National Exams May 2017
04-BS-1, Mathematics
3 hours Duration

Notes:

1. If doubt exists as to the interpretation of any question, the candidate is urged to include a clear statement of any assumptions made along with their answer.
2. Any APPROVED CALCULATOR is permitted. This is a CLOSED BOOK exam. However, candidates are permitted to bring ONE AID SHEET written on both sides.
3. Any five questions constitute a complete paper. Only the first five questions as they appear in your answer book will be marked.
4. All questions are of equal value.

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Marking Scheme:

1. (a) 7 marks, (b) 7 marks, (c) 6 marks
2. 20 marks
3. 20 marks
4. 20 marks
5. (a) 10 marks, (b) 10 marks
6. 20 marks
7. 20 marks
8. 20 marks

1. Find the general solutions of the following differential equations:

   (a) $x^2y' + 2xy = 2\sin(3x)$,

   (b) $y' + 2xy^2 = 0$,

   (c) $3y'' + 5y' - 2y = 0$.

   Note that in each case, $'$ denotes differentiation with respect to $x$.

2. Find the general solution to the differential equation $2x^2y'' + xy' - 3y = 4/x^2$.
   Note that $'$ denotes differentiation with respect to $x$.

3. Find the maximum and minimum values of $f(x, y, z) = x + y - z$ over the sphere $x^2 + y^2 + z^2 = 1$.

4. Solve the initial value problem

$$\begin{aligned} x' &= 3x + y, & x(0) &= 1, \\ y' &= -2x + y, & y(0) &= 0. \end{aligned}$$

5. Let $P$ be the plane passing through the three points (2,1,-2), (1,2,0) and (1,0,-1).

   (a) Find an equation representing the plane $P$.

   (b) Find the line of intersection between the plane $P$ and the plane $x + y - 2z = 3$

6. Let $C$ be the curve formed by the intersection of the cylinder $x^2 + y^2 = 1$ and the plane $z = 1 + y$, and let $\mathbf{v}$ be the vector function $\mathbf{v} = 4z\mathbf{i} - 2x\mathbf{j} + 2x\mathbf{k}$. Evaluate the line integral $\oint_C \mathbf{v} \cdot d\mathbf{r}$. Assume a clockwise orientation for the curve when viewed from above.

7. Let $S$ be the boundary of the region enclosed by the paraboloid $z = x^2 + y^2 - 2$ and the plane $z = 2$ and let

$$\mathbf{F}(x, y, z) = xy^2\mathbf{i} + 2xyz\mathbf{j} - xz^2\mathbf{k}.$$

   Evaluate the surface integral $\iint_S \mathbf{F} \cdot \mathbf{n}\, dA$, where $\mathbf{n}$ is the unit outward normal on $S$.

8. Compute the response of the damped mass-spring system modelled by

$$y'' + 3y' + 2y = r(t), \qquad y(0) = 0, \qquad y'(0) = 0,$$

   where $r$ is the square wave

$$r(t) = \begin{cases} 1, & 1 \le t < 2, \\ 0, & \text{otherwise}, \end{cases}$$

   and $'$ denotes differentiation with respect to time.