National Exams May 2016

98-Ind-A1

Operations Research

3 hours duration

NOTES:

1 If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.

2 Any non-communicating calculator is permitted. This is an Open Book exam. Note to candidates: You must indicate the type of calculator being used, i.e. write the name and model designation of the calculator, on the first left hand sheet of the exam workbook.

3 There may be more questions than you are able to answer in the allotted time. Although the total value of the questions is 160, any marks achieved will be considered toward the 100 total requirements.

20 1. Two investments with varying cash flows are available as follows:

Investment	Oct. 1, yr 0	Oct. 1, yr +1	Oct. 1, yr +2	Oct. 1, yr +3
A	($6,000)	($5,000)	$7,000	$9,000
B	($8,000)	($3,000)	$9,000	$7,000

Any fraction of an investment can be purchased this Oct. 1st when you will have $10,000 available for investment, and a year later you will have $7,000 available. Assuming that the prevailing interest rate is 4%, formulate, but do not solve, the linear programming model to determine how the money should be used to maximize its Net Present Value.

20 2. Perform two iterations of the Revised Simplex method for the following linear program:

Max $2x_1-x_2+x_3$ s.t. $3x_1+x_2+x_3 \leq 60$; $x_1-x_2+2x_3 \leq 10$; $x_1+x_2-x_3 \leq 20$; all $x_i \geq 0$.

20 3. The ABC electric company owns 3 hydro-electric power generation stations: A, B and C. The stations are located at reservoirs with dams across the Pristine River. Station A is located 10 Km upstream from station B, and station B is located 20 Km upstream from station C. Water is measured in units of ML and electricity in units of MW. The only entry of water into the Pristine River is at station A, 100 ML in each hour. Water travels down the river at an average speed of 10 Km/hr. For each hour the station manager has to decide how much of the water arriving at the station is: a) "used" to generate electricity and then allowed to proceed downstream, b) "spilled" to proceed downstream without producing electricity, and c) "stored" in the reservoir for later use or spill. Each plant has a different generating efficiency determined by the drop in elevation at each plant; the larger the drop, the larger the efficiency. The generating efficiencies at A, B, and C, are respectively 1.5, 4.2 and 8.5 MW/ML. Each plant also has a different maximum capacity determined by the size of the generating units at each plant. The maximum capacities at A, B, and C are respectively 50, 100 and 150 MW of electricity for each hour. ABC's hourly revenue is calculated as the product of the hourly MW production at each plant times the hourly price of electricity (λi). As a matter of policy ABC always returns each reservoir by the end of the planning horizon to the same volume it had at the beginning of the planning horizon.

Draw a labelled Network Flow model that will determine the optimal generation and water release policy that maximizes the revenue over an 8-hour planning horizon. Do not solve.

20 4. A paper mill produces jumbo reels of paper which are 150 cm. wide, and which then need to be cut into reels of standard smaller widths to meet the market demand. The demand for the upcoming season is as follows:

Standard Widths (in cm)	30	45	60	90
Demand (in number of rolls)	200	150	100	50

The mill cuts the jumbo reels to meet the demand with minimal waste. Waste is defined as any leftover portion of the cut jumbo reels that cannot be used to meet the demand. Waste is incurred by either trim loss or surplus. Trim loss is incurred, for example, if a jumbo reel is cut into three 45 cm. reels. The left-over 15 cm. reel needs to be disposed as there is no demand for this size. Surplus is incurred, for example, if 100 jumbo reels are each cut into two 45 cm. and one 60 cm. reels. The demand for 60 cm. reels will be fully satisfied, but there will be a 50 reel (2*100 – 150) surplus of 45 cm. reels. The combinations of various standard widths and trim waste that can be obtained from a jumbo reel are called patterns. The table below describes pattern #1.

Pattern	Standard Reel Sizes				Trim waste
	30	45	60	90	
1	5	0	0	0	0

Formulate the integer programming model that finds the number of jumbo reels to be cut according to each pattern, to meet demand with minimal waste

20 5. Thanks to a gift from her family on her 60th birthday, Delma Britt will definitely be going on a major trip worth $10,000 in 6 months, and as long as she is well, she will enjoy every penny. The main risk is that she may not be able to enjoy the trip because of a medical problem. She believes there is an 80% chance she will feel just fine, but a 20% chance she will not be able to enjoy the trip. She could check into the Malo Clinic at a cost of $2000, for a thorough screening and cure at an additional $1000. The Malo clinic specializes in her condition and she is sure they can screen her and if need be treat her within a one-month period. It costs $2000 to be screened by the Malo clinic, and an additional $1000 to be treated and cured, in time for her trip. To help her decide whether or not she should go to the Malo clinic she can undergo a medical examination at a cost of $500, or even go for a second opinion for an additional $400. Because it takes up to four months to get the appointments she must decide on the medical exams right away. Each examination has a 90% chance of detecting the problem right away, if she indeed has a problem, but also a 40% chance of a false positive, i.e. telling her she has the condition when she does not have it. Draw and solve a decision tree to guide Delma on what she should do.

20 6. A two-runway (one runway for landing, one runway for taking off) airport is being designed for propeller-driven aircraft. The time to land an airplane is known to be exponentially distributed, with a mean of 1-1/2 minutes. If airplane arrivals are assumed to occur at random, what arrival rate can be tolerated if the average wait in the sky is not to exceed 3 minutes?

20 7. The number of crimes in each of a city's three police precincts depends on the number of patrol cars assigned to each precinct (per the table below). Five patrol cars are available. The problem is to determine how many patrol cars should be assigned to each precinct to minimize the overall number of crimes. Solve the problem using dynamic programming

	No. Of Patrol Cars Assigned to Precinct					
	0	1	2	3	4	5
Precinct 1	25	17	10	4	1	0
Precinct 2	36	27	20	14	12	11
Precinct 3	30	20	12	8	6	5

20 8. During month t (t = 1, 2, . . . , 60), expert safecracker Dirk Stack knows that he will be offered a role in a bank job that will pay him d dollars. There is, however, a probability p_t that month t's job will result in his capture. If Dirk is captured, all his money will be lost. Dirk's goal is to maximize his expected asset position at the end of month 60. Formulate a dynamic programming recursion that will help Dirk accomplish his goal. At the beginning of month 1, Dirk has $50,000.