National Exams May 2014
04-BS-1, Mathematics
3 hours Duration

Notes:

1. If doubt exists as to the interpretation of any question, the candidate is urged to include a clear statement of any assumptions made along with their answer.
2. Any APPROVED CALCULATOR is permitted. This is a CLOSED BOOK exam. However, candidates are permitted to bring ONE AID SHEET written on both sides.
3. Any five questions constitute a complete paper. Only the first five questions as they appear in your answer book will be marked.
4. All questions are of equal value.

Marking Scheme:

1. (a) 10 marks, (b) 10 marks
2. 20 marks
3. 20 marks
4. (a) 8 marks, (b) 12 marks
5. 20 marks
6. 20 marks
7. 20 marks
8. 20 marks

1. For each of the following differential equations, find the general solution, $y(x)$.

   (a) $y'' + 9y = \sec 3x$

   (b) $y'' - y' - 6y = 3x^2 + e^{-2x}$

   Note that $'$ denotes differentiation with respect to x.

2. Find the maximum and minimum values of $f(x, y, z) = x + y - z$ over the sphere $x^2 + y^2 + z^2 = 1$.

3. Find the line tangent to the intersection of the surfaces

$$3x^2 + 2y^2 - 2z = 1$$

   and

$$x^2 + y^2 + z^2 - 4y - 2z + 2 = 0$$

   at the point $(1, 1, 2)$.

4. Let $A = \begin{pmatrix} 3 & 1 \\ -2 & 1 \end{pmatrix}$.

   (a) Find the eigenvalues and eigenvectors of A.

   (b) Solve the initial value problem

$$\begin{aligned} x' &= 3x + y, & x(0) &= 1, \\ y' &= -2x + y, & y(0) &= 0. \end{aligned}$$

5. Evaluate the surface integral $\iint_S \mathbf{F} \cdot dS$ where $\mathbf{F}(x, y, z) = yz\mathbf{i} - 2xy\mathbf{j} + 3z\mathbf{k}$ and S is the surface of the region bounded above by the paraboloid $z = 4 - x^2 - y^2$ and below by the plane $z = 0$.

6. Find the volume of the region bounded by the paraboloid $z = \frac{7}{4} + \frac{1}{4}(x^2 + y^2)$ and the plane $z = 4$ that lies outside the cone $z^2 - 4x^2 - 4y^2 = 0$.

7. Let C be the curve formed by the intersection of the cylinder $x^2 + y^2 = 9$ and the plane $z = 1 + y - 2x$, travelled clockwise as viewed from the positive z-axis, and let $\mathbf{v}$ be the vector function $\mathbf{v} = 4z\mathbf{i} - 2y\mathbf{j} + 2y\mathbf{k}$. Evaluate the line integral $\oint_C \mathbf{v} \cdot d\mathbf{r}$.

8. Compute the response of the damped mass-spring system modelled by

$$y'' + 3y' + 2y = r(t), \qquad y(0) = 0, \qquad y'(0) = 0,$$

   where r is the square wave

$$r(t) = \begin{cases} 1, & 1 \le t < 2, \\ 0, & \text{otherwise}, \end{cases}$$

   and $'$ denotes differentiation with respect to time.