National Exams

07-Elec-B1, Digital Signal Processing

May 2013

3 Hours Duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.

2. Approved calculator is permitted. This is a CLOSED BOOK EXAM, but one aid sheet is allowed written on both sides

3. There are five questions, however, FOUR(4) questions constitute a complete paper. The first four questions as they appear in the answer book will be marked.

4. All questions are of equal value.

5. Clarity and organization of the answer are important.

1. **(25 marks total)** Let $u[n] = \{a, b, c\}$ and $v[n] = \{0, 1\}$ be two finite-length, real-valued sequences.

   (a) **(3 marks)** Determine $z_l[n] = u[n] \circledast v[n]$, the linear convolution of the sequences $u[n]$ and $v[n]$.

   (b) **(3 marks)** Determine $z_c[n] = u[n] \textcircled{3} v[n]$, the 3-point circular convolution of the sequences $u[n]$ and $v[n]$.

   (c) **(5 marks)** Let $\{x[n]\} = \{x[0], x[1]\}$ be a 2-element sequence and let $\{X[k]\} = \{X[0], X[1]\}$ be the 2-point Discrete Fourier Transform (DFT) of $\{x[n]\}$. Determine the signal flow diagram for a 2-input, 2-output system, the "box", where $\{x[n]\}$ is the input and $\{X[k]\}$ is the output.

   (d) **(4 marks)** Assume that you are ONLY allowed to perform scalar multiplications external to the "box" you determined in part (c). Show how you would use the "box" such that its output is $x[n]$ when $\{X[k]\}$ is the input.

   (e) **(5 marks)** Determine $z_l[n]$ using 2-point DFT and 2-point IDFT operations. (You may want to use what you derived in parts (c) and (d).)

   (f) **(5 marks)** Let $v[n]$ be a length L_v sequence, and let $u[n]$ be a right-hand sequence (a sequence that is non-zero for $n \leq 0$). You are asked to calculate $z_l[n] = u[n] \circledast v[n]$ using "block filtering" (i.e., using either the overlap-add or the overlap-save method) implemented via L-point DFT/IDFT operations. Determine the minimum allowed value of L.

2. **(25 marks total)** Given the z-plane pole/zero plot, associated with a 2nd-order IIR digital filter, in Figure 1.

   (a) **(7 marks)** What is the $H(z)$ transfer function, in terms of z^{-1} and z^{-2}, filter having two poles and a single zero on the z-plane shown in the Figure 1.?

   (b) **(8 marks)** Draw the Direct Form I block diagram of the $H(z)$ filter that implements the transfer function arrived at in Part (a) of this problem.

   (c) **(10 marks)** Draw at least two new block diagrams of the $H(z)$ filter that eliminates one of the multipliers in the Direct Form I block diagram.

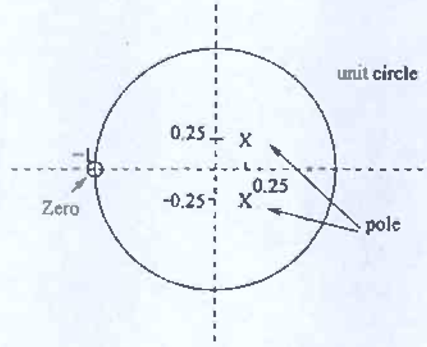

Figure 1:

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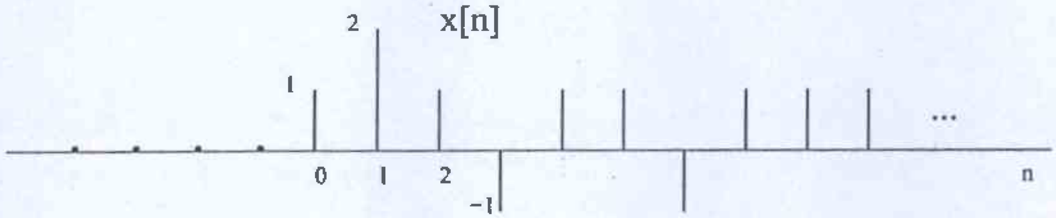
3. **(25 marks total)** Consider a linear time-invariant system whose system transfer function $H(z)$ is

$$H(z) = \frac{z^{-2}}{(1 - \frac{1}{2}z^{-1})(1 - 2z^{-1})}$$

(a) **(7 marks)** Suppose the system is known to be stable. Determine the output $y[n]$ when the input $x[n]$ is the unit step sequence.

(b) **(8 marks)** Suppose the region of convergence of $H(z)$ includes $z = \infty$. Determine $y[n]$ evaluated at $n = 2$ when $x[n]$ is as shown in the below Figure.



(c) **(10 marks)** Suppose we wish to recover $x[n]$ from $y[n]$ by processing $y[n]$ with an LTI system whose impulse response is given by $h_i[n]$. Determine $h_i[n]$. Does $h_i[n]$ depend on the region of convergence of $H(z)$?, explain.

4. **(25 marks total)** Suppose we have two four-point sequences $x[n]$ and $h[n]$ as follows:

$$x[n] = \cos(\frac{\pi n}{2}), \quad n = 0, 1, 2, 3, \qquad h[n] = (\frac{1}{2})^n, \quad n = 0, 1, 2, 3.$$

   (a) **(5 marks)** Calculate the four-point DFT $X[k]$.

   (b) **(5 marks)** Calculate the four-point DFT $H[k]$.

   (c) **(7 marks)** Calculate $y[n] = x[n] ④ h[n]$ by doing the circular convolution directly.

   (d) **(8 marks)** Calculate $y[n]$ of part (c) by multiplying the DFTs of $x[n]$ and $h[n]$ and performing an inverse DFT.

5. **(25 marks total)** Consider the system shown in Figure 2. The input to this system is the bandlimited signal whose Fourier transform is shown in Figure 3. with $\Omega_0 = \frac{\pi}{T}$. The discrete-time LTI system in Figure 2. has the frequency response shown in Figure 4.

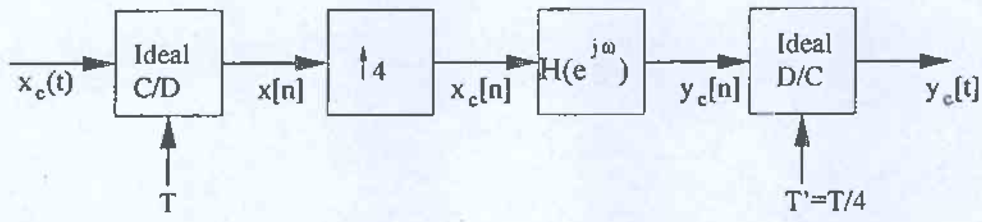


Figure 2:

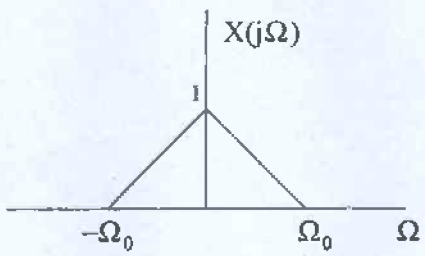


Figure 3:

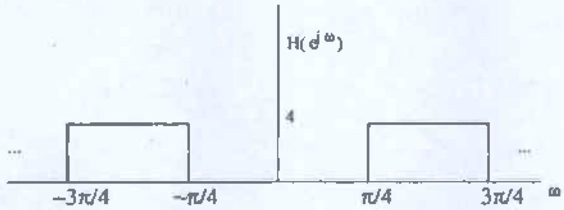


Figure 4:

(a) **(16 marks)** Sketch the Fourier transforms $X(e^{j\omega})$, $X_c(e^{j\omega})$, $Y_c(e^{j\omega})$, and $Y_c(j\Omega)$.

(b) **(9 marks)** For the general case when $X_c(j\Omega) = 0$ for $|\Omega| \geq \pi/T$, express $Y_c(j\Omega)$ in terms of $X_c(j\Omega)$. Also, give a general expression for $y_c(t)$ in terms of $x_c(t)$ when $x_c(t)$ is band-limited in this manner.