

National Exams May 2014

07-Elec-A2, Systems & Control

3 hours duration

NOTES:

1. This is a **CLOSED BOOK EXAM**. Only an approved Casio or Sharp calculator is permitted. Candidates are allowed to use a single-sided, handwritten, 8.5 by 11" formula and notes sheet. Otherwise, there are no restrictions on the content of the formula sheet. The sheet has to be signed and submitted together with the examination paper.
2. If doubt exists as to interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
3. Five (5) questions constitute a complete paper. **YOU ARE REQUIRED TO COMPLETE QUESTION 1 AND QUESTION 2.** Choose three (3) more questions out of the remaining six. Clearly indicate answers to which questions should be marked - otherwise, only the first five answers provided will be marked. Each question is of equal value. Weighting of topics within each question is indicated. Partial marks will be given. Clearly show steps taken in answering each question.
4. **Use exam booklets to answer the questions - clearly indicate which question is being answered.**

YOUR MARKS		
QUESTIONS 1 AND 2 ARE COMPULSORY:		
Question 1	20	
Question 2	20	
CHOOSE THREE OUT OF THE REMAINING SIX QUESTIONS:		
Question 3	20	
Question 4	20	
Question 5	20	
Question 6	20	
Question 7	20	
Question 8	20	
TOTAL:	<u>100</u>	

A Short Table of Laplace Transforms

Laplace Transform	Time Function
1	$\sigma(t)$
$\frac{1}{s}$	$1(t)$
$\frac{1}{(s)^2}$	$t \cdot 1(t)$
$\frac{1}{(s)^{k+1}}$	$\frac{t^k}{k!} \cdot 1(t)$
$\frac{a}{s+a}$	$e^{-at} \cdot 1(t)$
$\frac{a}{(s+a)^2}$	$te^{-at} \cdot 1(t)$
$\frac{a}{s(s+a)}$	$(1 - e^{-at}) \cdot 1(t)$
$\frac{a}{s^2 + a^2}$	$\sin at \cdot 1(t)$
$\frac{s}{s^2 + a^2}$	$\cos at \cdot 1(t)$
$\frac{s^2 + a^2}{s+a}$	$e^{-at} \cdot \cos bt \cdot 1(t)$
$\frac{b}{(s+a)^2 + b^2}$	$e^{-at} \cdot \sin bt \cdot 1(t)$
$\frac{a^2 + b^2}{s[(s+a)^2 + b^2]}$	$\left(1 - e^{-at} \cdot \left(\cos bt + \frac{a}{b} \cdot \sin bt\right)\right) \cdot 1(t)$
$\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$	$\frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \cdot \sin\left(\omega_n\sqrt{1-\zeta^2}t\right) \cdot 1(t)$
$\frac{\omega_n^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}$	$\left(1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \cdot \sin\left(\omega_n\sqrt{1-\zeta^2}t + \cos^{-1}\zeta\right)\right) \cdot 1(t)$
$F(s) \cdot e^{-Ts}$	$f(t-T) \cdot 1(t)$
$F(s+a)$	$f(t) \cdot e^{-at} \cdot 1(t)$
$sF(s) - f(0+)$	$\frac{df(t)}{dt}$
$\frac{1}{s} F(s)$	$\int_{0+}^{+\infty} f(t) dt$

Question 1 (Compulsory)

Stability Analysis in Frequency Domain: Nyquist Stability Criterion, Polar Plots, Gain and Phase Margins.

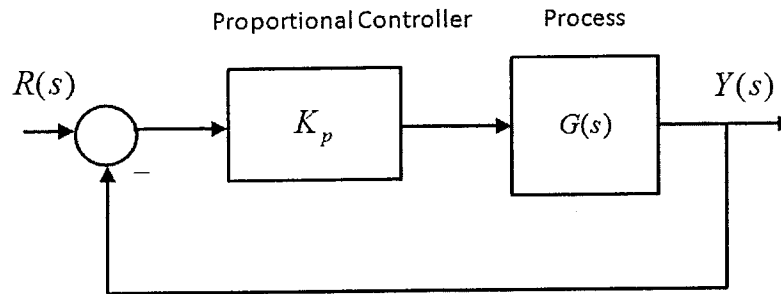


Figure Q1.1

Consider a unit feedback control system under Proportional Control (gain K_p), shown in Figure Q1.1. Your task is to investigate the stability of the closed loop system in the frequency domain by applying the Nyquist Criterion of Stability to find:

1. The value (or values) of the Proportional Controller Gain, $K_p = K_{crit}$, at which the closed loop system becomes marginally stable, and the corresponding frequency (or frequencies) of marginally stable oscillations, ω_{osc} .
2. The range (or ranges) of safe operating gains for the Proportional Controller in Figure Q1.1.

Figure Q1.2 shows an open-loop frequency response of the process, $K_p G(s)$, when the Controller Gain K_p is set to 1. Use this information to create a corresponding polar plot and next a complete Nyquist Contour for this system. The plot does not need to be to-scale but you should indicate all relevant coordinates of crossovers with the Real and Imaginary axes. Next, apply the Nyquist Criterion of Stability to answer the above questions.

NOTE: You do not need to know $G(s)$ to create the polar plot from the information contained in the following frequency response. Please note that, since it is difficult to read off from Figure Q1.2 the exact value of the open loop DC gain, it is provided here as: $G_{open}(0) = \frac{2000}{24} = 83.33 \frac{V}{V} = +38.4 \text{ dB}$.

Bode Diagram

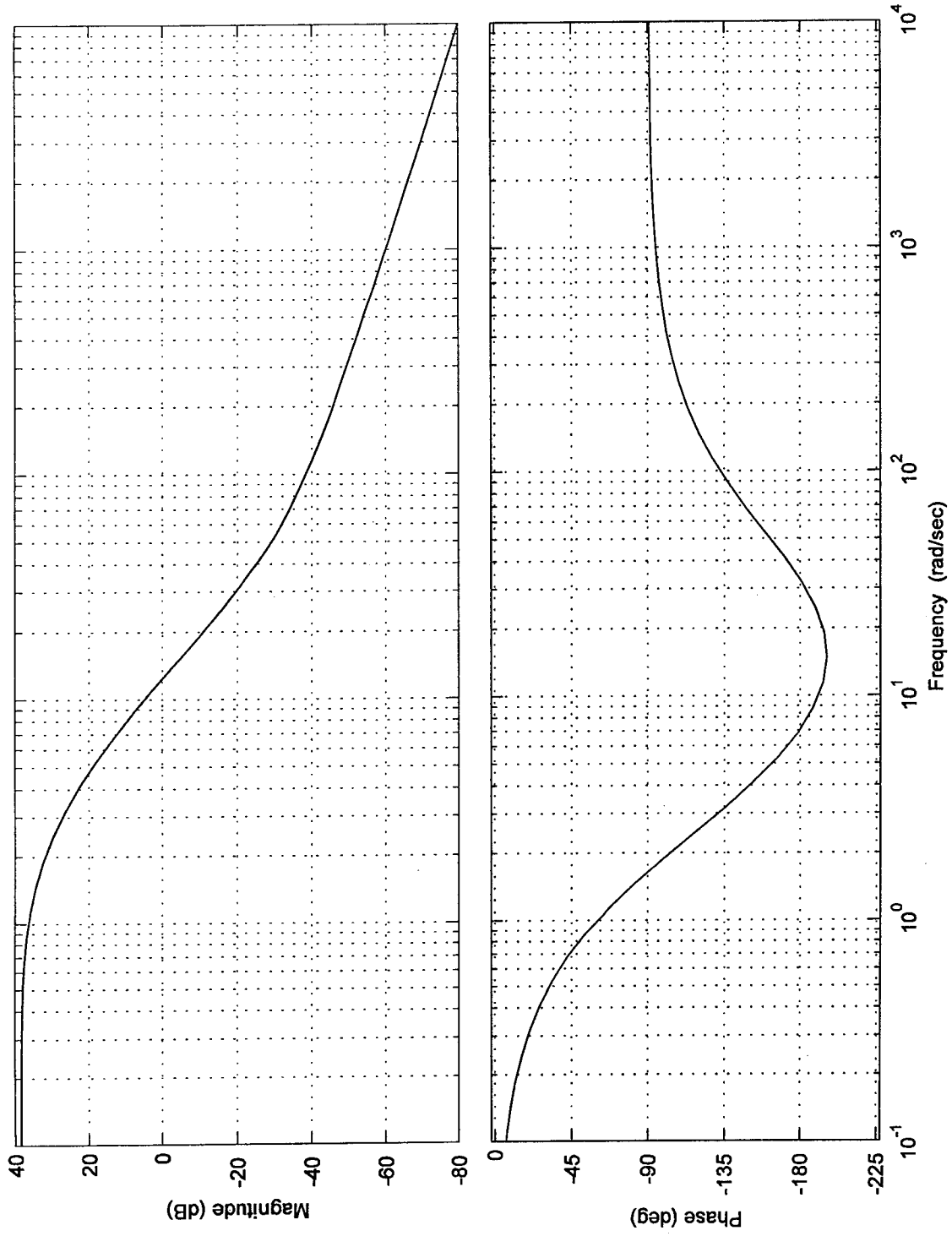


Figure Q1.2

Question 2 (Compulsory)

Stability Analysis in s-Domain: Routh-Hurwitz Criterion and Root Locus.

Consider again the unit feedback control system under Proportional Control (gain K_p), shown in Figure Q1.1, where the process $G(s)$ is described by the following transfer function:

$$G(s) = \frac{(s + 40)(s + 50)}{(s + 2)(s + 2)(s + 6)}$$

Your task is to investigate the stability of the closed loop system using s-domain analysis by finding:

1. The value of the Proportional Controller Gain(s), $K_p = K_{crit}$, at which the closed loop system becomes marginally stable, and the corresponding frequency (frequencies) of marginally stable oscillations, ω_{osc} .
2. The range (or ranges) of safe operating gains for the Proportional Controller in Figure Q1.1;

Sketch a Root Locus plot for the system (the plot does not need to be to-scale). Only rough estimates of break-in and break-away points are required - no calculations are necessary but please provide a rationale for your estimates. Focus on the overall shape of the plot and particularly on the crossovers with the Imaginary Axis. To find the crossovers, either apply the Routh-Hurwitz Criterion of Stability, or substitute $s = j\omega$ into the closed loop characteristic equation. Either approach will allow you to determine the critical value (or values) of the gain, K_{crit} , at which the system is marginally stable, and the corresponding frequency (or frequencies) of marginally stable oscillations, ω_{osc} .

HINT 1: The shape of the Root Locus plot will help you interpret the results and to correctly determine the range (or ranges) of the stable gains K_p .

HINT 2: Your results should be consistent with the frequency domain analysis of the system stability, as performed in Question 1.

Question 3

Basic System Representation using Transfer Functions; Second Order Model, Step Response Specifications.

Consider the electric circuit, a two-port, shown in Figure Q3.2, where its components have the following values: $R_1 = 100\Omega$, $R_2 = 10\Omega$, $C = 10\mu F$, and $L = 5mH$.

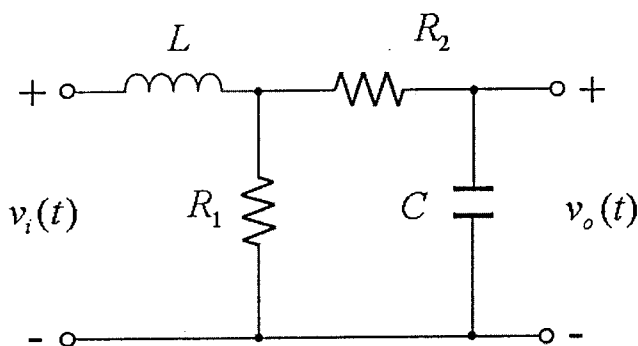


Figure Q3.2

Find the transfer function of this two-port, $G(s) = \frac{V_o(s)}{V_i(s)}$ and answer the following questions:

1. What kind of a filter does this transfer function represent?
2. What is the expected DC gain of this filter?
3. If input voltage $v_i(t)$ was a normalized unit step function, provide your estimate of the Percent Overshoot (PO) in the output voltage $v_o(t)$.
4. If resistance R_2 is adjustable, find its value such that the expected Percent Overshoot in the step response of this filter is $PO = 15\%$.

Question 4

Proportional + Integral + Derivative (PID) Controller Design in s-Domain, Dominant Poles Model, Step Response Specifications.

Consider a closed loop control system working with a relatively slow hydraulic process, as shown in Figure Q4.1.

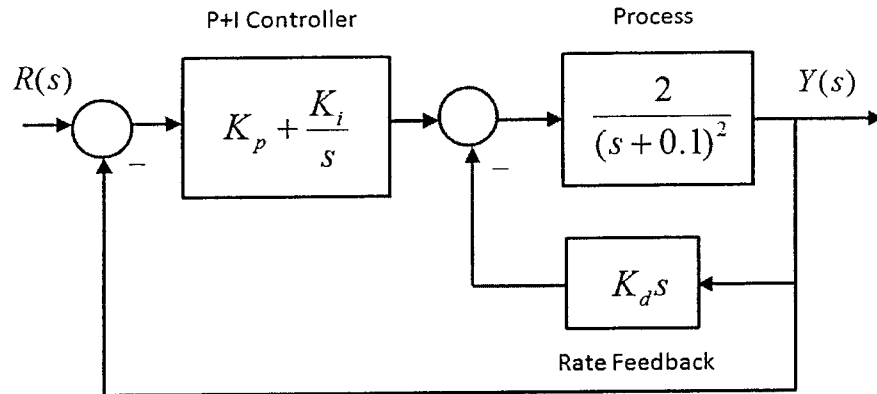


Figure Q4.1

The desired step response specifications are as follows:

- Percent Overshoot, $PO = 10\%$
- Settling time within 2% of the steady state value, $T_{settle(\pm 2\%)} = 5$ seconds
- Steady State Error, $e_{ss\%} = 0$

The system is to operate under a modified PID Control, with Rate Feedback replacing the Derivative term. Your task is to calculate the PID Controller parameters, K_p , K_d and K_i so that the above specifications are met. In order to do so, follow the steps outlined next.

PART A (5 marks)

Find the closed loop transfer function, $G_{cl}(s)$, in terms of controller gains K_p , K_d and K_i .

PART B (5 marks)

Choose the location for the two dominant poles of the closed loop based on a standard second order dominant poles model that is described by the following transfer function:

$$G_m(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Compute the model parameters i.e. ω_n , ζ , as appropriate, given the time response specifications.

PART C (10 marks)

Find the PID Controller parameters K_p , K_d and K_i so that the design benefits from a pole-zero cancellation, thus resulting in a closed loop transfer function identical to that of the above model, $G_{cl}(s) = G_m(s)$.

HINT: This can be accomplished by placing the third closed loop pole at the EXACT location of the closed loop zero, known from PART A.

Question 5

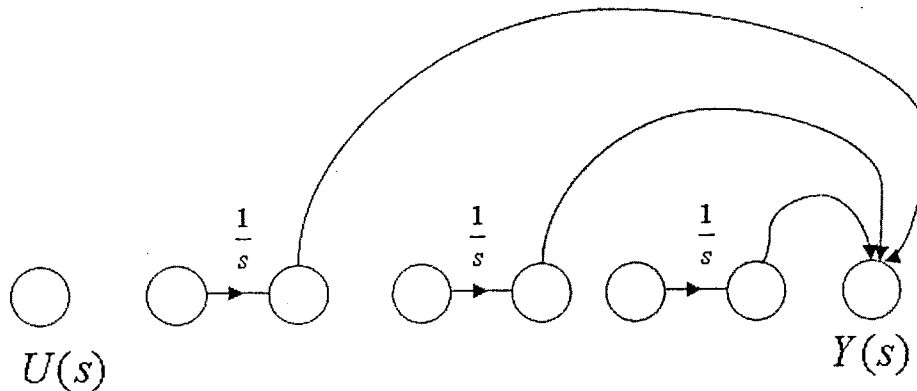
System Representation - Signal Flow Graphs and Mason's Gain Formula; Proportional Control: Steady State Error Analysis, Stability - Routh-Hurwitz Criterion.

PART A (10 marks)

Consider the following transfer function of a certain process $G(s)$:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{2s + 50}{(s + 2)(s + 3)(s + 4)}$$

Complete a signal flow graph diagram shown below so that it will represent $G(s)$. Justify your sketch by applying the Mason's Gain formula to verify the resulting transfer function.



PART B (10 marks)

Assume that the process $G(s)$ from PART A is going to work in a unit feedback closed loop system under Proportional Control. Find the practical range of values for the Controller Gain K_p that would meet these two conditions:

- Safe, stable operation of the system;
- Tracking of a reference signal such that the steady state error, $e_{ss(step)\%}$, does not exceed 8%.

Question 6

Lag Control, Gain and Phase Margins, Second Order Dominant Poles Model from Frequency Response Plots, Step Response Specifications, Steady State Errors and Error Constants.

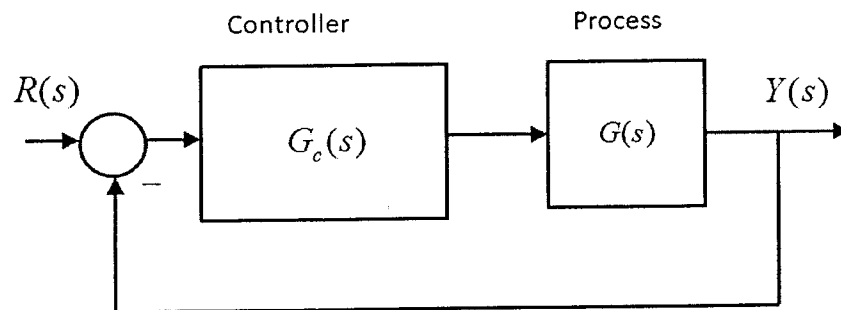


Figure Q6.1

Consider a certain unit feedback closed loop system, shown in Figure Q6.1. Open loop frequency response plots for the process $G(s)$ only (i.e. without controller) are shown in Figure Q6.2.

PART A (10 marks)

First, consider the uncompensated closed loop system i.e. where $G_c(s) = 1$. Assume that the system dynamics can be adequately represented by a second order system model with a transfer function $G_m(s)$ as shown below:

$$G_m(s) = K_{dc} \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Determine appropriate parameters of the model (i.e. K_{dc} , ω_n , ζ) and estimate the following closed loop step response specifications: Percent Overshoot, PO, Rise Time ($T_{rise(10\%-90\%)}$), Settling Time, $T_{settle(\pm 2\%)}$, and Steady State Errors for a step input ($e_{ss(step)\%}$) and a ramp input ($e_{ss(ramp)}$).

PART B (10 marks)

Next, assume that the compensated closed loop system would operate under a Lag Controller described by the following transfer function:

$$G_c(s) = K_c \frac{\alpha\tau s + 1}{\alpha\tau s + 1} \quad \alpha < 1$$

Design a Lag Compensator such that for the compensated system the following performance specifications can be achieved:

- Percent Overshoot $PO \leq 15\%$
- Ramp error $e_{ss(\text{ramp})} \leq 0.1$

Make sure to show the approximate shape of the compensated open loop frequency response in Figure Q6.2.

Bode Diagram

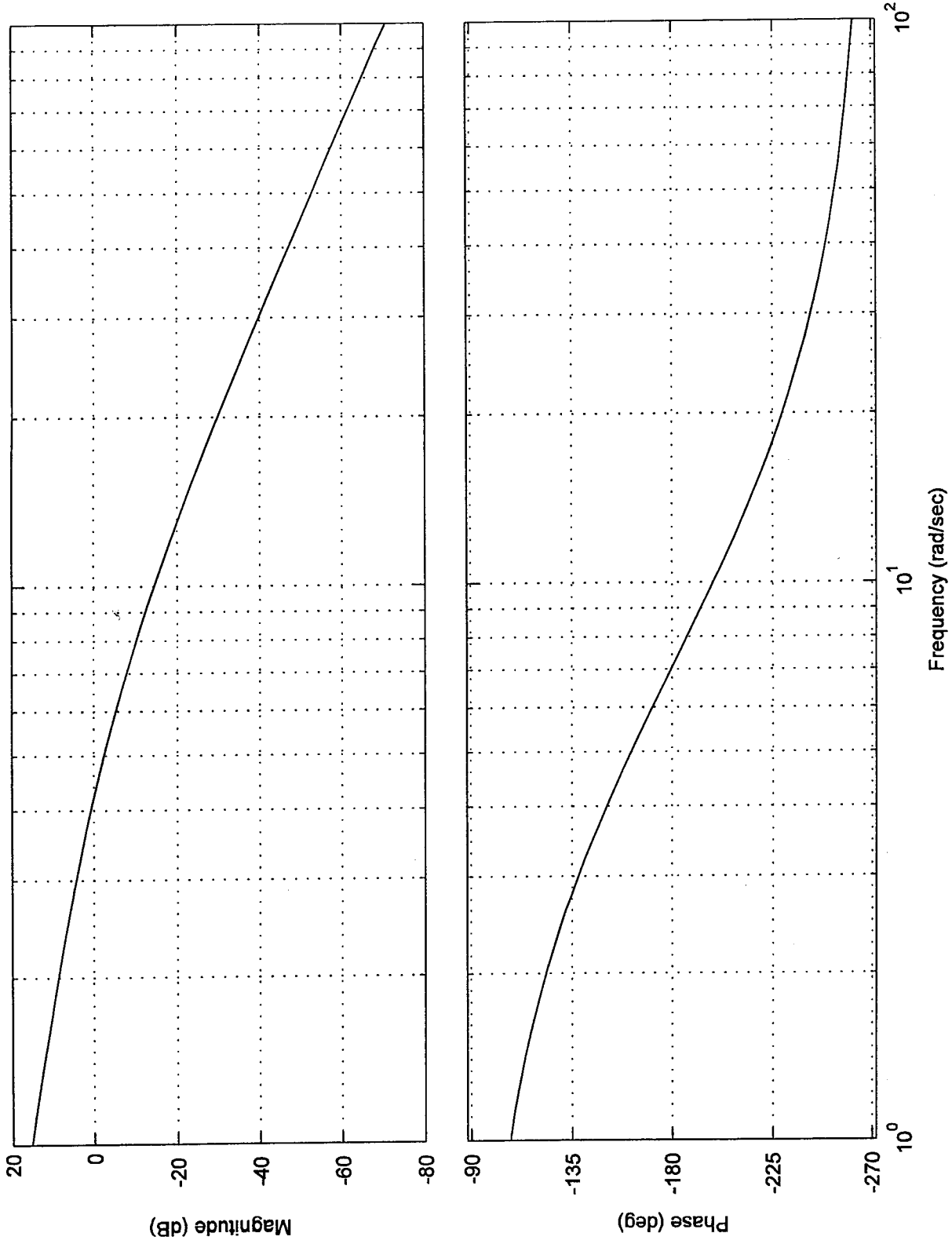


Figure Q6.2

Question 7

State Space Model, System Eigenvalues, Stability, Pole Placement by State Feedback Method, Steady State Errors to Step Inputs.

Consider a linear process described by the signal flow graph in Figure Q7.1:

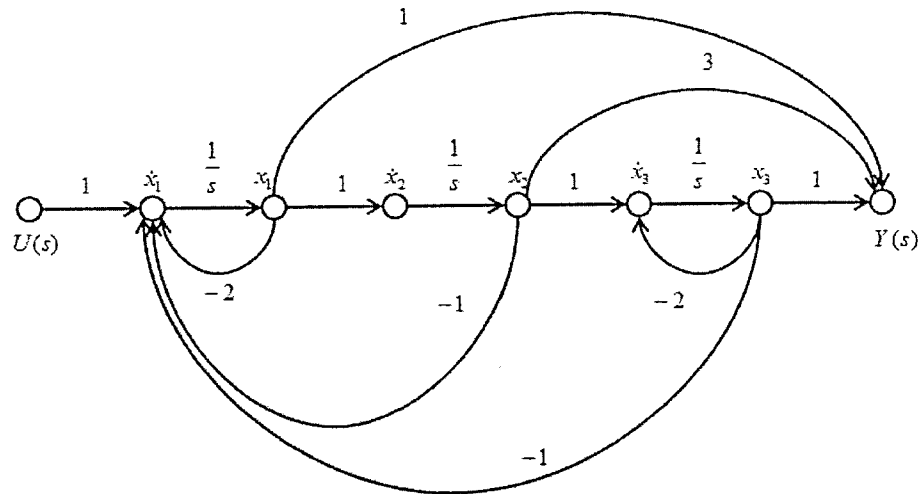


Figure Q7.1

PART A (10 marks)

Derive a set of the corresponding state equations - follow the choice of state variables as indicated in Figure Q7.1. Check if the process is Controllable and Observable.

PART B (10 marks)

A control system is to be built around the process by utilizing a state-variable feedback according to the following equation:

$$u = K \cdot (r - \mathbf{k}^T \cdot \mathbf{x})$$

Determine the values of the gain constant K and the state feedback vector \mathbf{k} so that the closed loop system will have poles at: -3 and at: $-2 \pm j3$, and the steady-state error to a step input signal r is to be zero.

Question 8

Analytical Calculations of System Responses, Step Response Specifications, Basic Analytical Relationships for a Standard Second Order System.

PART A (10 marks)

Consider the following transfer function of a certain process $G(s)$:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{2s + 5}{(s + 2)^2(s + 4)}$$

Find an analytical expression for a Step Response of the process $G(s)$.

PART B (10 marks)

Consider a standard second order under-damped system, described by the following transfer function:

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

where ω_n represents the frequency of natural oscillations of the system, and ζ represents its damping ratio.

Demonstrate how two well-known formulae describing relationships between system parameters and the system step response are derived:

$$PO = 100 \cdot \left(e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} \right)$$

$$T_{\text{settle}(\pm 2\%)} = \frac{4}{\zeta\omega_n}$$

HINT: Use the Laplace Table entries to help with the derivations.