NATIONAL EXAMS May 2014
07-Elec-B2 Advanced Control Systems

3 hours duration

NOTES:

1. If a doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.

2. Candidates may use one of two calculators, a Casio FX-991 or a Sharpe EL-540.

3. This is a closed-book examination. Tables of Laplace and z-transforms are attached as page 4 and page 5.

4. Any four questions constitute a complete paper. Only the first four questions as they appear in your answer book will be marked.

5. All questions are of equal value.

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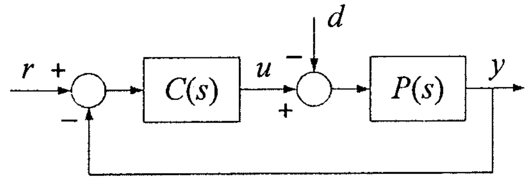
1. Consider the following system with $C(s)=\dfrac{sK_1+K_2}{s}$ and $P(s)=\dfrac{1}{s(s+3)}$.

(a) Let $K_2=0$. Find a value for K_1, say $K_1=K_1^0$, such that the overshoot at $y(t)$ is 10% when there is a step change at $r(t)$ with $d(t)=0$.



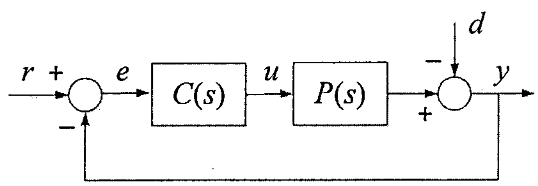
(b) Let $K_1=K_1^0$. Find the maximum value of K_2, say $K_2=K_2^{\max}$, for closed loop stability.

(c) Let $K_2=K_2^{\max}/2$. Determine the steady state tracking error when $r(t)$ is a ramp with slope 2 and $d(t)=0$. Then determine the steady state value of the control input, $u(t)$, when $r(t)=0$ and $d(t)=$ a unit step.

2. Consider the system, $y(s)=G(s)u(s)$, $G(s)=\dfrac{\alpha-s}{(1+s)^2}$.

(a) Find a state space model for the system taking $y(t)$ as one of the state variables.
(b) Justify the conditions under which the system is controllable and observable.
(c) Let $\alpha=1$. Design a state feedback controller such that the closed loop poles are -3, -2.

3. Consider the feedback system below with $P(s)=\dfrac{4(s+3)}{s^2+0.2s+2}$.



(a) Determine a feedback controller, $C(s)$, such that the closed loop transfer function relating r to y is given by $\dfrac{32}{8s^2+6s+32}$. Note: $C(s)$ must be proper, i.e., the degree of the numerator must be greater than or equal to the degree of the numerator.
(b) Determine the gain and phase margin of the feedback design.
(c) Determine the steady state value of u when d is a unit step, $r=0$, and $n=0$.

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4. Measurements of the frequency response for an unknown but stable first order system are recorded as follows,

Frequency	Gain	Phase Shift
0 rad/s	9.543 db	0
1 rad/s	5.563 db	-108.4 deg
2 rad/s	4.228 db	-139.4 deg

(a) Determine the transfer function, $P(s)$. *Note*: the transfer function of the system may have both numerator and denominator terms.

(b) Draw the associated unit step response being careful to identify the key features.

(c) Justify whether the system is stable or not when a controller, $C(s) = 1/s$, is cascaded with $P(s)$ in a negative (unity) feedback loop.

5. The discrete plant, $P(z) = \dfrac{z-1.5}{z(z-0.5)}$ is to be controlled with a proportional feedback controller.

(a) Determine the range of the proportional gain for stability.

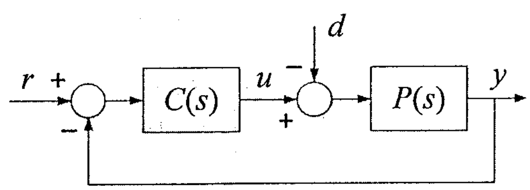
(b) Sketch the root locus.

(c) $P(z)$ is obtained by uniform sampling of a continuous time plant, $P_c(s)$, that is driven by a zero order hold (or ideal digital to analog converter). Determine $P_c(s)$.

6. Consider the (continuous time) feedback system below with, $C(s) = K,\ \ P(s) = \dfrac{e^{-s/3}}{s}$.

(a) Determine the range of K that results in closed loop stability.

(b) Determine the phase margin when $K = 1$ and sketch the associated Nyquist plot.



(c) The system is stable and operating with disturbance, $d(t) = 0.3$, and set-point, $r(t) = 1$. Determine the tracking error, $e(t) = r(t) - y(t)$, as a function of K.

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Inverse Laplace Transforms	
$F(s)$	$f(t)$
$\dfrac{A}{s+\alpha}$	$Ae^{-\alpha t}$
$\dfrac{C+jD}{s+\alpha+j\beta}+\dfrac{C-jD}{s+\alpha-j\beta}$	$2e^{-\alpha t}\left(C\cos\beta t+D\sin\beta t\right)$
$\dfrac{A}{(s+\alpha)^{n+1}}$	$\dfrac{At^n e^{-\alpha t}}{n!}$
$\dfrac{C+jD}{(s+\alpha+j\beta)^{n+1}}+\dfrac{C-jD}{(s+\alpha-j\beta)^{n+1}}$	$\dfrac{2t^n e^{-\alpha t}}{n!}\left(C\cos\beta t+D\sin\beta t\right)$

Inverse z-Transforms	
$F(z)$	$f(nT)$
$\dfrac{Kz}{z-a}$	Ka^n
$\dfrac{(C+jD)z}{z-re^{j\varphi}}+\dfrac{(C-jD)z}{z-re^{-j\varphi}}$	$2r^n\left(C\cos n\varphi-D\sin n\varphi\right)$
$\dfrac{Kz}{(z-a)^r}$, $r=2,3\ldots$	$\dfrac{Kn(n-1)\ldots(n-r+2)}{(r-1)!a^{r-1}}a^n$

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Table of Laplace and z-Transforms
(*h* denotes the sample period)

$f(t)$	$F(s)$	$F(z)$
unit impulse	1	1
unit step	$\frac{1}{s}$	$\frac{z}{z-1}$
$e^{-\alpha t}$	$\frac{1}{s+\alpha}$	$\frac{z}{z-e^{\alpha h}}$
t	$\frac{1}{s^2}$	$\frac{hz}{(z-1)^2}$
$\cos \beta t$	$\frac{s}{s^2+\beta^2}$	$\frac{z(z-\cos\beta h)}{z^2-2z\cos\beta h+1}$
$\sin \beta t$	$\frac{\beta}{s^2+\beta^2}$	$\frac{z\sin\beta h}{z^2-2z\cos\beta h+1}$
$e^{-\alpha t}\cos \beta t$	$\frac{s+\alpha}{(s+\alpha)^2+\beta^2}$	$\frac{z(z-e^{-\alpha h}\cos\beta h)}{z^2-2ze^{-\alpha h}\cos\beta h+e^{-2\alpha h}}$
$e^{-\alpha t}\sin \beta t$	$\frac{\beta}{(s+\alpha)^2+\beta^2}$	$\frac{ze^{-\alpha h}\sin\beta h}{z^2-2ze^{-\alpha h}\cos\beta h+e^{-2\alpha h}}$
$t\,f(t)$	$-\frac{dF(s)}{ds}$	$-zh\frac{dF(z)}{dz}$
$e^{-\alpha t}f(t)$	$F(s+\alpha)$	$F(ze^{\alpha h})$