National Exams May 2018

17-Ind-A6, Systems Simulation

3 hours duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.

2. This is a closed book exam. Candidates are permitted to use one of the two permitted calculators (Sharp or Casio models).

3. Candidates are permitted to have an aid sheet consisting of one 8.5" x 11.0" sheet of paper. Writing is permitted on both sides of the paper.

4. This exam consists of three sections (A, B, and C). Within each section, candidates will be given a choice of questions to answer. Please read the instructions for each section carefully. A breakdown of questions and marks is as follows:

Section A:	Do 2 of 3 Questions.	Total marks: 20
Section B:	Do 2 of 3 Questions.	Total marks: 30
Section C:	Do 1 of 2 Questions.	Total marks: 20
Exam:	5 Questions.	Total marks: 70

4. The value of each question is listed in the exam. Remember to check the instructions for each section. DO NOT ATTEMPT TO DO ALL QUESTIONS.

5. Statistical tables are provided.

Part A: Concepts

Complete **two** of the following **four** sets of questions. Do NOT attempt all questions. Please note that all questions have the same value.

1. Consider the following function:

$$f(x) = \begin{cases} \frac{2x}{25} & 0 \le x \le 5 \\ 0 & Otherwise \end{cases}$$

a) Prove that f(x) is a valid probability function.
b) Develop an inverse-transformation for this function.
c) Assume a multiplicative congruential random number generator with parameters: a = 23, m = 100, and x_0 = 17. Generate two random variates from the function for f(x).

10 Marks

2. Now assume that we wish to test a data set to determine if it has come from the pdf f(x) = 2x/25 for 0<= x <= 5. We have calculated the inverse transform as required in Q1 and applied the same MCG from Q1 (a = 11, m = 100, and x_0 = 17) to generate 100 data points for our sample. A frequency chart for the data appears below:

Bin	Count
0.5	21
1.0	10
1.5	10
2.0	10
2.5	10
3.0	5
3.5	10
4.0	5
4.5	10
5.0	10

a. Prove, using a Chi-Squared test, whether that this data does not come from the pdf f(x) = 2x/25 for 0<= x <= 5.
b. Assume that we have correctly calculated the inverse transform for f(x). Can you think of any reason(s) why this data doesn't fit f(x)? What should we do to correct the problem?

10 Marks

3. Consider the following pdf:

$$q(x) = \begin{cases} \frac{3x^2}{2} & -1 \leq x \leq 1 \\ 0 & Otherwise \end{cases}$$

   a. Set up an acceptance-rejection algorithm for this distribution. Use the majorizing function g(x) = 3/2.
   b. Assuming a linear congruential generator with parameters a = 21, m = 100, c = 13, and x_0 = 7, generate two random variates from the distribution q(x).
   c. Can you make any comments about the potential period of the LCG in part b?

10 Marks

4. Using an LCG generator with parameters a = 21, m = 100, c = 13 and x_0 =7 generate (if possible) a random variate from each of the following distributions. For each sub-question, you may restart the LCG stream (i.e. feel free to reuse the same random number for each sub-question)
   a. Normal (15,5)
   b. Gamma (5,4)
   c. Poisson (2)
   d. Weibull (2,2)
   e. Triangular (1, 2, 5)

10 Marks

Part B: Methods

Complete **two** of the following **three** sets of questions.
Do NOT attempt all questions.
Please note that all questions have the same value.

Preamble: Nico Case is an industrial engineer working in Vancouver, BC. Ms. Case is responsible for the design and development of a new warehousing operation for a national manufacturer of yoga clothing. Nico has several design alternatives available, each with differing types of equipment, production strategy, cost, and potential throughput. Accordingly, Ms. Case has decided to build a simulation model to guide her analysis. The following questions relate to this study.

1. Ms. Case is interested in setting the warmup period for her model. She has run a pilot run of her simulation and output hourly throughput for her system. She receives the following data:

Time	Rep 1	Rep 2	Rep 3
1	3	5	4
2	49	40	37
3	20	19	19
4	22	24	24
5	24	20	20
6	22	25	25
7	25	20	22
8	25	21	24
9	21	20	24
10	21	24	25

a) Define what is meant by "transient" as opposed to "steady-state" for a simulation model.
b) Why is it important to identify a warmup period for a simulation model?
c) Using Welch's technique and a moving window with w = 1, estimate the warmup period for this model.

15 Marks

2. Ms. Case must determine a replication length and run time strategy for her simulation. She believes that a 30-day period, after the identification of the transient, is sufficient for her purposes and makes some sense, given the historical data she has available for a similar system that was installed by the Ontario Liquor Control Board in their warehouse in Whitby, Ontario.

Nico has completed a pilot run of 10 replications of 30 simulated days, using the run strategy of batch means. She receives the following output:

Rep	Throughput
1	3500
2	3851
3	4522
4	4756
5	4872
6	4020
7	4618
8	4968
9	4632
10	4548

a) Use the data from the pilot runs to prove to Nico that a 30-day replication length is insufficient.

b) Provide an algorithm (and advice) for determining a proper run length for this model.

c) Compare the run strategies of "batch means" and "replication/deletion". How are these implemented? What are the advantages/disadvantages of each?

d) Assume that Ms. Case wishes to get around the problem in part (a) by implementing a "replication/deletion" strategy with a run length of 30 days. Would this solve the problem? Why or why not?

15 Marks

3. After determining an appropriate warmup period, run length, and run strategy, Nico Case conducts a set of experiments with her model.
   a. In the first set of experiments, she runs her model for 5 replications of one-year, following a 30 day warm up, under the strategy of batch means. Assume that a one-year period is sufficient for statistical purposes. The monthly throughput from her model is 6,000 units/month and the standard deviation is 50. Approximately how many replications are required if Ms. Case wants her experiments to be powered to detect a difference in monthly throughput of 15 units, 19 times out of 20?
   b. Assume that Ms. Case has run her model for 5 replications of 30 days, following a 30-day warmup period. The average monthly throughput from her model is N(6104, 70). The system vendor indicates that a similar system installed in Whitby, ON is known to have a monthly throughput of 6053 units. No standard deviation is provided by the vendor. What can you say about Nico's model, with relation to the vendor estimates?

c. As a final test of validity, Ms. Case adjusts her model to make it representative of her firm's current warehouse layout and logistics. She runs her model for 6 replications of one year and calculates the monthly throughput. She also obtains a sample of data from her firm's warehouse for the previous 6 months.

	Simulation	Warehouse
Month 1	4040	4060
Month 2	4020	4060
Month 3	4005	4051
Month 4	4001	4024
Month 5	4046	4028
Month 6	3977	4061

Is there any evidence that the simulation is representative of the existing warehouse?

15 Marks

Part C: Applications

Complete **one** of the following **two** sets of questions. Do NOT attempt all questions. Please note that all questions have the same value.

1. Nico Case, the Industrial Engineer from Part B, has just completed an experiment with five different scenarios. She has run her scenarios for five different replications, each of 12 months. She obtains the following output, which represents monthly throughput:

	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5
	3993	3979	3965	4007	4052
	3992	3999	3988	4006	4125
	4007	4021	4017	3991	4013
	4010	4015	3971	3995	4014
	3988	4000	4025	3994	4042
Sum	19990	20014	19966	19993	20246
Average	3998.0	4002.8	3993.2	3998.6	4049.2
Variance	96.5	267.2	723.2	54.3	2088.7

a) Which, if any, of the scenarios is provably the best (where larger is better)? Use an alpha of 0.05.
b) Outline all assumptions that underlie your analysis in (a).
c) There are at least two ways of analyzing the dataset above to determine which of the policies is best. Identify one other method for completing this analysis. Describe in your own words (but do not calculate) how you would complete this other analysis.

To aid you in your calculations, you may assume that the overall sum of the data is 100,209 and that the sample average is 4008 with a variance of 982.3233.

20 Marks

2. Ms. Case is interested in setting up a screening experiment to identify significant (and insignificant) factors affecting warehouse throughput. Three factors have been identified:

- Relief practice for staff (Mass relief vs. tag relief)
- Machine availability (3 or 4 automatic storage and retrieval systems)
- Staffing levels (30 or 35 FTEs)

Assume that Ms. Strange has completed five replications of the eight runs laid out in the factorial design matrix in (a). You get the following results:

Run	Rep 1	Rep 2	Rep 3	Rep 4	Rep 5	Sum	Avg	Var
1	4014	4016	4011	4015	4018	20074	4024.38	6.70
2	4021	4014	4012	4014	4013	20074	4025.74	12.70
3	4014	4016	4017	4018	4015	20080	4027.57	2.50
4	4017	4014	4014	4018	4015	20078	4029.88	3.30
5	4033	4034	4034	4034	4032	20167	4033.45	0.80
6	4034	4032	4032	4032	4036	20166	4033.47	3.20
7	4031	4031	4031	4034	4032	20159	4033.60	1.70
8	4034	4037	4034	4035	4037	20177	4044.40	2.30

d) Determine which of three factors are significant, using an Analysis of Variance (ANOVA) model and an α value of 0.05.
e) Can you say anything about interaction between any of the factors?
f) Assuming that more throughput is desirable, determine the overall best setting of the three factors to optimize performance

To assist in your calculations, you may assume that the Grand Sum of overall runs and replications is 160,975 and that the variance of the entire sample is 88.86.

20 Marks

Normal Table

Areas Under the Normal Curve

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.40	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.30	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.20	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.10	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.00	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.90	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.80	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.70	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.60	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.50	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.40	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.30	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.20	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.10	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.00	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.90	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.80	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.70	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.60	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.50	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.40	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.30	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.20	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.10	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.00	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.90	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.80	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.70	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.60	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.50	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.40	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.30	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.20	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.10	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.00	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Areas Under the Normal Curve

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.00	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.10	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.20	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.30	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.40	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.50	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.60	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.70	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.80	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.90	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.00	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.10	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.20	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.30	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.40	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.50	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.60	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.70	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.80	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.90	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.00	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.10	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.20	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.30	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.40	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.50	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.60	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.70	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.80	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.90	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.00	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.10	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.20	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.30	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.40	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998