NATIONAL EXAMINATIONS MAY 2014

04-BS-5 ADVANCED MATHEMATICS

3 Hours duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper a clear statement of any assumption made.

2. Candidates may use one of the approved Casio or Sharp calculators. This is a Closed Book Exam. However, candidates are permitted to bring **ONE** aid sheet (8.5"x11") written on both sides.

3. Any five (5) questions constitute a complete paper. Only the first five answers as they appear in your answer book will be marked.

4. All questions are of equal value.

Marking Scheme

1. 20 marks
2. (A) 14 marks ; (B) 6 marks
3. 20 marks
4. (A) 10 marks ; (B) 10 marks
5. 20 marks
6. (A) 5 marks ; (B) 8 marks ; (C) 7 marks
7. (a) 10 marks ; (b) 10 marks

1. The following Sturm-Liouville problem is regular on the interval [0,1].

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + (1+\lambda)y = 0 \quad ; \quad y(0) = 0 \;;\; y(1) = 0$$

Find the eigenvalues and, corresponding to each eigenvalue, find an eigenfunction.

2.(A) Find the Fourier series expansion of the periodic function F(x) of period p=2.

$$F(x) = \begin{cases} x & 0 < x \le 1 \\ 1 & 1 < x \le 2 \end{cases}$$

2.(B) By letting x=0 or x=1 in the result obtained in (A) prove that

$$\frac{\pi^2}{8} = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$$

3.Consider the following function

$$f(x) = \begin{cases} x+2 & -2 \le x < -1 \\ 1 & -1 \le x \le 1 \\ 2-x & 1 < x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

Find the Fourier transform F(ω) of f(x).

Note: $$F(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x)\exp(-i\omega x)dx$$

4.(A) It is believed that the n points (x_i,y_i) obtained in an experiment can be fitted by an equation having the following form

$$y=\frac{a}{x}+\frac{b}{x^3}$$

Use the method of least-squares to prove that the coefficients a and b are equal to D_1/D_S and D_2/D_S respectively where

$$D_1=\sum\frac{y_i}{x_i}\sum\frac{1}{x_i^6}-\sum\frac{y_i}{x_i^3}\sum\frac{1}{x_1^4} \quad ; \quad D_2=\sum\frac{y_i}{x_i^3}\sum\frac{1}{x_1^2}-\sum\frac{y_i}{x_i}\sum\frac{1}{x_i^4}$$

$$D_S=\sum\frac{1}{x_i^2}\sum\frac{1}{x_i^6}-\left(\sum\frac{1}{x_i^4}\right)^2$$

Note: All the summations are carried out from $i=1$ to $i=n$.

4(B) Find the Legendre polynomial that fits the following set of four points.

x	−1	0	1	2
F(x)	0	−2	−4	0

5. The following results were obtained in a certain experiment:

x	1.00	1.50	2.00	2.50	3.00	3.50	4.00	4.50	5.00
y	10.60	11.25	15.20	17.50	21.85	26.25	32.30	39.15	46.90

Use Romberg's algorithm to obtain an approximate value of the area bounded by the unknown curve represented by the table and the lines x = 1.00, x = 5.00 and the x-axis.

Note: The Romberg algorithm produces a triangular array of numbers, all of which are numerical estimates of the definite integral $\int_a^b f(x)dx$. The array is denoted by the following notation:

R(1,1)			
R(2,1)	R(2,2)		
R(3,1)	R(3,2)	R(3,3)	
R(4,1)	R(4,2)	R(4,3)	R(4,4)

where

$$R(1,1)=\frac{H_1}{2}[f(a)+f(b)]$$

$$R(k,1)=\frac{1}{2}\left[R(k-1,1)+H_{k-1}\sum_{n=1}^{n=2^{k-2}}f(a+(2n-1)H_k)\right]; \qquad H_k=\frac{b-a}{2^{k-1}}$$

$$R(k,j)=R(k,j-1)+\frac{R(k,j-1)-R(k-1,j-1)}{4^{j-1}-1}$$

6.(A) The equation $6^x - 13x = 0$ has a root between $\alpha=1$ and $\beta=2$. Use the method of bisection four times to find a better approximation of this root.
6.(B) Use Newton's method three times to find a better approximation of the root under consideration in (A). Start with the last value you obtained in (A). (Note: Carry seven digits in your calculations)
6.(C) The equation $\ln(x+5) - x^2 + 5x + 6 = 0$ can be written in the form $x = g(x)$ in several obvious ways. One of the forms is

$$x = 0.2\,[x^2 - 6 - \ln(x+5)]$$

Apply the method of fixed-point iteration six times to find the root that is close to $x_0 = -1$. Explain clearly why this form converges to the root. (Note: Carry six digits in your computations).

7.(a)The positive definite matrix $A = \begin{bmatrix} 16 & -8 & 12 \\ -8 & 5 & -6 \\ 12 & -6 & 13 \end{bmatrix}$ can be written as the product of a lower triangular matrix $L = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix}$ and its transpose L^T.

(b) Use the results obtained in (a) to solve the following system of three linear equations:

$$\begin{aligned} 16x_1 - 8x_2 + 12x_3 &= -16 \\ -8x_1 + 5x_2 - 6x_3 &= 11 \\ 12x_1 - 6x_2 + 13x_3 &= -8 \end{aligned}$$