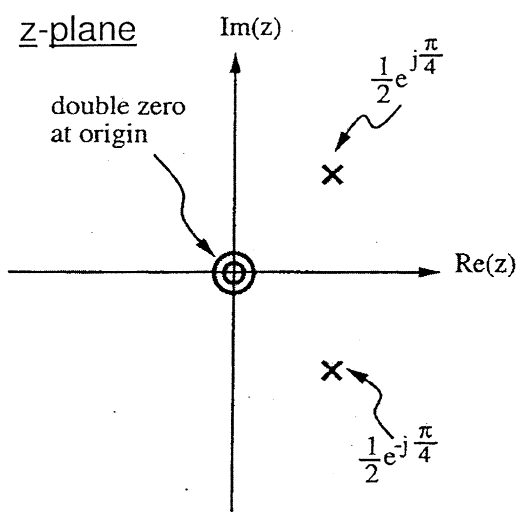
3.- A causal and stable LTI system has system function H(z). The pole-zero plot for H(z) is shown in the figure below.

(a) What is the region of convergence (ROC) for H(z)? Justify answer.

(b) Is the system impulse response h[n] real? Justify your answer.

(c) What is the pole-zero plot for the z-transform of $\left(\frac{1}{2}\right)^n h[n]$?

(d) What is the pole-zero plot for the z-transform of $\left(\frac{j}{2}\right)^n h[n]$?



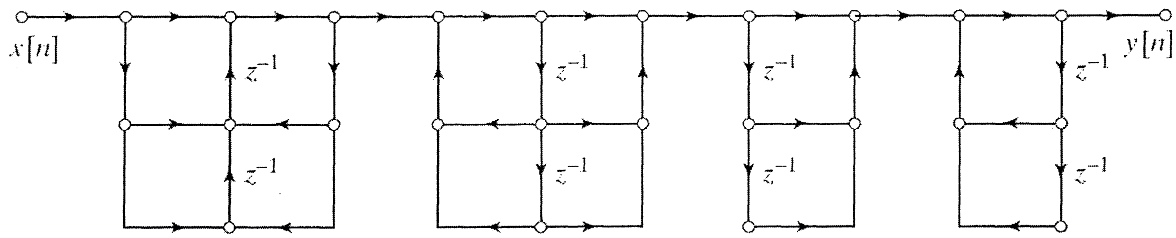
Pole-zero plot for H(z)

Additional information (not necessarily required): $H(1) = \frac{4}{5-2\sqrt{2}}$.

4.- An LTI system with system function

$$H(z) = \frac{0.2(1+z^{-1})^6}{\left(1-2z^{-1}+\frac{7}{8}z^{-2}\right)\left(1+z^{-1}+\frac{1}{2}z^{-2}\right)\left(1-\frac{1}{2}z^{-1}+z^{-2}\right)}$$

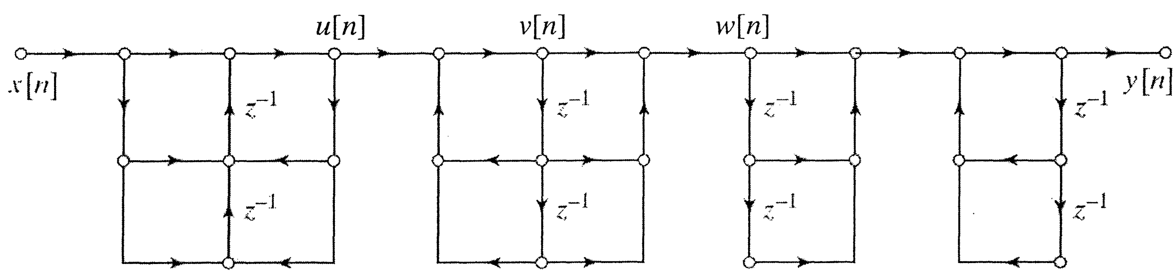
is to be implemented using a flow graph of the form shown in the figure below



(a) Fill in all the coefficients in the diagram above.
Is your solution unique? Explain

(b) Identify the structure for each section in cascade displayed in the flow graph.

(c) Using the coefficient assignment you established in part (a), for the node variables defined below $u[n]$, $v[n]$, $w[n]$ and output $y[n]$, write the set of difference equations that is represented by the flow graph.



5.- It is suggested that if you have a fast Fourier transform (FFT) subroutine for computing an N-point discrete Fourier transform (DFT), the inverse N-point DFT of a sequence $X[k]$ can be computed using this subroutine as follows:

1. Swap the real and imaginary parts of each DFT value $X[k]$.
2. Apply the FFT subroutine to this input sequence.
3. Swap the real and imaginary parts of the output sequence.
4. Scale the resulting sequence by $1/N$ to obtain the sequence $x[n]$, corresponding to the inverse DFT of $X[k]$.

Determine whether this procedure works as claimed.
If it does show why, if it doesn't propose a simple modification that will make it work.

Hint: Swapping of real & imaginary parts of a complex number A can be achieved through: $(-jA)^*$, where $(.)^*$ the star operator stands for complex conjugate

6.- The Kaiser window method is used for designing a highpass filter with cutoff frequency $\omega_c = 0.6\pi\ rad/sample$. From the Kaiser formulas seen below values of $\beta = 3.86$ and $M = 51$ are found to satisfy the filter specifications except in the neighborhood of π where the error rapidly increases well beyond the specified tolerance.

$$\beta = \begin{cases} 0.1102(A - 8.7), & A > 50, \\ 0.5842(A-21)^{0.4} + 0.07886(A - 21), & 21 \le A \le 50, \\ 0.0, & A < 21. \end{cases}$$

$where\ A = -20\log_{10}\delta$, and

$$M = \frac{A - 8}{2.285\Delta\omega}$$

$where\ \Delta\omega\ is\ the\ transition\ band\ width\ in\ the\ design\ specifications.$

(a) Provide the filter specifications, including the values of the tolerance δ, the stopband corner frequency ω_s and the passband corner frequency ω_p.

(b) What else is required in order to obtain a final design that satisfies the filter specifications for all values of ω? Explain.

TABLE 7.2 COMPARISON OF COMMONLY USED WINDOWS

Type of Window	Peak Side-Lobe Amplitude (Relative)	Approximate Width of Main Lobe	Peak Approximation Error, $20\log_{10}\delta$ (dB)	Equivalent Kaiser Window, β	Transition Width of Equivalent Kaiser Window
Rectangular	−13	$4\pi/(M+1)$	−21	0	$1.81\pi/M$
Bartlett	−25	$8\pi/M$	−25	1.33	$2.37\pi/M$
Hann	−31	$8\pi/M$	−44	3.86	$5.01\pi/M$
Hamming	−41	$8\pi/M$	−53	4.86	$6.27\pi/M$
Blackman	−57	$12\pi/M$	−74	7.04	$9.19\pi/M$

Additional Information

(Not all of this information is necessarily required today!)

DTFT Synthesis Equation	DTFT Analysis Equation
$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}d\omega$	$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$
Parseval's Theorem	**DFT Exponential Factor**
$E = \sum_{n=-\infty}^{\infty}\lvert x[n]\rvert^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi}\lvert X(e^{j\omega})\rvert^2 d\omega$	$W_N = e^{-j(2\pi/N)}$
Z-transform of a sequence $x[n]$	**Z-transform Properties**
$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$	$x[-n] \overset{Z}{\leftrightarrow} X(1/z),\quad ROC = \frac{1}{R_x}$ $z_o^n x[n] \overset{Z}{\leftrightarrow} X(z/z_o),\quad ROC = \lvert z_o\rvert R_x$

TABLE 3.2 SOME z-TRANSFORM PROPERTIES

Property Number	Section Reference	Sequence	Transform	ROC
		$x[n]$	$X(z)$	R_x
		$x_1[n]$	$X_1(z)$	R_{x_1}
		$x_2[n]$	$X_2(z)$	R_{x_2}
1	3.4.1	$ax_1[n] + bx_2[n]$	$aX_1(z) + bX_2(z)$	Contains $R_{x_1} \cap R_{x_2}$
2	3.4.2	$x[n - n_0]$	$z^{-n_0}X(z)$	R_x, except for the possible addition or deletion of the origin or ∞
3	3.4.3	$z_0^n x[n]$	$X(z/z_0)$	$\lvert z_0\rvert R_x$
4	3.4.4	$nx[n]$	$-z\frac{dX(z)}{dz}$	R_x
5	3.4.5	$x^*[n]$	$X^*(z^*)$	R_x
6		$\mathcal{Re}\{x[n]\}$	$\frac{1}{2}[X(z) + X^*(z^*)]$	Contains R_x
7		$\mathcal{Im}\{x[n]\}$	$\frac{1}{2j}[X(z) - X^*(z^*)]$	Contains R_x
8	3.4.6	$x^*[-n]$	$X^*(1/z^*)$	$1/R_x$
9	3.4.7	$x_1[n] * x_2[n]$	$X_1(z)X_2(z)$	Contains $R_{x_1} \cap R_{x_2}$

Properties of the Discrete Fourier Transform

	Finite-Length Sequence (Length N)	N-point DFT (Length N)
1.	$x[n]$	$X[k]$
2.	$x_1[n], x_2[n]$	$X_1[k], X_2[k]$
3.	$ax_1[n] + bx_2[n]$	$aX_1[k] + bX_2[k]$
4.	$X[n]$	$Nx[((-k))_N]$
5.	$x[((n-m))_N]$	$W_N^{km}X[k]$
6.	$W_N^{-\ell n}x[n]$	$X[((k-\ell))_N]$
7.	$\sum_{m=0}^{N-1} x_1(m)x_2[((n-m))_N]$	$X_1[k]X_2[k]$
8.	$x_1[n]x_2[n]$	$\frac{1}{N}\sum_{\ell=0}^{N-1} X_1(\ell)X_2[((k-\ell))_N]$
9.	$x^*[n]$	$X^*[((-k))_N]$
10.	$x^*[((-n))_N]$	$X^*[k]$
11.	$\mathcal{R}e\{x[n]\}$	$X_{ep}[k] = \frac{1}{2}\{X[((k))_N] + X^*[((-k))_N]\}$
12.	$j\mathcal{J}m\{x[n]\}$	$X_{op}[k] = \frac{1}{2}\{X[((k))_N] - X^*[((-k))_N]\}$
13.	$x_{ep}[n] = \frac{1}{2}\{x[n] + x^*[((-n))_N]\}$	$\mathcal{R}e\{X[k]\}$
14.	$x_{op}[n] = \frac{1}{2}\{x[n] - x^*[((-n))_N]\}$	$j\mathcal{J}m\{X[k]\}$
Properties 15–17 apply only when $x[n]$ is real.		
15.	Symmetry properties	$\begin{cases} X[k] = X^*[((-k))_N] \\ \mathcal{R}e\{X[k]\} = \mathcal{R}e\{X[((-k))_N]\} \\ \mathcal{J}m\{X[k]\} = -\mathcal{J}m\{X[((-k))_N]\} \\ \lvert X[k]\rvert = \lvert X[((-k))_N]\rvert \\ \sphericalangle\{X[k]\} = -\sphericalangle\{X[((-k))_N]\} \end{cases}$
16.	$x_{ep}[n] = \frac{1}{2}\{x[n] + x[((-n))_N]\}$	$\mathcal{R}e\{X[k]\}$
17.	$x_{op}[n] = \frac{1}{2}\{x[n] - x[((-n))_N]\}$	$j\mathcal{J}m\{X[k]\}$

TABLE 3.1 SOME COMMON z-TRANSFORM PAIRS

Sequence	Transform	ROC
1. $\delta[n]$	1	All z
2. $u[n]$	$\dfrac{1}{1-z^{-1}}$	$\lvert z\rvert > 1$
3. $-u[-n-1]$	$\dfrac{1}{1-z^{-1}}$	$\lvert z\rvert < 1$
4. $\delta[n-m]$	z^{-m}	All z except 0 (if $m > 0$) or ∞ (if $m < 0$)
5. $a^n u[n]$	$\dfrac{1}{1-az^{-1}}$	$\lvert z\rvert > \lvert a\rvert$
6. $-a^n u[-n-1]$	$\dfrac{1}{1-az^{-1}}$	$\lvert z\rvert < \lvert a\rvert$
7. $na^n u[n]$	$\dfrac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert > \lvert a\rvert$
8. $-na^n u[-n-1]$	$\dfrac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert < \lvert a\rvert$
9. $[\cos\omega_0 n]u[n]$	$\dfrac{1-[\cos\omega_0]z^{-1}}{1-[2\cos\omega_0]z^{-1}+z^{-2}}$	$\lvert z\rvert > 1$
10. $[\sin\omega_0 n]u[n]$	$\dfrac{[\sin\omega_0]z^{-1}}{1-[2\cos\omega_0]z^{-1}+z^{-2}}$	$\lvert z\rvert > 1$
11. $[r^n\cos\omega_0 n]u[n]$	$\dfrac{1-[r\cos\omega_0]z^{-1}}{1-[2r\cos\omega_0]z^{-1}+r^2z^{-2}}$	$\lvert z\rvert > r$
12. $[r^n\sin\omega_0 n]u[n]$	$\dfrac{[r\sin\omega_0]z^{-1}}{1-[2r\cos\omega_0]z^{-1}+r^2z^{-2}}$	$\lvert z\rvert > r$
13. $\begin{cases} a^n, & 0 \le n \le N-1, \\ 0, & \text{otherwise} \end{cases}$	$\dfrac{1-a^N z^{-N}}{1-az^{-1}}$	$\lvert z\rvert > 0$

Initial Value Theorem:

If $x[n]$ is a causal sequence, *i.e.* $x[n] = 0$, $\forall n < 0$, then

$$x[0] = \lim_{z\to\infty} X(z)$$