National Exams May 2018

17-Phys-A1, Classical Mechanics

3 hours duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.

2. This is a CLOSED BOOK EXAM.
   One of two calculators is permitted - any Casio or Sharp approved model.

3. FIVE (5) questions constitute a complete exam paper.
   The first five questions as they appear in the answer book will be marked.

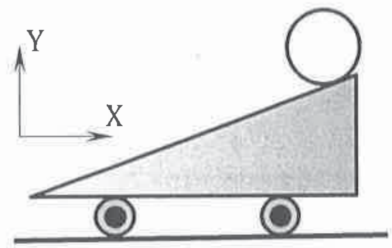
4. Each question is of equal value.

5. Most questions require an answer in essay format. Clarity and organization of the answer are important.

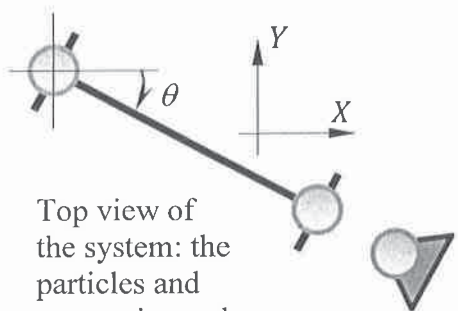
MARKING SCHEME:

1. (a) 6 marks (b) 6 marks (c) 8 marks
2. (a) 3 marks (b) 3 marks (c) 3 marks (d) 3 marks (e) 5 marks (f) 3 marks
3. 20 marks
4. (a) 3 marks (b) 3 marks (c) 3 marks (d) 3 marks (e) 3 marks (f) 5 marks
5. 20 marks
6. 20 marks

1. A disk, of 1-*m* diameter and mass m_d = 2-*kg* mass, was initially at rest on top of a slanted surface of 30-degree and mass M_s = 5-*kg*; see Figure. The slanted surface is free to roll without slipping after the disk is released. Assume the motion maintains its plane-of-motion in the X-Y plane as shown in the Figure. Neglect the mass of the wheels of the slanted surface and answer the following questions. (a) What type of constraint is best suited to describe the motion of the disk as it rolls down the slope? (b) How many degrees-of-freedom are needed to describe the motion of the disk? (c) Analyze the conservation of linear momentum of the disk and the surface considered as one unit.



2. Consider the planar motion of two identical particles of mass m connected by a, massless, rigid rod of length l on the horizontal plane. The support to the particle has a particular geometrical shape that only allows the particle to move perpendicular to the rod. This is achieved by a support with an flat base (see Figure). Assume no friction between the support base and the horizontal plane. Let the position of the particles be written as (x_1, y_1) and (x_2, y_2) and the position of the centre be (x, y) where $x = (x_1 + x_2)/2, y = (y_1 + y_2)/2$. It is important to note that, due to the supports, there is a constraint force which prevents the system moving in an arbitrary path on the plane. Answer the following questions.



Top view of the system: the particles and connecting rod

Side view of the particle and its flat-base support

(a) Write down the holonomic constraints in $x_1, y_1, x_2, y_2,$ x, y and $\dot{\theta}$, the angular rate of rotation of the bar.

(b) Write down the nonholonomic constraint and explain whether it is scleronomic or rheonomic.

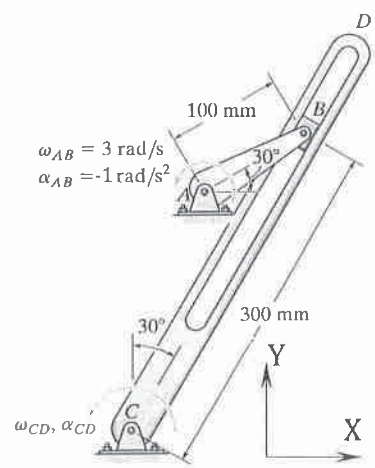
(c) Find the kinetic energy of the system based on the centre location of the bar.

(d) Use Lagrange's equation with Lagrange multiplier to find the equations of motion.

(e) Find the constraint force and comment on its physical meaning.

(f) Comment on the motion path of the bar.

3. The mechanical system shown in the Figure consists of a crank AB, a slider B and a slotted rod CD. At the instant shown, the crank has an angular velocity and acceleration, $\vec{\omega}_{AB} = 3\vec{k}\, rad/s$, $\vec{\alpha}_{AB} = -1\vec{k}\, rad/s$, respectively. Determine the velocity and acceleration of the slider B relative to the slotted rod at this instant.



4. A triangular block of mass $m_A = 2$ lb is held at rest on a smooth inclined plane by a stop block at S. A bullet of mass $m_B = 0.1$ lb is fired at the block horizontally. Assume the bullet is embedded in the block after impact. Let the magnitude of the average impact force between the block and the bullet be F_{imp}, the velocity of the block right before (with superscript '–') and right after (with superscript '+') the impact as $\vec{v}_A^- = \vec{0}$, $\vec{v}_A^+$, respectively. Note the block only moves in the x–axis direction shown in the figure. The velocity of the bullet right before the impact, denoted as $\vec{v}_B^-$, is 200 m/s. Assume the impact lasted $\Delta t = 0.015$ sec.

(a) Write down the impulse momentum equation for only the triangular block A in the (lower-case) x–y coordinate system. Express your answer in m_A, m_B, $\vec{v}_A^+$, and forces which exist to define the impulse during the impact. Please note the result must be in vector form.

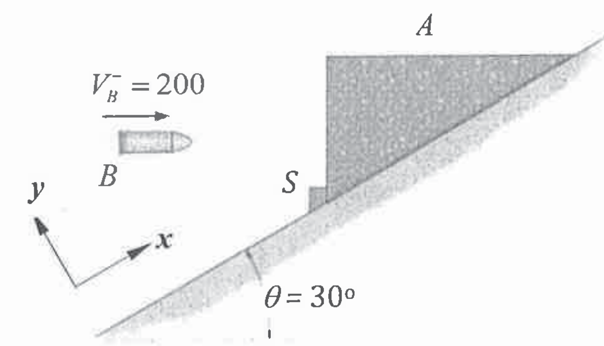
(b) Write down the impulse momentum equation for both the bullet B and the triangular block A in the x–y coordinate system. Express your answer in m_A, m_B, $\vec{v}_A^+$, $\vec{v}_B^-$, $\vec{v}_B^+$ and forces which exist to define the impulse during the impact. Please note the result must be in vector form.

(c) During the impact, which of the forces applied to the triangular block change from non-impulsive to impulsive?

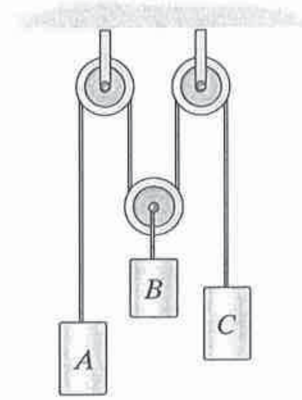
(d) For further analysis, it is possible to ignore the impulse contributed by the weights of A and B ($m_A g$, $m_B g$). Explain why that is the case.

(e) Find $\vec{v}_A^+$ and F_{imp}.

(f) Assume the bullet bounces off the triangular block and the coefficient of restitution is e. Let the velocity of the bullet before and after the impact be $\vec{v}_B^- = V_{Bx}^- \vec{\imath} + V_{By}^- \vec{\jmath}$, $\vec{v}_B^+ = V_{Bx}^+ \vec{\imath} + V_{By}^+ \vec{\jmath}$, respectively, and that of the block right after the impact be $\vec{v}_A^+ = V_{Ax}^+ \vec{\imath}$, where $\vec{\imath}, \vec{\jmath}$ are the basis vectors of the x–y coordinate system,. Write down the equation for the coefficient of restitution in terms of V_{Bx}^-, V_{By}^-, V_{Bx}^+, V_{By}^+, V_{Ax}^+, θ. There is no numeric value to compute for this sub question.)



5. In the cable-pulley system shown here, block A is moving upwards at a speed of 5 m/s and block C is moving downwards at a speed of 2.5 m/s. What is the speed of block B?



6. At the instant shown, the cable attached to point B snaps. The rod has a mass of 1 kg. Determine the angular acceleration of the rod AB at this moment.