National Exams May 2014

07-Mec-A3, SYSTEM ANALYSIS AND CONTROL

3 hours duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.

2. Candidates may use a Casio **or** Sharp approved calculator. This is a **closed book** exam. No aids other than semi-log graph papers are permitted.

3. Any four questions constitute a complete paper. Only the first four (4) questions as they appear in your answer book will be marked.

4. All questions are of equal value.

Question 1:

A unity feedback system has an open-loop transfer function

$$G(s) = \frac{200K}{s(s+2)(s+5)}$$

Obtain a Bode plot for this system when $K = 1$ and find the phase and gain margins.

Question 2:

Find the range of the parameter α for which all roots of the given characteristic equations are in the left half-pane.

(a) $s^3 + s^2 + s + \alpha = 0$

(b) $s^3 + s^2 + \alpha s + 1 = 0$

(c) $s^3 + \alpha s^2 + s + 1 = 0$

(d) $\alpha s^3 + s^2 + s + 1 = 0$

Question 3:

Consider the system with the open-loop function

$$KG(s)H(s) = \frac{50K}{(s+1)(s+2)(s+10)}$$

(a) Sketch the root locus for this system for both positive and negative K.

(b) Locate all crossings of the $j\omega$-axis by the root locus, and find the value of K at each of these crossings.

(c) From the results in (b), state the complete range of K for which the system is stable.

Question 4:

Suppose that, in the system in Figure 1, the controller is a proportional type, that is, $G_c(s) = K_p$.

(a) Find the transient-response terms for the case that $K_p = 0.05$.

(b) Find the transient-response terms for the case that $K_p = 0.5$.

(c) Find the minimum value of K for which the transient response will have the fastest decay.

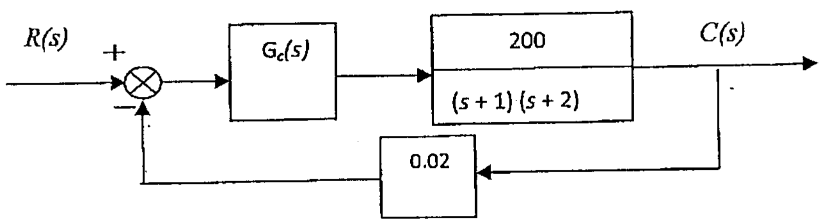

Figure 1

Question 5:

Consider the system shown in Figure 2. For each case given, find the steady-state error for (i) a unit step input; (ii) a unit ramp input. Assume in each case that the closed-loop system is stable.

(a) $G(s) = \dfrac{10}{(s+1)(s+2)}$

(b) $G(s) = \dfrac{10}{s(s+1)(s+5)}$

(c) $G(s) = \dfrac{5(s+2)}{s^2(s+6)}$

(d) $G(s) = \dfrac{6s^2 + 2s + 10}{s(s^2+3)}$

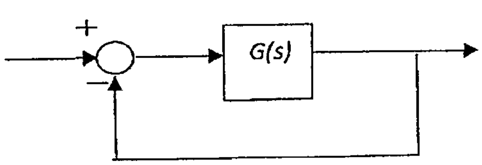

Figure 2

Question 6:

For the system of Figure 3, the input $r(t) = 3\cos 0.4t$ is applied at $t = 0$.

(a) Find the steady-state system response.

(b) Find the range of time t for which the system is in steady state.

(c) Find the steady-state response for the input $r(t) = 3\cos 4t$.

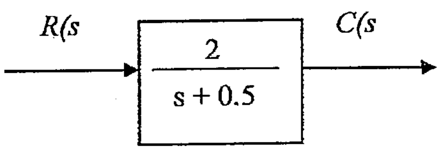

Figure 3

Laplace Transform Table

Laplace Transform $F(s)$	Time function $f(t)$
1	Unit-impulse function $\delta(t)$
$\frac{1}{s}$	Unit-step function $u_s(t)$
$\frac{1}{s^2}$	Unit-ramp function t
$\frac{n!}{s^{n+1}}$	t^n (n = positive integer)
$\frac{1}{s+\alpha}$	$e^{-\alpha t}$
$\frac{1}{(s+\alpha)^2}$	$te^{-\alpha t}$
$\frac{n!}{(s+\alpha)^{n+1}}$	$t^n e^{-\alpha t}$ (n = positive integer)
$\frac{1}{(s+\alpha)(s+\beta)}$	$\frac{1}{\beta-\alpha}(e^{-\alpha t}-e^{-\beta t})$ $(\alpha \neq \beta)$
$\frac{s}{(s+\alpha)(s+\beta)}$	$\frac{1}{\beta-\alpha}(\beta e^{-\beta t}-\alpha e^{-\alpha t})$ $(\alpha \neq \beta)$
$\frac{1}{s(s+\alpha)}$	$\frac{1}{\alpha}(1-e^{-\alpha t})$
$\frac{1}{s(s+\alpha)^2}$	$\frac{1}{\alpha^2}(1-e^{-\alpha t}-\alpha te^{-\alpha t})$
$\frac{1}{s^2(s+\alpha)}$	$\frac{1}{\alpha^2}(\alpha t-1+e^{-\alpha t})$
$\frac{1}{s^2(s+\alpha)^2}$	$\frac{1}{\alpha^2}\left[t-\frac{1}{\alpha}+\left(t+\frac{2}{\alpha}\right)e^{-\alpha t}\right]$

Laplace Transform Table (continued)

Laplace Transform $F(s)$	Time Function $f(t)$
$\frac{s}{(s+\alpha)^2}$	$(1-\alpha t)e^{-\alpha t}$
$\frac{\omega_n^2}{s^2+\omega_n^2}$	$\sin \omega_n t$
$\frac{s}{s^2+\omega_n^2}$	$\cos \omega_n t$
$\frac{\omega_n^2}{s(s^2+\omega_n^2)}$	$1-\cos \omega_n t$
$\frac{\omega_n^2(s+\alpha)}{s^2+\omega_n^2}$	$\omega_n\sqrt{\alpha^2+\omega_n^2}\sin(\omega_n t+\theta)$ where $\theta=\tan^{-1}(\omega_n/\alpha)$
$\frac{\omega_n}{(s+\alpha)(s^2+\omega_n^2)}$	$\frac{\omega_n}{\alpha^2+\omega_n^2}e^{-\alpha t}+\frac{1}{\sqrt{\alpha^2+\omega_n^2}}\sin(\omega_n t-\theta)$ where $\theta=\tan^{-1}(\omega_n/\alpha)$
$\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$	$\frac{\omega_n}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin \omega_n\sqrt{1-\zeta^2}\,t \quad (\zeta<1)$
$\frac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$	$1-\frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}\,t+\theta)$ where $\theta=\cos^{-1}\zeta \quad (\zeta<1)$
$\frac{s\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$	$\frac{-\omega_n^2}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}\,t-\theta)$ where $\theta=\cos^{-1}\zeta \quad (\zeta<1)$
$\frac{\omega_n^2(s+\alpha)}{s^2+2\zeta\omega_n s+\omega_n^2}$	$\omega_n\sqrt{\frac{\alpha^2-2\alpha\zeta\omega_n+\omega_n^2}{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}\,t+\theta)$ where $\theta=\tan^{-1}\frac{\omega_n\sqrt{1-\zeta^2}}{\alpha-\zeta\omega_n} \quad (\zeta<1)$
$\frac{\omega_n^2}{s^2(s^2+2\zeta\omega_n s+\omega_n^2)}$	$t-\frac{2\zeta}{\omega_n}+\frac{1}{\omega_n\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}\,t+\theta)$ where $\theta=\cos^{-1}(2\zeta^2-1) \quad (\zeta<1)$

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Semi-Logarithmic
4 Cycles x 10 to the Inch

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