National Exams May 2016

04-BS-6: Mechanics of Materials

3 hours duration

Notes:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper a clear statement of any assumptions made.

2. Candidates may use one of two calculators, the Casio or Sharp approved models.

   This is a Closed Book exam. However candidates are permitted to bring the following into the examination room:

   - ONE aid sheet 8.5" x 11" hand-written on both sides containing notes and formulae. **Example problems and solutions to problems are not allowed!**

3. Any FIVE (5) questions (out of 8 given) constitute a complete paper. Only the first five questions as they appear in your answer book will be marked.

4. All questions are of equal value.

5. Information on geometric properties of wide flange or W shape sections is attached at the end of this exam.

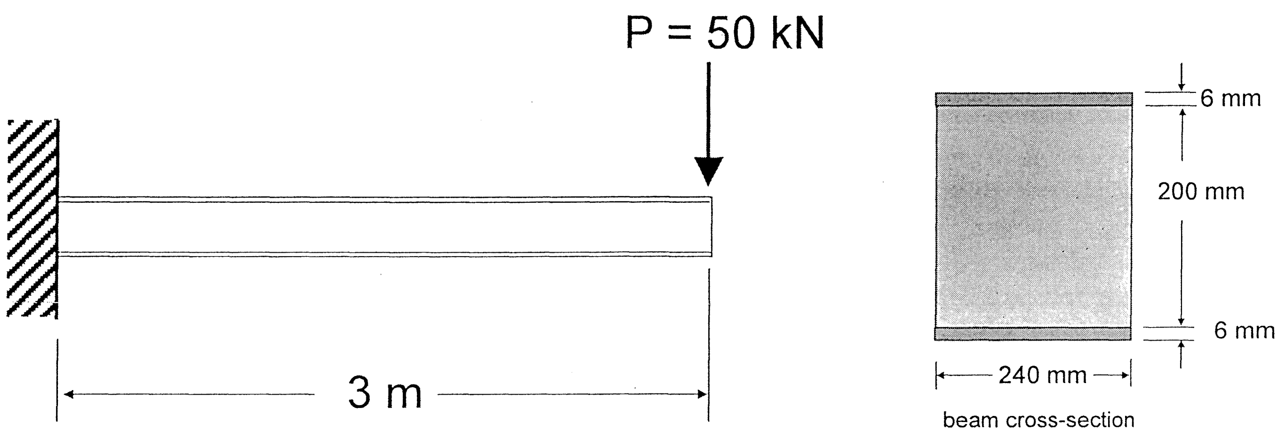
NOTE: The aid sheet must be handed in with the exam!

Your exam will not be marked if you do not hand in an aid sheet, unless there is a signed statement by the exam invigilator stating that no aid sheet was used for the exam.

Question 1: The cantilevered beam shown below is a sandwich beam with a plastic core and aluminum alloy faces. The member is subjected to a concentrated load at the free end. The plastic core (240 mm x 200 mm in cross-section) has an elastic modulus of 100 GPa and allowable normal stress of 220 MPa, while the 6 mm thick aluminum face plates have an elastic modulus of 75 GPa and allowable normal stress of 260 MPa.

[18 marks] (a) Determine whether the composite beam can support the loading shown. (remember to check for failure in each material)

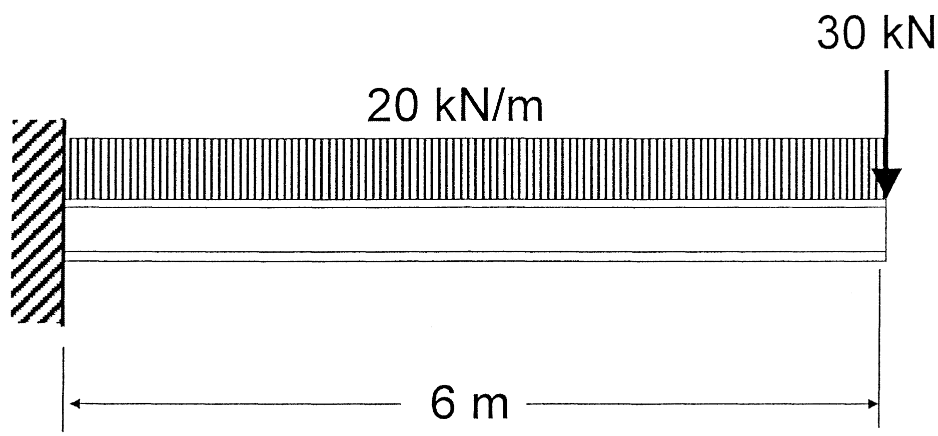
[2 marks] (b) Give the maximum load P the beam can support without causing failure.



Question 2: A cantilevered beam is subjected to a uniformly distributed load in addition to a concentrated load acting at the free end of the member. The beam is a wide flange W610 x 125 section and is made of steel with an allowable normal stress of 240 MPa and allowable shear stress of 60 MPa. The elastic modulus of the steel equals 200 GPa. Refer to the attached table for section properties.

[19 marks] (a) Determine the deflection and slope at the free end of the beam using the method of integration.

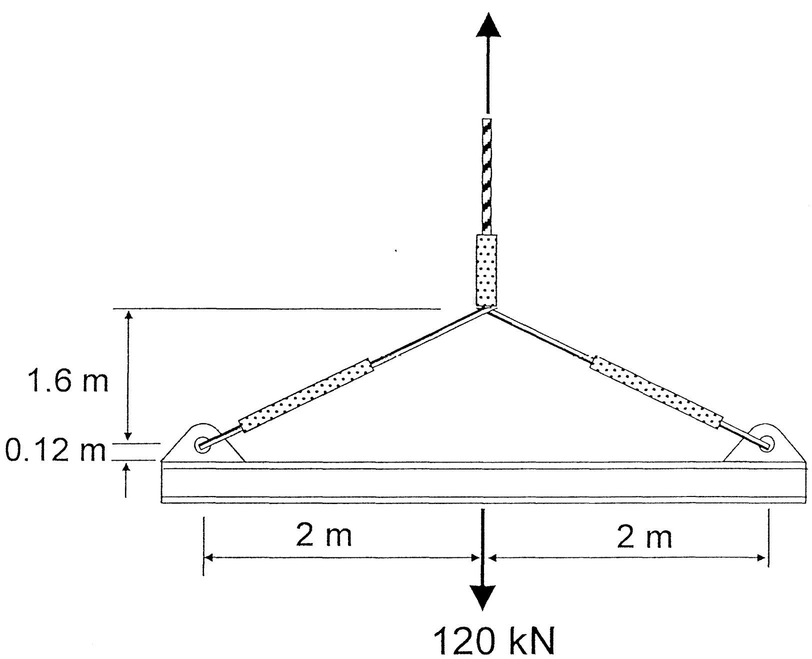
[1 mark] (b) Sketch the deflected shape of the beam and indicate whether the beam satisfies an allowable deflection limit of L/120 (where L equals the span of the beam).

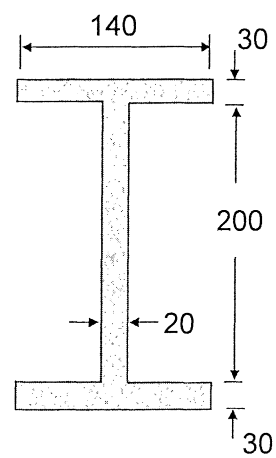


Question 3: [20 marks] A steel spreader beam is used to support a vertical load of 120 kN as shown. The spreader beam is supported with steel cables attached to a connection plate at each end of the beam. The beam has the cross section given and is made of steel with a normal yield stress of 350 MPa and yield stress in shear of 60 MPa. The elastic modulus of the steel equals 200 GPa.

(a) Compute the distribution of **normal stress** in the spreader beam at the location of the point load (that is, at midspan). Show this distribution on a sketch and make sure to show maximum and minimum values of stress.

(b) Compute the maximum **shear stress** in the spreader beam at the same location. Also sketch the distribution of shear stress on the section.



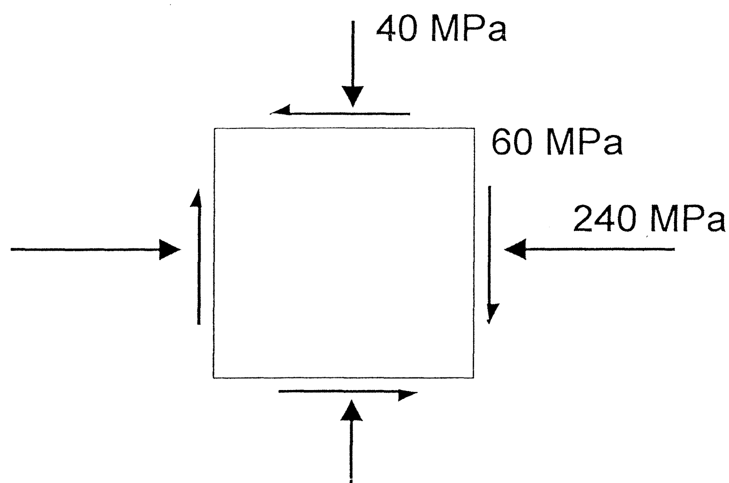


beam cross-section
(all dimensions in mm)

Question 4: **[20 marks]** For an element in a state of plane stress subjected to the normal and shear stresses shown below, use the Mohr's circle solution (***not*** the transformation equations) to determine the following:

(a) the principal stresses and orientation of the principal planes, showing your answer on a properly oriented element.

(b) the maximum in-plane shear stress (and associated normal stresses) and orientation of the corresponding planes. Once again, show your answer on a sketch of a properly oriented element.

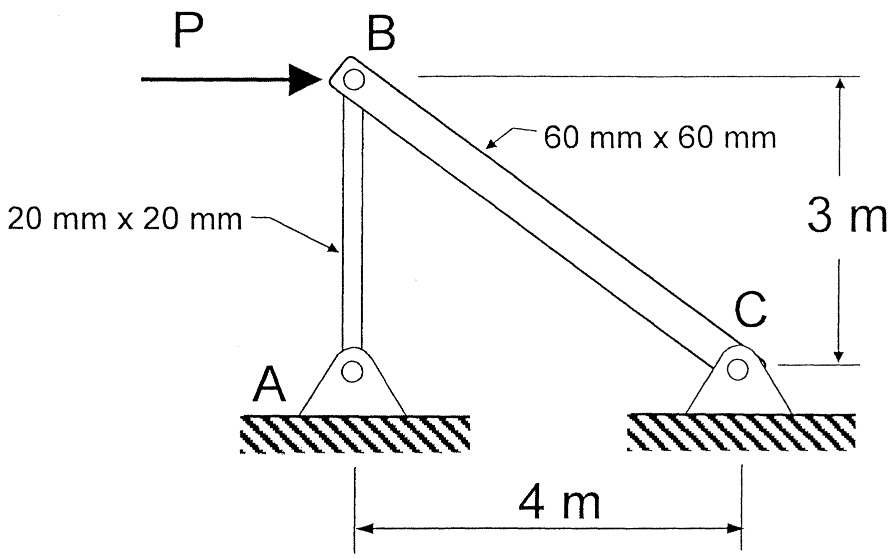


WARNING! Credit will **only** be given for a **solution using Mohr's circle**. Not the stress transformation equations. This means that you need to draw a Mohr's circle based on the stress components given in this problem. Remember to show numbers on your circle. Your calculations must be based on the geometry of your circle. So use your calculator. In other words, you are expected to use trigonometry to construct your Mohr's circle. Do not give a graphical solution that is scaled off!

The stress transformation equations can only be used to check your answer.

Question 5: The truss type structure shown below has been designed to resist a horizontal load P.
[20 marks] The inclined strut BC has a cross-section area 60 mm x 60 mm and is made of steel with an elastic modulus of 200 GPa and allowable yield strength of 340 MPa. The vertical member AB has a cross-section area 20 mm x 20 mm and is also made of steel with an elastic modulus of 200 GPa and allowable yield strength of 340 MPa.

Determine the largest load P that can be applied to the structure. Use a safety factor of 2 against buckling and consider buckling in the plane of the structure only. Do not use a safety factor for yielding of the steel. Assume all members are pin connected.

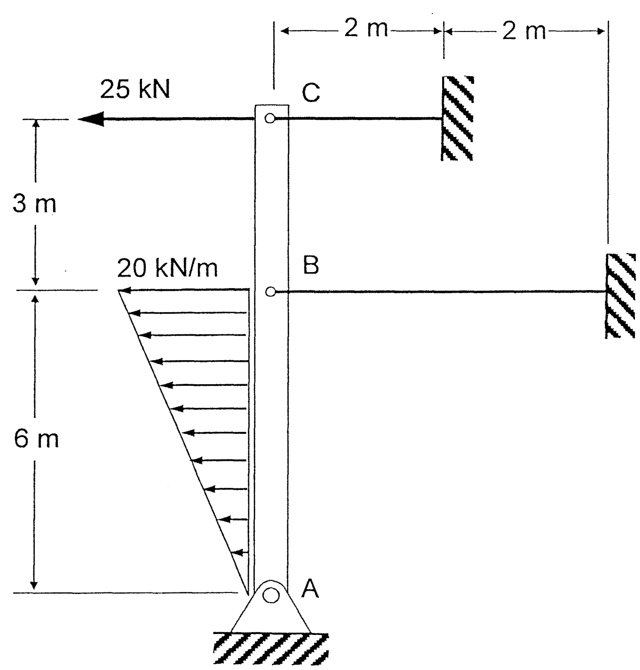


Question 6: A rigid vertical bar (ABC) is supported by a 20 mm diameter pin at A and two 12 mm diameter cables at points B and C. The cable at B has a length of 4 m and the cable at C has a length of 2 m. Both cables are made of steel with a yield strength of 400 MPa and elastic modulus of 200 GPa. The bar is loaded with a triangularly distributed load having a maximum intensity of 20 kN/m two-thirds up the bar (at B) plus a concentrated load of 25 kN acting at the top of the bar (at C).

[12 marks] (a) find the forces developed in each cable

[4 marks] (b) find the corresponding horizontal displacement at the top of the bar (point C)

[4 marks] (c) find the shear stress in the pin at A given that the pin is loaded in double shear.

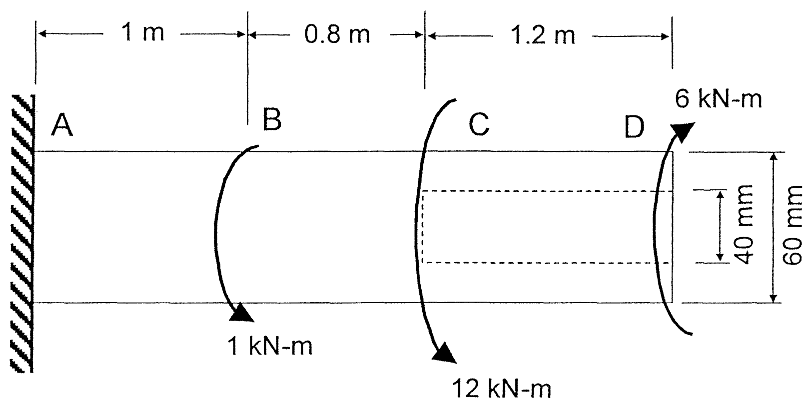


Question 7: A circular shaft is fixed at the left end (point A) and subjected to three torques (points B, C and D) acting as shown below. Part of the shaft (CD) is hollow and the entire shaft is made of aluminum with G = 25 GPa and a yield stress τ_y of 200 MPa. Dimensions (diameter and length) and magnitude of the torques are given in the diagram.

[12 marks] (a) determine the maximum shear stress in the shaft and sketch the variation of shear stress along the shaft radius for the cross-section where the stress is maximum.

[6 marks] (b) find the angle of twist at the end of the shaft (point D) and give your answer in degrees.

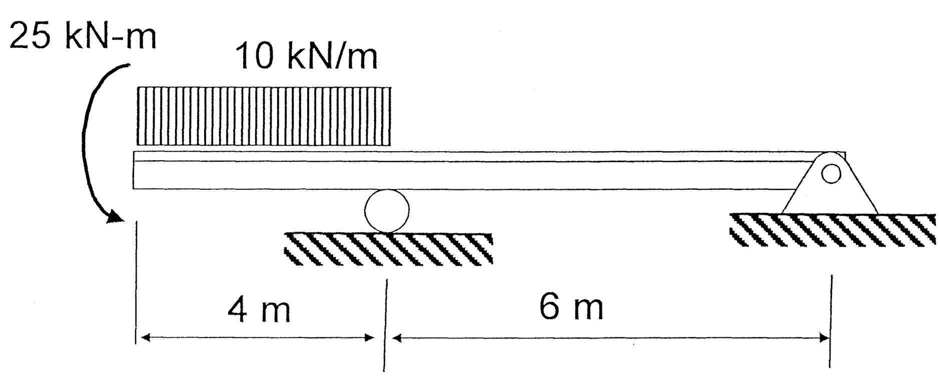
[2 marks] (c) what would happen if the loads on the shaft were doubled?

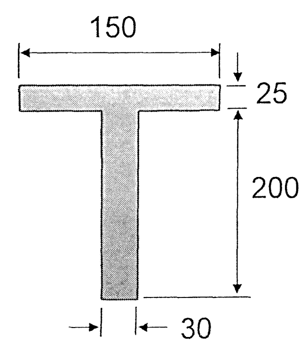


Question 8: A simply supported beam with an overhang is subjected to a uniformly distributed load acting on the overhang in addition to a couple acting at the end of the overhang as shown. The beam has the cross-section given and is made of steel with a yield strength of 350 MPa and shear stress at yield of 75 MPa. The elastic modulus of the steel is 200 GPa.
[20 marks]

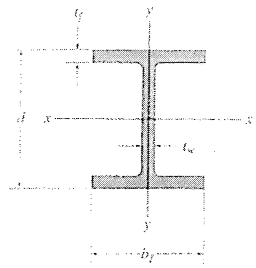
Determine the SHEAR FORCE and BENDING MOMENT along the length of the beam as a function of x. In other words, find V(x) and M(x) for the beam.

Then draw the corresponding shear force and bending moment diagrams for the beam (label all critical points and show your work by indicating exactly how you obtained your answers).





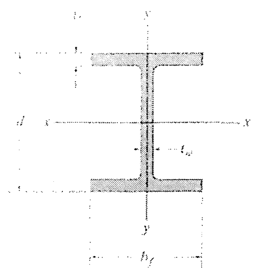
beam cross-section
(all dimensions in mm)



Wide-Flange Sections or W Shapes SI Units

Designation	Area A	Depth d	Web thickness t_w	Flange width b_f	Flange thickness t_f	x–x axis I	x–x axis S	x–x axis r	y–y axis I	y–y axis S	y–y axis r
mm × kg/m	mm^2	mm	mm	mm	mm	10^6 mm^4	10^3 mm^3	mm	10^6 mm^4	10^3 mm^3	mm
W610 × 155	19 800	611	12.70	324.0	19.0	1 290	4 220	255	108	667	73.9
W610 × 140	17 900	617	13.10	230.0	22.2	1 120	3 630	250	45.1	392	50.2
W610 × 125	15 900	612	11.90	229.0	19.6	985	3 220	249	39.3	343	49.7
W610 × 113	14 400	608	11.20	228.0	17.3	875	2 880	247	34.3	301	48.8
W610 × 101	12 900	603	10.50	228.0	14.9	764	2 530	243	29.5	259	47.8
W610 × 92	11 800	603	10.90	179.0	15.0	646	2 140	234	14.4	161	34.9
W610 × 82	10 500	599	10.00	178.0	12.8	560	1 870	231	12.1	136	33.9
W460 × 97	12 300	466	11.40	193.0	19.0	445	1 910	190	22.8	236	43.1
W460 × 89	11 400	463	10.50	192.0	17.7	410	1 770	190	20.9	218	42.8
W460 × 82	10 400	460	9.91	191.0	16.0	370	1 610	189	18.6	195	42.3
W460 × 74	9 460	457	9.02	190.0	14.5	333	1 460	188	16.6	175	41.9
W460 × 68	8 730	459	9.14	154.0	15.4	297	1 290	184	9.41	122	32.8
W460 × 60	7 590	455	8.00	153.0	13.3	255	1 120	183	7.96	104	32.4
W460 × 52	6 640	450	7.62	152.0	10.8	212	942	179	6.34	83.4	30.9
W410 × 85	10 800	417	10.90	181.0	18.2	315	1 510	171	18.0	199	40.8
W410 × 74	9 510	413	9.65	180.0	16.0	275	1 330	170	15.6	173	40.5
W410 × 67	8 560	410	8.76	179.0	14.4	245	1 200	169	13.8	154	40.2
W410 × 53	6 820	403	7.49	177.0	10.9	186	923	165	10.1	114	38.5
W410 × 46	5 890	403	6.99	140.0	11.2	156	774	163	5.14	73.4	29.5
W410 × 39	4 960	399	6.35	140.0	8.8	126	632	159	4.02	57.4	28.5
W360 × 79	10 100	354	9.40	205.0	16.8	227	1 280	150	24.2	236	48.9
W360 × 64	8 150	347	7.75	203.0	13.5	179	1 030	148	18.8	185	48.0
W360 × 57	7 200	358	7.87	172.0	13.1	160	894	149	11.1	129	39.3
W360 × 51	6 450	355	7.24	171.0	11.6	141	794	148	9.68	113	38.7
W360 × 45	5 710	352	6.86	171.0	9.8	121	688	146	8.16	95.4	37.8
W360 × 39	4 960	353	6.48	128.0	10.7	102	578	143	3.75	58.6	27.5
W360 × 33	4 190	349	5.84	127.0	8.5	82.9	475	141	2.91	45.8	26.4

Wide-Flange Sections or W Shapes FPS Units



Wide-Flange Sections or W Shapes SI Units

Designation	Area A	Depth d	Web thickness t_w	Flange width b_f	Flange thickness t_f	x–x axis I	x–x axis S	x–x axis r	y–y axis I	y–y axis S	y–y axis r
mm × kg/m	mm^2	mm	mm	mm	mm	10^6 mm^4	10^3 mm^3	mm	10^6 mm^4	10^3 mm^3	mm
W310 × 129	16 500	318	13.10	308.0	20.6	308	1940	137	100	649	77.8
W310 × 74	9 480	310	9.40	205.0	16.3	165	1060	132	23.4	228	49.7
W310 × 67	8 530	306	8.51	204.0	14.6	145	948	130	20.7	203	49.3
W310 × 39	4 930	310	5.84	165.0	9.7	84.8	547	131	7.23	87.6	38.3
W310 × 33	4 180	313	6.60	102.0	10.8	65.0	415	125	1.92	37.6	21.4
W310 × 24	3 040	305	5.59	101.0	6.7	42.8	281	119	1.16	23.0	19.5
W310 × 21	2 680	303	5.08	101.0	5.7	37.0	244	117	0.986	19.5	19.2
W250 × 149	19 000	282	17.30	263.0	28.4	259	1840	117	86.2	656	67.4
W250 × 80	10 200	256	9.40	255.0	15.6	126	984	111	43.1	338	65.0
W250 × 67	8 560	257	8.89	204.0	15.7	104	809	110	22.2	218	50.9
W250 × 58	7 400	252	8.00	203.0	13.5	87.3	693	109	18.8	185	50.4
W250 × 45	5 700	266	7.62	148.0	13.0	71.1	535	112	7.03	95	35.1
W250 × 28	3 620	260	6.35	102.0	10.0	39.9	307	105	1.78	34.9	22.2
W250 × 22	2 850	254	5.84	102.0	6.9	28.8	227	101	1.22	23.9	20.7
W250 × 18	2 280	251	4.83	101.0	5.3	22.5	179	99.3	0.919	18.2	20.1
W200 × 100	12 700	229	14.50	210.0	23.7	113	987	94.3	36.6	349	53.7
W200 × 86	11 000	222	13.00	209.0	20.6	94.7	853	92.8	31.4	300	53.4
W200 × 71	9 100	216	10.20	206.0	17.4	76.6	709	91.7	25.4	247	52.8
W200 × 59	7 580	210	9.14	205.0	14.2	61.2	583	89.9	20.4	199	51.9
W200 × 46	5 890	203	7.24	203.0	11.0	45.5	448	87.9	15.3	151	51.0
W200 × 36	4 570	201	6.22	165.0	10.2	34.4	342	86.8	7.64	92.6	40.9
W200 × 22	2 860	206	6.22	102.0	8.0	20.0	194	83.6	1.42	27.8	22.3
W150 × 37	4 730	162	8.13	154.0	11.6	22.2	274	68.5	7.07	91.8	38.7
W150 × 30	3 790	157	6.60	153.0	9.3	17.1	218	67.2	5.54	72.4	38.2
W150 × 22	2 860	152	5.84	152.0	6.6	12.1	159	65.0	3.87	50.9	36.8
W150 × 24	3 060	160	6.60	102.0	10.3	13.4	168	66.2	1.83	35.9	24.5
W150 × 18	2 290	153	5.84	102.0	7.1	9.19	120	63.3	1.26	24.7	23.5
W150 × 14	1 730	150	4.32	100.0	5.5	6.84	91.2	62.9	0.912	18.2	23.0