National Exams December 2016

04-BS-6: Mechanics of Materials

3 hours duration

Notes:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper a clear statement of any assumptions made.

2. Candidates may use one of two calculators, the Casio or Sharp approved models.

   This is a Closed Book exam. However candidates are permitted to bring the following into the examination room:

   - ONE aid sheet 8.5" x 11" hand-written on both sides containing notes and formulae. **Example problems and solutions to problems are not allowed!**

3. Any FIVE (5) questions (out of 8 given) constitute a complete paper. Only the first five questions as they appear in your answer book will be marked.

4. All questions are of equal value.

5. Information on geometric properties of wide flange or W shape sections is attached at the end of this exam. Note that this information may not be required.

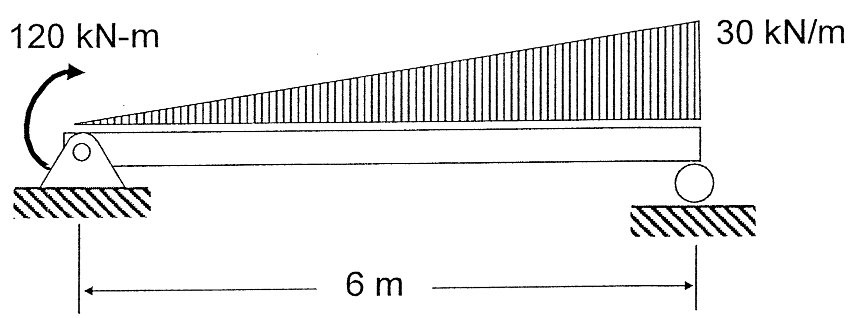
NOTE: The aid sheet must be handed in with the exam!

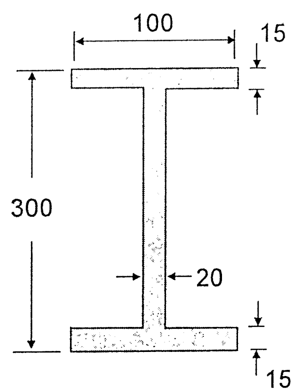
Your exam will not be marked if you do not hand in an aid sheet, unless there is a signed statement by the exam invigilator stating that no aid sheet was used for the exam.

Question 1: **[20 marks]** A simply supported beam supports a triangularly distributed load and a couple acting at the left support as shown. The beam has the cross-section given and is made of steel with a yield strength of 350 MPa and shear stress at yield of 75 MPa. The elastic modulus of the steel is 200 GPa.

Determine the SHEAR FORCE and BENDING MOMENT along the length of the beam as a function of x. In other words, find V(x) and M(x) for the beam.

Then draw the corresponding shear force and bending moment diagrams for the beam (label all critical points and show your work by indicating exactly how you obtained your answers).





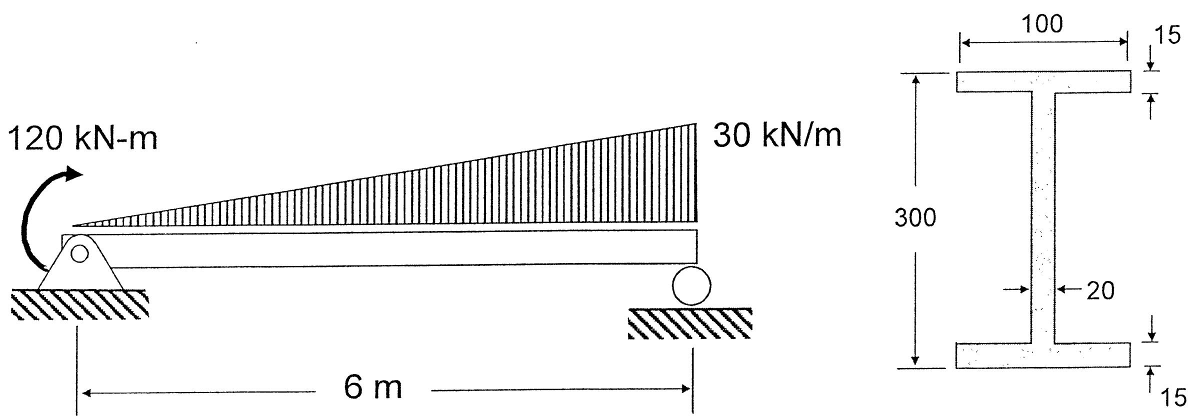
beam cross-section
(all dimensions in mm)

Question 2: A simply supported beam supports a triangularly distributed load and a couple
[20 marks] acting at the left support as shown. The beam has the cross-section given and is made of steel with a yield strength of 350 MPa and shear stress at yield of 75 MPa. The elastic modulus of the steel is 200 GPa.

(a) Determine the maximum deflection using the method of integration.

(b) Determine the slope at the left support using the method of integration.

(c) Sketch the deflected shape of the beam and indicate whether the beam satisfies an allowable deflection limit of L/120 (where L equals the span of the beam).



beam cross-section
(all dimensions in mm)

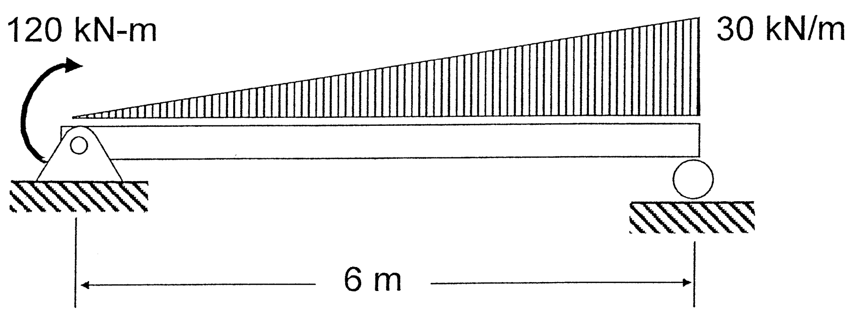
Question 3: A simply supported beam supports a triangularly distributed load and a couple
[20 marks] acting at the left support as shown. The beam has the cross-section given and is made of steel with a yield strength of 350 MPa and shear stress at yield of 75 MPa. The elastic modulus of the steel is 200 GPa.

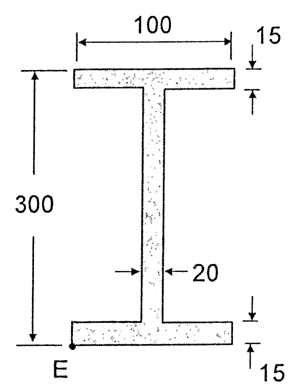
Determine the following:

(a) maximum normal stress in the beam

(b) maximum shear stress in the beam

(c) shear stress at the tip of the flange (point E) at a section located 2 m from the left support. To receive marks you must give reasons to justify your answer.





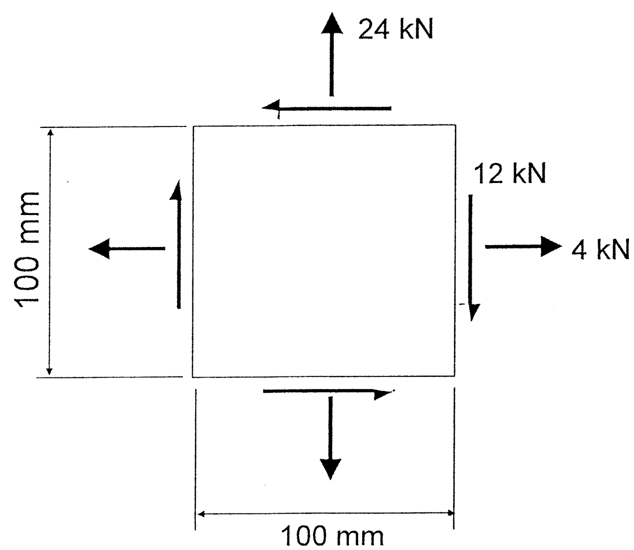
beam cross-section
(all dimensions in mm)

Question 4: A steel plate 100 mm x 100 mm in plan and 2 mm thick is subjected to the axial and shear forces as shown. Ignore stress concentrations from the forces.

[20 marks] Use the Mohr's circle solution (***not*** the transformation equations) to determine the following:

(a) the principal stresses and orientation of the principal planes, showing your answer on a properly oriented element.

(b) the maximum in-plane shear stress (and associated normal stresses) and orientation of the corresponding planes. Once again, show your answer on a sketch of a properly oriented element.



Careful: Mohr's circle involves stresses .... Not forces.

WARNING! Credit will **only** be given for a **solution using Mohr's circle**. Not the stress transformation equations. This means that you need to draw a Mohr's circle based on the stress components given in this problem. Remember to show numbers on your circle. Your calculations must be based on the geometry of your circle. So use your calculator. In other words, you are expected to use trigonometry to construct your Mohr's circle. Do not give a graphical solution that is scaled off!

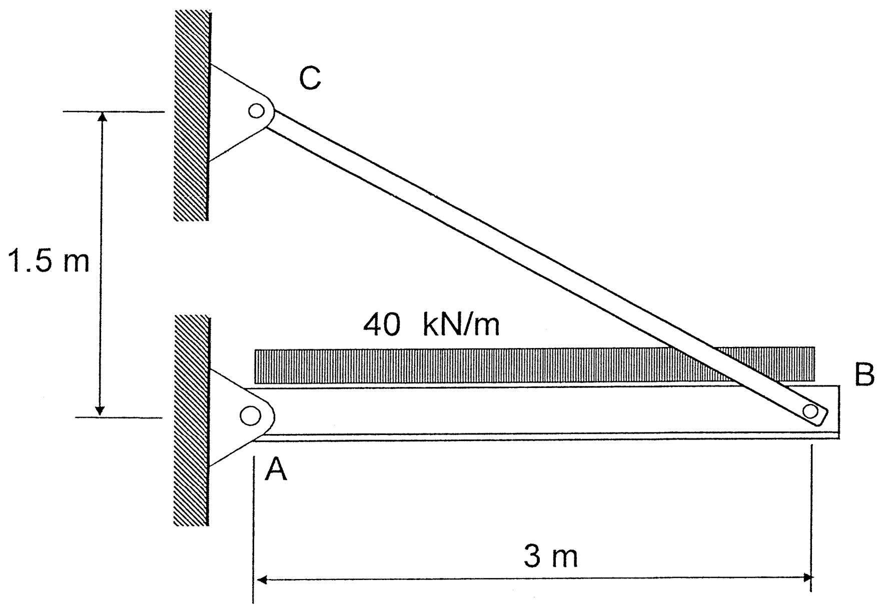
The stress transformation equations can only be used to check your answer.

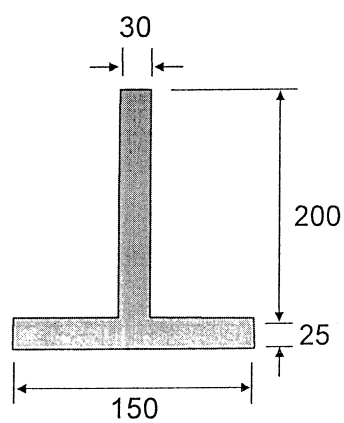
Question 5: A uniformly distributed load of 40 kN/m is applied to a horizontal beam AB that is **[20 marks]** supported by a pinned connection at A and an inclined tension member at B. The beam has the cross section shown and is made of steel with a normal yield stress of 350 MPa and yield stress in shear of 60 MPa. The elastic modulus of the steel equals 200 GPa. The pinned connections at A and B are located at the centroid of the beam cross-section.

(a) Compute the distribution of **normal stress** in the I-beam at a section located **1 m from the support at A**. Show this distribution on a sketch and make sure to show maximum and minimum values of stress.

(b) Compute the maximum **shear stress** in the I-beam at the same section located **1 m from the support at A.**

(c) What happens when the pinned connections at A and B are not located at the centroid of the beam cross-section.



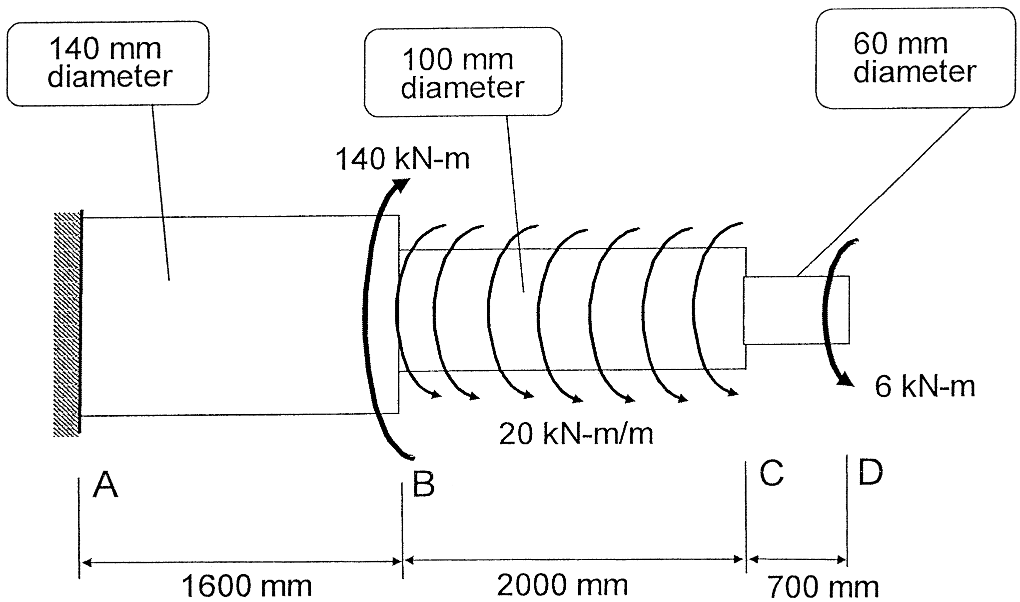


beam cross-section
(all dimensions in mm)

Question 6: A stepped steel shaft with G = 80 GPa and τ_y = 250 MPa is subjected to the
[20 marks] torques shown (note that one of the torques is a distributed load). Dimensions (length and diameter) are also given.

(a) Determine the maximum shear stress in the stepped shaft, and sketch the corresponding variation of shear stress along the shaft radius at this location.

(b) Determine the rotation (in degrees) at the free end of the shaft.

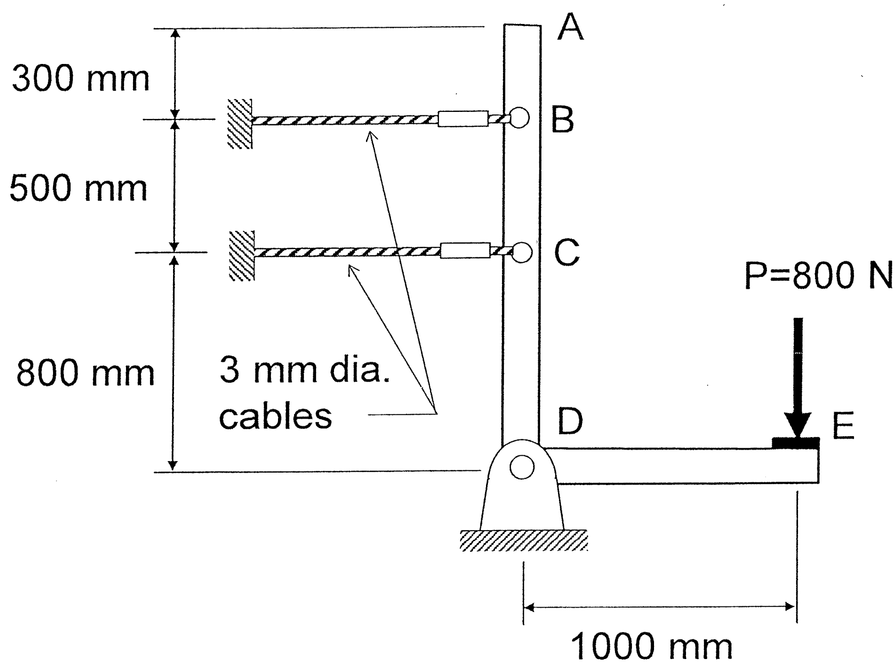


Question 7: **[20 marks]** A rigid L-shaped link (ABCDE) is supported by a 12 mm diameter pin at D and two 3 mm diameter cables at points B and C. Both cables have a length of 1000 mm and are made of steel with a yield strength of 400 MPa and elastic modulus of 200 GPa. The L-shaped link is loaded with a concentrated load at E as shown.

(a) find the forces developed in each cable

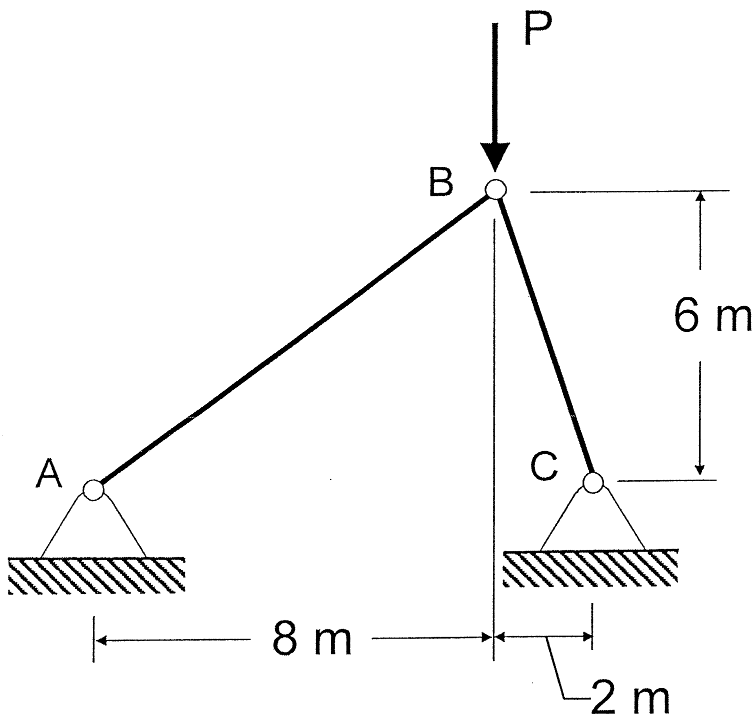
(b) find the corresponding horizontal displacement at the top of the link (point A)

(c) find the shear stress in the pin at D given that the pin is loaded in double shear.

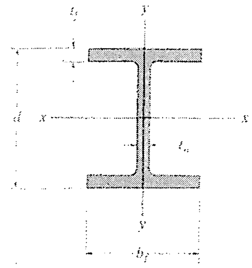


Question 8 **[20 marks]** Determine the largest load P that can be applied to the truss structure below given that members AB and BC are made of 120 mm outside diameter hollow steel pipes with a 10 mm wall thickness. Both steel pipes are pinned at their ends.

Consider in-plane buckling only for the compression members and use a factor of safety of 2 for the Euler buckling load. Do not use a safety factor for yielding of the steel. The steel used in the rods has an allowable yield strength equal to 240 MPa and elastic modulus of 200 GPa.



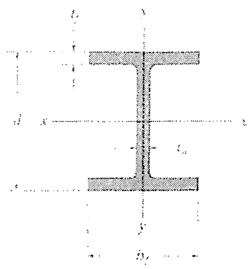
Note: $A_{circle} = \pi r^2$ and $I_{circle} = \pi r^4 / 4$



Wide-Flange Sections or W Shapes SI Units

Designation	Area A	Depth d	Web thickness t_w	Flange width b_f	Flange thickness t_f	x–x axis I	x–x axis S	x–x axis r	y–y axis I	y–y axis S	y–y axis r
mm × kg/m	mm^2	mm	mm	mm	mm	$10^6\ mm^4$	$10^3\ mm^3$	mm	$10^6\ mm^4$	$10^3\ mm^3$	mm
W610 × 155	19 800	611	12.70	324.0	19.0	1 290	4 220	255	108	667	73.9
W610 × 140	17 900	617	13.10	230.0	22.2	1 120	3 630	250	45.1	392	50.2
W610 × 125	15 900	612	11.90	229.0	19.6	985	3 220	249	39.3	343	49.7
W610 × 113	14 400	608	11.20	228.0	17.3	875	2 880	247	34.3	301	48.8
W610 × 101	12 900	603	10.50	228.0	14.9	764	2 530	243	29.5	259	47.8
W610 × 92	11 800	603	10.90	179.0	15.0	646	2 140	234	14.4	161	34.9
W610 × 82	10 500	599	10.00	178.0	12.8	560	1 870	231	12.1	136	33.9
W460 × 97	12 300	466	11.40	193.0	19.0	445	1 910	190	22.8	236	43.1
W460 × 89	11 400	463	10.50	192.0	17.7	410	1 770	190	20.9	218	42.8
W460 × 82	10 400	460	9.91	191.0	16.0	370	1 610	189	18.6	195	42.3
W460 × 74	9 460	457	9.02	190.0	14.5	333	1 460	188	16.6	175	41.9
W460 × 68	8 730	459	9.14	154.0	15.4	297	1 290	184	9.41	122	32.8
W460 × 60	7 590	455	8.00	153.0	13.3	255	1 120	183	7.96	104	32.4
W460 × 52	6 640	450	7.62	152.0	10.8	212	942	179	6.34	83.4	30.9
W410 × 85	10 800	417	10.90	181.0	18.2	315	1 510	171	18.0	199	40.8
W410 × 74	9 510	413	9.65	180.0	16.0	275	1 330	170	15.6	173	40.5
W410 × 67	8 560	410	8.76	179.0	14.4	245	1 200	169	13.8	154	40.2
W410 × 53	6 820	403	7.49	177.0	10.9	186	923	165	10.1	114	38.5
W410 × 46	5 890	403	6.99	140.0	11.2	156	774	163	5.14	73.4	29.5
W410 × 39	4 960	399	6.35	140.0	8.8	126	632	159	4.02	57.4	28.5
W360 × 79	10 100	354	9.40	205.0	16.8	227	1 280	150	24.2	236	48.9
W360 × 64	8 150	347	7.75	203.0	13.5	179	1 030	148	18.8	185	48.0
W360 × 57	7 200	358	7.87	172.0	13.1	160	894	149	11.1	129	39.3
W360 × 51	6 450	355	7.24	171.0	11.6	141	794	148	9.68	113	38.7
W360 × 45	5 710	352	6.86	171.0	9.8	121	688	146	8.16	95.4	37.8
W360 × 39	4 960	353	6.48	128.0	10.7	102	578	143	3.75	58.6	27.5
W360 × 33	4 190	349	5.84	127.0	8.5	82.9	475	141	2.91	45.8	26.4

WIDE-FLANGE SECTIONS OR W SHAPES FPS UNITS



Wide-Flange Sections or W Shapes SI Units

Designation	Area A	Depth d	Web thickness t_w	Flange width b_f	Flange thickness t_f	x–x axis I	x–x axis S	x–x axis r	y–y axis I	y–y axis S	y–y axis r
mm × kg/m	mm^2	mm	mm	mm	mm	10^6 mm^4	10^3 mm^3	mm	10^6 mm^4	10^3 mm^3	mm
W310 × 129	16 500	318	13.10	308.0	20.6	308	1940	137	100	649	77.8
W310 × 74	9 480	310	9.40	205.0	16.3	165	1060	132	23.4	228	49.7
W310 × 67	8 530	306	8.51	204.0	14.6	145	948	130	20.7	203	49.3
W310 × 39	4 930	310	5.84	165.0	9.7	84.8	547	131	7.23	87.6	38.3
W310 × 33	4 180	313	6.60	102.0	10.8	65.0	415	125	1.92	37.6	21.4
W310 × 24	3 040	305	5.59	101.0	6.7	42.8	281	119	1.16	23.0	19.5
W310 × 21	2 680	303	5.08	101.0	5.7	37.0	244	117	0.986	19.5	19.2
W250 × 149	19 000	282	17.30	263.0	28.4	259	1840	117	86.2	656	67.4
W250 × 80	10 200	256	9.40	255.0	15.6	126	984	111	43.1	338	65.0
W250 × 67	8 560	257	8.89	204.0	15.7	104	809	110	22.2	218	50.9
W250 × 58	7 400	252	8.00	203.0	13.5	87.3	693	109	18.8	185	50.4
W250 × 45	5 700	266	7.62	148.0	13.0	71.1	535	112	7.03	95	35.1
W250 × 28	3 620	260	6.35	102.0	10.0	39.9	307	105	1.78	34.9	22.2
W250 × 22	2 850	254	5.84	102.0	6.9	28.8	227	101	1.22	23.9	20.7
W250 × 18	2 280	251	4.83	101.0	5.3	22.5	179	99.3	0.919	18.2	20.1
W200 × 100	12 700	229	14.50	210.0	23.7	113	987	94.3	36.6	349	53.7
W200 × 86	11 000	222	13.00	209.0	20.6	94.7	853	92.8	31.4	300	53.4
W200 × 71	9 100	216	10.20	206.0	17.4	76.6	709	91.7	25.4	247	52.8
W200 × 59	7 580	210	9.14	205.0	14.2	61.2	583	89.9	20.4	199	51.9
W200 × 46	5 890	203	7.24	203.0	11.0	45.5	448	87.9	15.3	151	51.0
W200 × 36	4 570	201	6.22	165.0	10.2	34.4	342	86.8	7.64	92.6	40.9
W200 × 22	2 860	206	6.22	102.0	8.0	20.0	194	83.6	1.42	27.8	22.3
W150 × 37	4 730	162	8.13	154.0	11.6	22.2	274	68.5	7.07	91.8	38.7
W150 × 30	3 790	157	6.60	153.0	9.3	17.1	218	67.2	5.54	72.4	38.2
W150 × 22	2 860	152	5.84	152.0	6.6	12.1	159	65.0	3.87	50.9	36.8
W150 × 24	3 060	160	6.60	102.0	10.3	13.4	168	66.2	1.83	35.9	24.5
W150 × 18	2 290	153	5.84	102.0	7.1	9.19	120	63.3	1.26	24.7	23.5
W150 × 14	1 730	150	4.32	100.0	5.5	6.84	91.2	62.9	0.912	18.2	23.0