NATIONAL EXAMINATIONS

May 2017

04-BS-7 MECHANICS OF FLUIDS

Three (3) hours duration

Notes to Candidates

1. This is a **Closed Book** examination.

2. Exam consists of two Sections. **Section A is Calculative (9 questions)** and **Section B is Analytical (4 questions).**

3. **Do seven (7) questions from Section A (Calculative) and three (3) questions from Section B (Analytical).** Note that the Analytical Questions do not require detailed calculations but do require full explanations.

4. **Ten (10) questions constitute a complete paper.** (Total 50 marks).

5. **All questions are of equal value.** (Each 5 marks).

6. If doubt exists as to the interpretation of any question, the candidate is urged to submit, with the answer paper, a clear statement of any assumptions made.

7. **Read the entire question before commencing** the calculations and **take note of hints or recommendations** given.

8. Either one of the approved **Casio** or **Sharp** calculators may be used.

9. **Reference information** for particular questions is given on pages 7 to 10. **All pages of questions attempted are to be returned with the Answer Booklet, showing diagrams generated or where readings were taken and which data was used. Candidates must write their names on these pages.**

10. **Constants** are given on page 11.

11. **Nomenclature** and **Reference Equations** are given on pages 12 to 15.

SECTION A CALCULATIVE QUESTIONS

Do seven of nine questions. Solutions to these questions must be set out logically with all intermediate answers and units given.

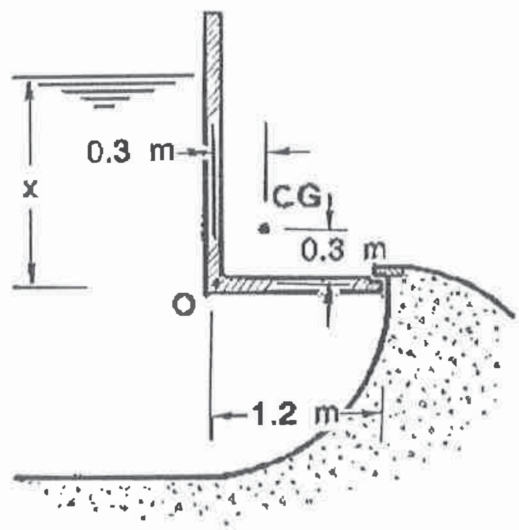
QUESTION 1

Calculate the capillary rise (height) of water in contact with air in a small clean glass channel of rectangular cross section 1 mm x 2 mm. The surface tension of water σ = 0.073 N/m and the contact angle θ = 0°. Explain how the contact angle affects the rise of liquid in a capillary tube.

(5 marks)

QUESTION 2

Refer to the illustration alongside of a reservoir gate designed to remain closed at low water levels but to flip open on rising water level to discharge a large volume of water.



The angle of the gate is 90° as shown, the mass of the gate is 800 kg, the horizontal width of the gate 2.0 m, the length of the small arm 1.2 m, and the centre of gravity 0.3 m to right of and 0.3 m above pivot O. Neglect friction at the pivot and thickness of the gate. If the depth of water x above the pivot is 2.0 m, state whether the gate will open or remain closed.

(5 marks)

QUESTION 3

A large balloon is filled with 9350 m^3 of helium at ground level where the ambient conditions are 100 kPa and 15°C. Determine the mass of helium required. When released, it rises to an altitude of 10 000 m (approximately 33 000 ft) where the atmospheric conditions are 24 kPa and -54°C.

(a) Determine the volume of the balloon at this altitude.

(b) If, when at 10 000 m, the sun heats the balloon to 15°C above ambient conditions at that altitude, determine the percentage increase in volume of the balloon.

(5 marks)

QUESTION 4

Refer to the Examination Paper Attachments Page 7 **Flow Coefficients**.

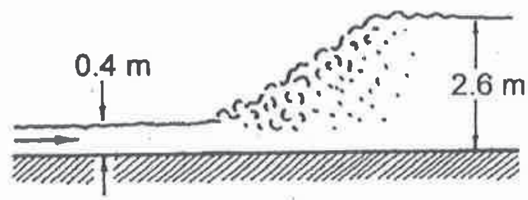
A VDI Orifice meter in conjunction with a differential pressure gauge is required to measure the flow of light fuel oil with a specific gravity of 0.94 in a pipeline. The pipeline is 50 mm in diameter and the orifice diameter is 25 mm. At the prevailing temperature the viscosity μ of the oil is 0.015 Ns/m^2. At an oil flow rate of 1.5 kg/s, determine the reading on a differential pressure gauge connected to the pipe upstream and downstream of the orifice. Refer to the figure on Page 7 for the flow coefficients of a VDI orifice meter.

(5 marks)

QUESTION 5

An hydraulic jump forms in a channel in which there is high velocity water flow. The channel is 2.4 m wide and the water depth is 0.4 m before the jump and 2.6 m after the jump

Determine the flow rate through the jump (m^3/s)



Hint: Use the momentum and continuity equations in the calculations.

(5 marks)

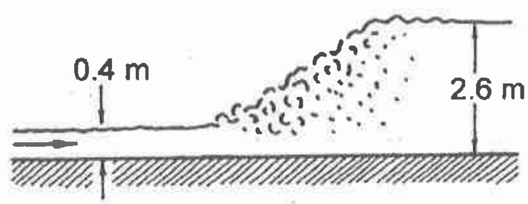
QUESTION 6

An hydraulic jump forms in a channel in which there is high velocity water flow. The channel is 2.4 m wide and the water depth is 0.4 m before the jump and 2.6 m after the jump. The flow rate in the channel is 9.4 m^3/s. Determine the following:

(a) Head loss in the jump (m)

(b) Energy loss in the jump (J/kg)

(c) Power loss in the jump (kW)



Hint: Use the energy equation in the calculations.

(5 marks)

QUESTION 7

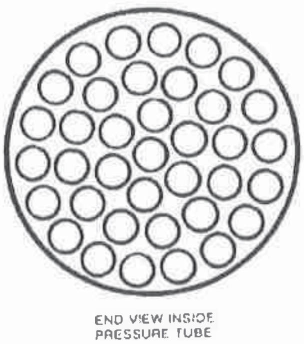
Refer to the Examination Paper Attachments Page 8 **Moody Diagram.**

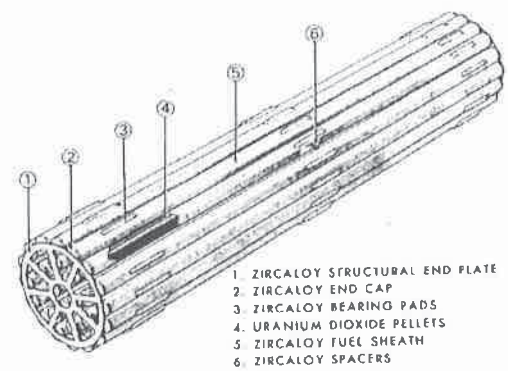
Air with a density ρ = 1.2 kg/m^3 and kinematic viscosity v = 1.6 X 10^{-5} m^2/s flows in a 300 mm by 450 mm rectangular ventilation air duct at the rate of 0.5 m^3/s. Determine the head loss per 100 m of duct. Assume that the wall of the duct is smooth. Use the Moody Diagram on Page 8 for the friction factor

(5 marks)

QUESTION 8

Refer to the Examination Paper Attachments Page 9 **Moody Diagram.**





The diagram on the right above shows a fuel bundle as used in a CANDU nuclear reactor. A typical reactor has a dozen such bundles in several hundred pressure tubes through which coolant (heavy water) flows to remove the heat generated in the fuel. The diagram on the left above shows the flow cross section with the fuel rods which hold the fuel pellets surrounded by coolant which in turn flows within a pressure tube.

Calculate the pressure drop in kPa within the pressure tube over the length of one of the bundles assuming the following specified and operational parameters:

Internal diameter of pressure tube	104.0 mm	
External diameter of fuel rods	13.1 mm	
Number of fuel rods	37	
Length of fuel bundle	495 mm	
Density of water	712 kg/m3	at ~300ºC
Viscosity of water	9.0 x 10^{-5} kg/ms	at ~300ºC
Coolant flow rate	24 kg/s	

Neglect the effect of the spacers between the fuel rods and the end plates as well as entrance and exit losses.

(5 marks)

QUESTION 9

Refer to the Examination Paper Attachments Page 10 **Drag Diagram for Solid Bodies.**

In order to determine the viscosity of an oil, a small steel ball is allowed to fall freely through the oil in a tall glass cylinder. The time taken for it to pass between two points is measured to obtain its terminal velocity.

A steel sphere with a specific gravity of 7.8 and a diameter of 6 mm is released in a cylinder of oil having a specific gravity of 0.825. The sphere is observed to have a terminal velocity of 0.5 m/s. Calculate the absolute viscosity of the oil. Refer to the attached Drag Coefficients for Spheres on Page 10 for empirical data.

(5 marks)

SECTION B GRAPHICAL AND ANALYTICAL QUESTIONS

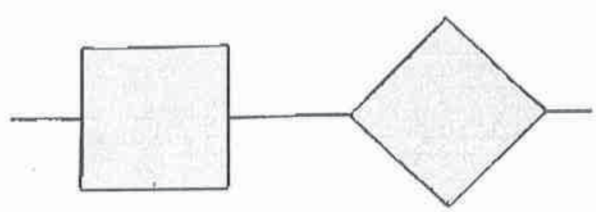
Do three of four questions. These questions do not require detailed calculations but complete written explanations must be given to support the answers where descriptive answers are required.

QUESTION 10

Explain the significance of the laminar sublayer. State what effect the laminar sublayer has on pipe flow if $\delta > \varepsilon$ and if $\delta < \varepsilon$, where δ is the thickness of the laminar sublayer and ε is the roughness of the pipe. Explain how the laminar sublayer thickness varies with flow velocity and how the resultant "apparent" roughness changes as velocity changes.

(5 marks)

QUESTION 11

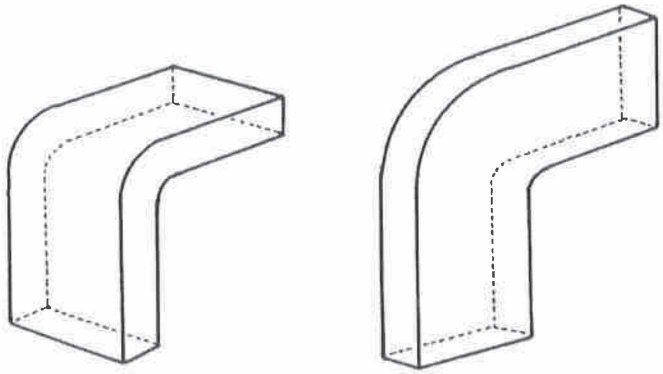


A **B**

A long bar of square cross section floats on water. If its specific gravity is one half that of water, determine its stable orientation (one flat side upwards as in **A** or edge between two flat sides upward as in **B**). For each of the two extreme conditions, determine the depth of the centre of buoyancy in terms of the width of one side of the square w. Explain how the depth of the centre of buoyancy can be expected to determine the most stable orientation.

(5 marks)

QUESTION 12

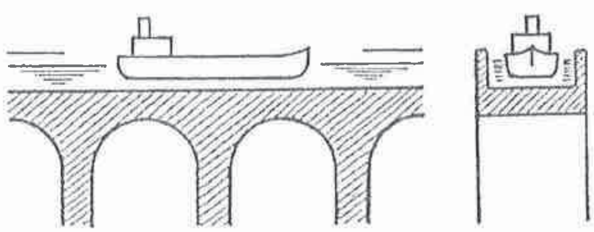


A **B**

Air flowing through a rectangular air conditioning duct is redirected around a right angled bend as in the sketches above. State which bend **A** or **B** will have the greater head loss assuming that both are of the same cross section and carry the same volumetric air flow. Explain why this is the case. Use a sketch showing the flow pattern around the bend to justify the explanation.

(5 marks)

QUESTION 13

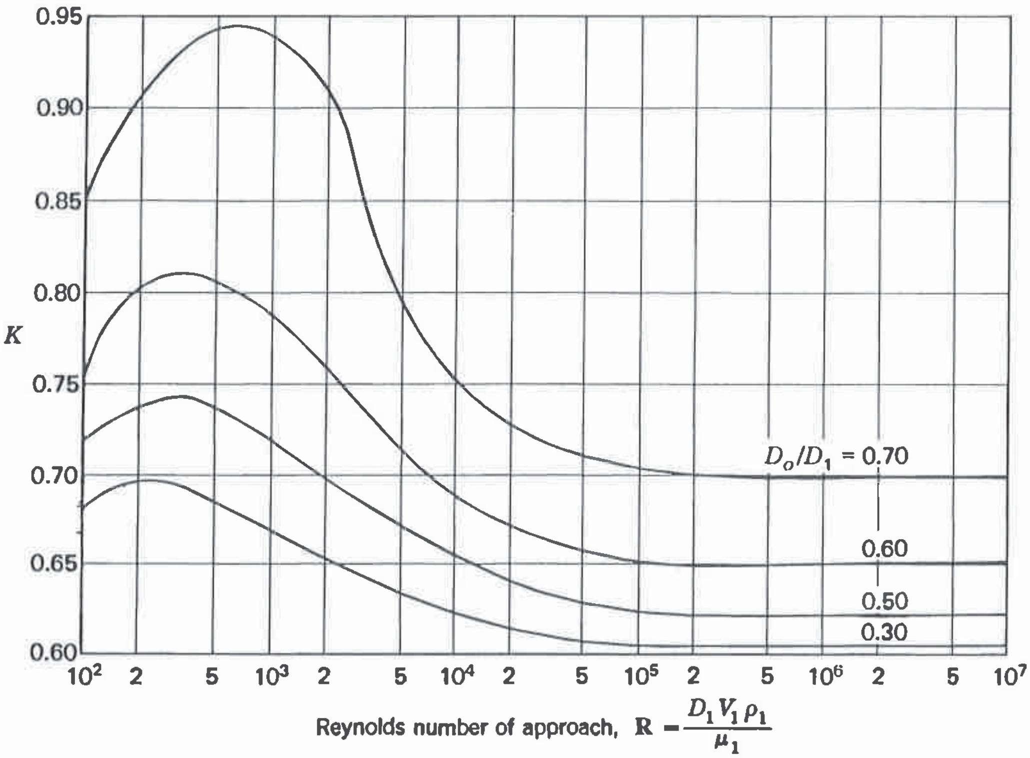


A barge 15 m long and 3 m wide is loaded such that its draught (depth of bottom below water surface) is 1.2 m. It sails in a long canal 5 m wide and 2 m deep. At one point the canal is taken across a valley by an aqueduct as shown in the sketch above. Determine the change in compressive force on the aqueduct pillars as the barge passes over them. Give a full explanation of your answer

(5 marks)

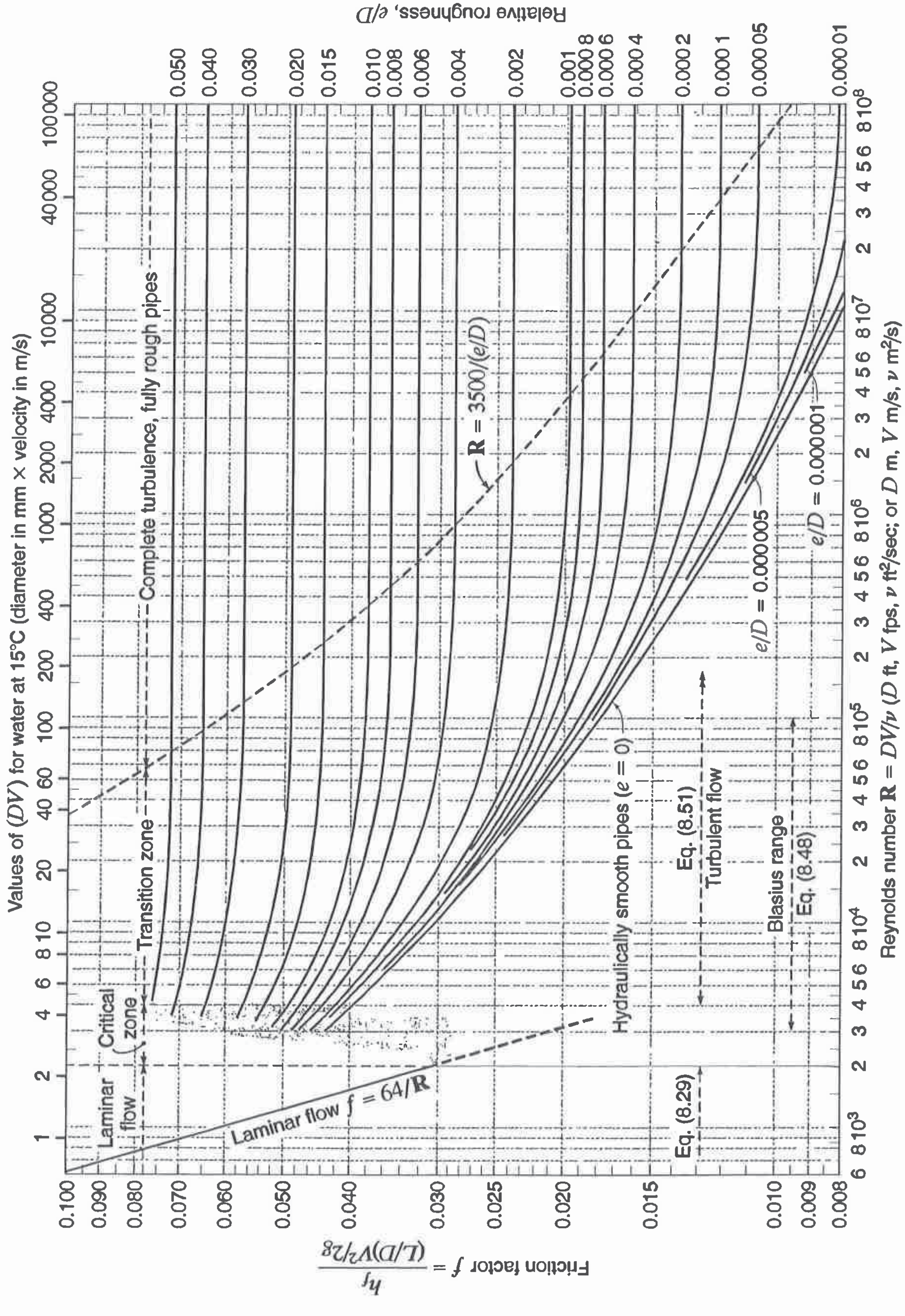
EXAMINATION PAPER ATTACHMENTS

QUESTION 4 FLOW COEFFICIENTS FOR VDI ORIFICE METER



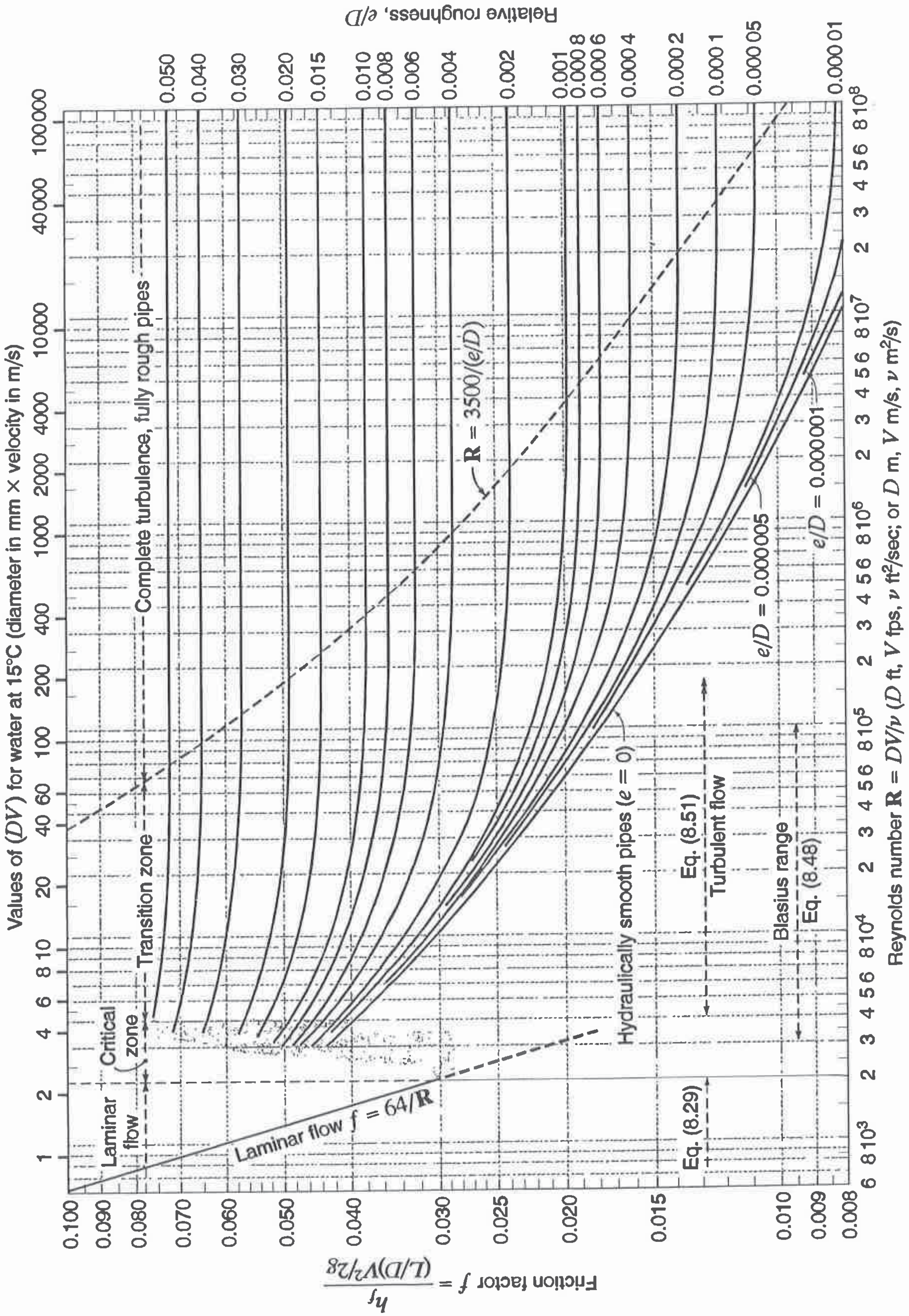
VDI orifice meter and flow coefficients for flange taps. (*Adapted from NACA Tech. Mem.* 952.)

QUESTION 7 MOODY DIAGRAM NAME ....



Moody chart for pipe friction factor (Stanton diagram).

QUESTION 8 MOODY DIAGRAM NAME ....



Moody chart for pipe friction factor (Stanton diagram).

QUESTION 9 DRAG DIAGRAM FOR SOLID BODIES **NAME ....**

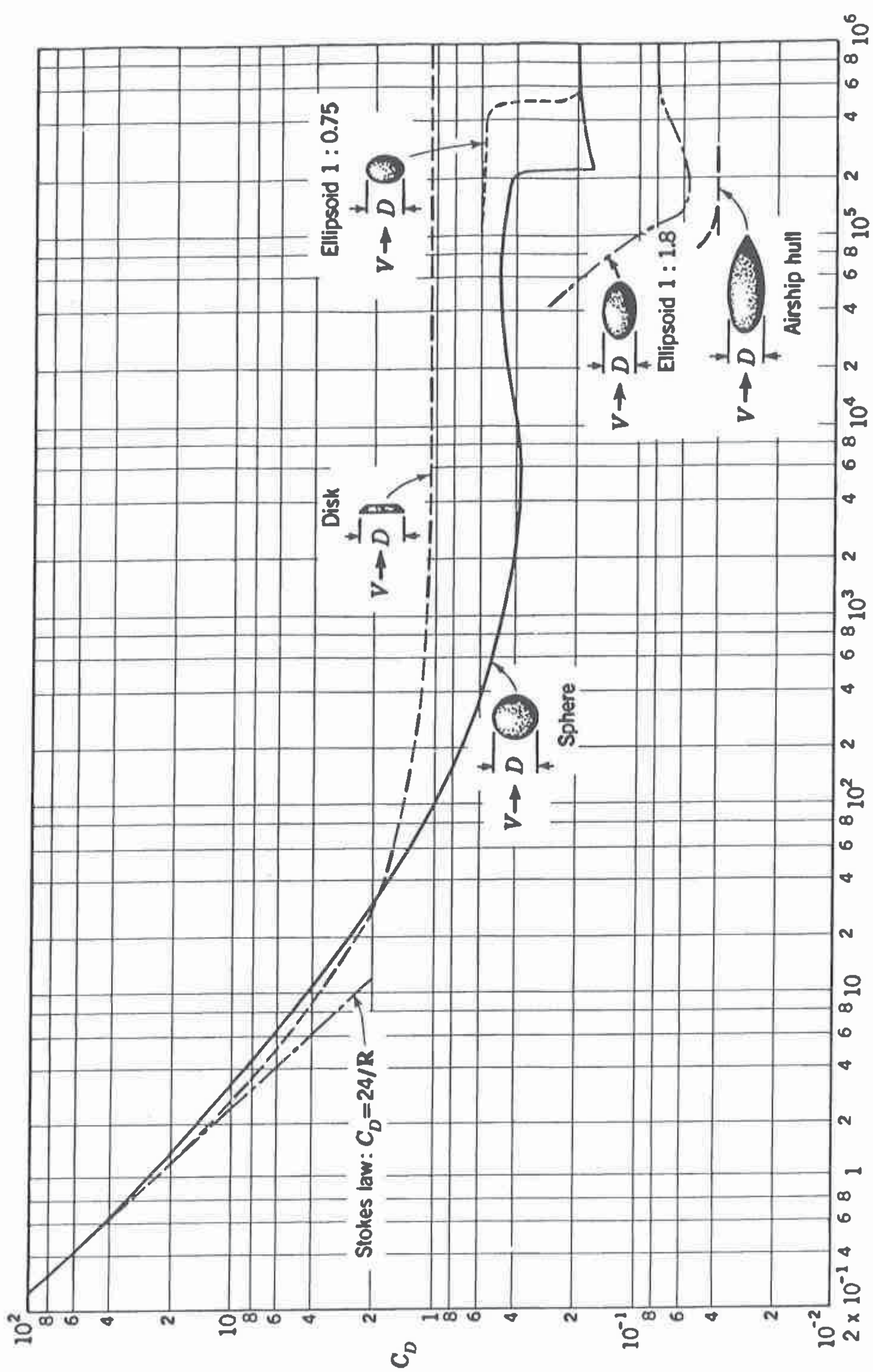


Figure 10.10 Drag coefficient for bodies of revolution. (Adapted from L. Prandtl, "Ergebnisse der aerodynamischen Versuchsanstalt zu Göttingen," p. 29, R. Oldenbourg, Munich and Berlin, 1923; and F. Eisner, "Das Widerstandsproblem," *Proc. 3d Internatn. Congr. Appl. Mech., p. 32, 1930.)*

04-BS-7 MECHANICS OF FLUIDS

GENERAL REFERENCE INFORMATION

CONSTANTS

In engineering calculations a high degree of accuracy is seldom attained due to the neglect of minor influences or the inaccuracy of available data. For consistency in calculations however the following reasonably accurate constants should be used:

Atmospheric Pressure p_o = 100 kPa
Gravitational Acceleration g = 9.81 m/s^2
Specific Gravity of Water = 1.00
Specific Gravity of Glycerine = 1.26
Specific Gravity of Mercury = 13.56
Specific Gravity of Benzene = 0.90
Specific Gravity of Carbon Tetrachloride = 1.59
Density of Water Δ = 1000 kg/m^3
Density of Sea Water Δ = 1025 kg/m^3
Density of Concrete Δ = 2400 kg/m^3
Density of Air Δ = 1.19 kg/m^3 (at 20EC), ρ = 1.21 kg/m^3 (at 15°C)
Absolute Viscosity of Water Φ = 1.0 x 10^{-3} Ns/m^2
Absolute Viscosity of Air Φ = 1.8 x 10^{-5} Ns/m^2
Surface Tension of Water Φ = 0.0728 N/m (at 20EC)
Specific Heat of Water c_p = 4.19 kJ/kg°C
Specific Heat of Air c_p = 1005 J/kg°C
Specific Heat of Air c_v = 718 J/kg°C
Gas Constant for Air R = 287 J/kgEK
Gas Constant for Helium R = 2077 J/kgEK
Gas Constant for Hydrogen R = 4120 J/kgEK

NOMENCLATURE FOR REFERENCE EQUATIONS (SI UNITS)

a	Width	m
A	Flow area, Surface area	m^2
CV	Calorific value	J/kg
c_p	Specific heat at constant pressure	J/kg°C
b	Width	m
D	Diameter	m
E	Energy	J
F	Force	N
g	Gravitational acceleration	m/s^2
h	System head	m
h_L	Head loss	m
H	Pump or turbine head	m
I	Moment of inertia	m^4
k	Ratio of specific heats	
k	Loss coefficient	
K	Constant	
L	Length	m
m	Mass	kg
M	Mass flow rate	kg/s
N	Rotational speed	rev/s
p	Pressure	Pa (N/m^2)
P	Power	W (J/s)
q	Specific heat	J/kg
Q	Flow rate	m^3/s
r	Radius	m
R	Specific gas constant	J/kg K
T	Temperature	K
U	Blade velocity	m/s
v	Specific volume	m^3/kg
V	Velocity	m/s
V	Volume	m^3
w	Specific work	J/kg
W	Work	J
y	Depth	m
z	Elevation	m
η	Efficiency	
μ	Dynamic viscosity	Ns/m^2
ν	Kinematic viscosity	m^2/s
ρ	Density	kg/m^3
σ	Surface tension	N/m
τ	Thrust	N
τ	Shear stress	N/m^2

REFERENCE EQUATIONS

Equation of State

$p\ v = R\ T$
$p = \rho\ RT$

Universal Gas Law

$p\ v^n = \text{constant}$

Compressibility

$\beta = - \Delta / V\ \Delta p$

Viscous Force and Viscosity

$F = \mu\ A\ du/dy$
$\mu = \tau / (du/dy)$
$\nu = \mu / \rho$

Capillary Rise and Internal Pressure due to Surface Tension

$h = (\sigma \cos \theta / \rho\ g) \times (\text{perimeter} / \text{area})$
$p = 2\ \sigma / r$

Pressure at a Point

$p = \rho\ g\ h$

Forces on Plane Areas and Centre of Pressure

$F = \rho\ g\ y_c\ A$
$y_p = y_c + I_c / y_c\ A$

Moments of Inertia

Rectangle: $I_c = b\ h^3 / 12$
Triangle: $I_c = b\ h^3 / 36$
Circle: $I_c = \pi\ D^4 / 64$

Surface Area of Solids

Sphere: $A = \pi\ D^2$

Volumes of Solids

Sphere: $V = \pi D^3 / 6$
Cone: $V = \pi D^2 h / 12$
Spherical Segment: $V = (3 a^2 + 3 b^2 + 4 h^2) \pi h / 2 g$

Continuity Equation

$$\rho_1 V_1 A_1 = \rho_2 V_2 A_2 = M$$

General Energy Equation

$$p_1 / \rho_1 g + z_1 + V_1^2 / 2 g + q_{in} / g + w_{in} / g$$
$$= p_2 / \rho_2 g + z_2 + V_2^2 / 2 g + h_L + q_{out} / g + w_{out} / g$$

Bernoulli Equation

$$p_1 / \rho g + z_1 + V_1^2 / 2 g = p_2 / \rho g + z_2 + V_2^2 / 2 g$$

Momentum Equation

Conduit: $F_R = p_1 A - p_2 A - M (V_2 - V_1)$
Free Jet: $F_R = - \rho Q (V_2 - V_1)$

Flow Measurement

Venturi Tube: $Q = [C A_2 / \{1 - (D_2 / D_1)^4\}^{1/2}][2 g \Delta h]^{1/2}$
Flow Nozzle: $Q = K A_2 [2 g \Delta h]^{1/2}$
Orifice Meter: $Q = K A_o [2 g \Delta h]^{1/2}$

Flow over Weirs

Rectangular Weir: $Q = C_d (2 / 3) [2 g]^{1/2} L H^{3/2}$

Power

Turbomachine: $P = \rho g Q H$
Free Jet: $P = ½ \rho Q V^2$
Moving Blades: $P = M \Delta V U$

Aircraft Propulsion

$F_{thrust} = M (V_{jet} - V_{aircraft})$
$P_{thrust} = M (V_{jet} - V_{aircraft}) V_{aircraft}$
$E_{jet} = ½ (V_{jet}^2 - V_{aircraft}^2)$
$P_{jet} = ½ M (V_{jet}^2 - V_{aircraft}^2)$

$E_{fuel} = CV_{fuel}$
$P_{fuel} = M_{fuel}\, CV_{fuel}$
$\eta_{thermal} = P_{jet} / P_{fuel}$
$\eta_{propulsion} = P_{thrust} / P_{jet} = 2\, V_{aircraft} / (V_{jet} + V_{aircraft})$
$\eta_{overall} = \eta_{thermal} \times \eta_{propulsion}$

Wind Power

$P_{total} = ½\, \rho\, A_T\, V_1^3$
$P_{max} = 8/27\, \rho\, A_T\, V_1^3$
$H_{max} = P_{max} / P_{total} = 16/27$

Reynolds Number

$Re = d\, V\, \rho / \mu$

Flow in Pipes

$h_L = f\, (L / D)\, (V^2 / 2\, g)$
$D_e = 4$ (flow area) / (wetted perimeter)
$D = D_e$ for non-circular pipes
$L = L_{total} + L_e$ for non-linear pipes
$(L / D) = 35\, k$ for $Re \sim 10^4$

Drag on Immersed Bodies

Friction Drag: $F_f = C_f\, ½\, \rho\, V^2\, B\, L \quad (B = \pi\, D)$
Pressure Drag: $F_p = C_p\, ½\, \rho\, V^2\, A$
Total Drag: $F_D = C_D\, ½\, \rho\, V^2\, \mathbf{A}$

Aircraft Wing: $F_L = C_L\, ½\, \rho\, V^2\, A_{wing}$
Aircraft Wing: $F_D = C_D\, ½\, \rho\, V^2\, A_{wing}$

Karmen Vortex Frequency

$f \approx 0.20\, (V / D)\, (1 - 20 / Re)$