National Exams December 2019

16-Mec-B12, Robot Mechanics

3 hours duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit, with the answer paper, a clear statement of any assumptions made.

2. This is a CLOSED BOOK EXAM with one 8.5"x11" formula sheet allowed written on both sides. The formula sheet must not hold any solutions of examples and must be handed in with the exam submission. A Casio or Sharp approved calculator is permitted.

3. FIVE (5) questions constitute a complete exam paper. There are two sections in this exam. There are five questions in the first section. The candidate will need to answer **ONLY three**. In the second section there are two questions which the candidate **MUST answer both**.

4. Each question is of equal value.

5. Question value in marks is shown in parentheses at the end of each question part.

6. Logical order, clarity, and organization of the solution steps are important.

Nomenclature:

The unit vectors of coordinate system A are denoted by $\hat{X}_A, \hat{Y}_A, \hat{Z}_A$.

The leading superscript such as A in ${}^{A}V$ indicates the coordinate system to which the vector V is referenced.

The leading subscript and superscript such as A and B in ${}^{B}_{A}T$ indicate the transformation of coordinate frame A relative to B by matrix T.

Section 1: Answer ONLY three out of five

(20)

1. A position vector is given by ${}^{A}P = \begin{bmatrix}10 & -4 & 3\end{bmatrix}^T$ and a velocity vector by ${}^{B}V = \begin{bmatrix}15 & 5 & 20\end{bmatrix}^T$. Given the Homogeneous Transformation

$$ {}^{A}_{B}T = \begin{bmatrix} 0.5 & 0 & -\sqrt{3}/2 & 5 \\ -\sqrt{3}/2 & 0 & -0.5 & -10\sqrt{3}/2 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}; $$

Compute:

a) ${}^{B}P$ **(12)**

b) ${}^{A}V$ **(8)**

(20)

2. The following frame definitions are given

$$ {}^{U}_{A}T = \begin{bmatrix} 0.866 & -0.5 & 0 & 11 \\ 0.5 & 0.866 & 0 & -1 \\ 0 & 0 & 1 & 8 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad {}^{B}_{A}T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.866 & -0.5 & 10 \\ 0 & 0.5 & 0.866 & -20 \\ 0 & 0 & 0 & 1 \end{bmatrix}, $$

$$ {}^{C}_{U}T = \begin{bmatrix} 0.866 & -0.5 & 0 & -3 \\ 0.433 & 0.75 & -0.5 & -3 \\ 0.25 & 0.433 & 0.866 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

a) Draw a frame diagram to show their arrangement graphically. **(5)**

b) Solve for ${}^{B}_{C}T$. **(15)**

(20)

3. A single-link robot with a rotary joint is motionless at $\theta_0 = -5°$. It is desired to move the joint in a smooth manner to $\theta = 80°$ in 4 seconds. Find the coefficients of a cubic which accomplishes this motion and brings the arm to rest at the goal. Plot the position, velocity, and acceleration of the joint as a function of time. Hint: a cubic has the following form:

$$\theta(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

(20)

4. A 4R manipulator is shown schematically in Figure 1. The nonzero link parameters are $a_1 = 1$, $\alpha_2 = 45°$, $d_3 = \sqrt{2}$, and $a_3 = \sqrt{2}$, and the mechanism is pictured in the configuration corresponding to $\Theta = [0, 90°, -90°, 0]^T$. Each joint has ±180° as limits. Find all values of θ_3 such that ${}^{0}P_{4ORG} = [1.1, 1.5, 1.707]^T$.

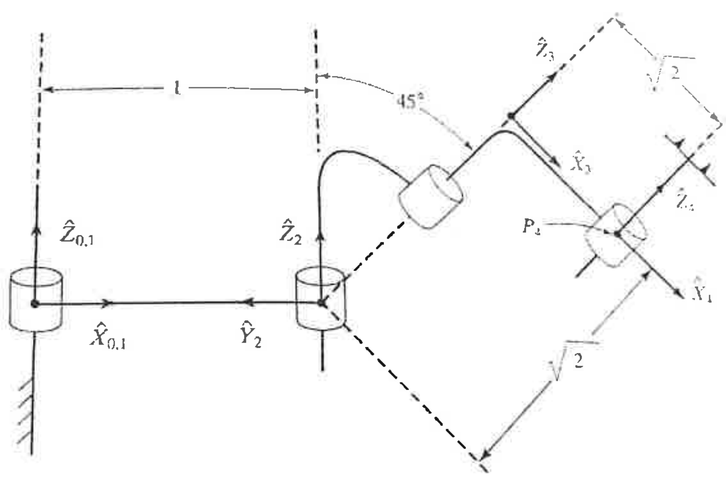

Figure 1

(20)

5. The single-degree-of-freedom manipulator in Figure 2 has total mass m=1, with the center of mass at

$${}^{1}P_C = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}, \text{ and has inertia tensor } {}^{C}I_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

From rest at t=0, the joint angle θ_1 moves in accordance with the time function $\theta_1(t) = bt + ct^2$ in radians. Find the angular acceleration of the link and the linear acceleration of the center of mass in terms of frame {1} as a function of t.

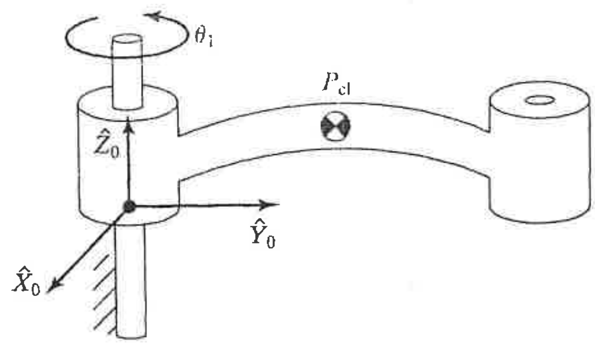

Figure 2

Section 2: Answer both questions

(20)
6. The arm with three degrees of freedom shown in Figure 3 has joints 1 and 2 perpendicular and joints 2 and 3 parallel. As pictured, all joints are at their zero location. Note that the positive sense of the joint angle is indicated.

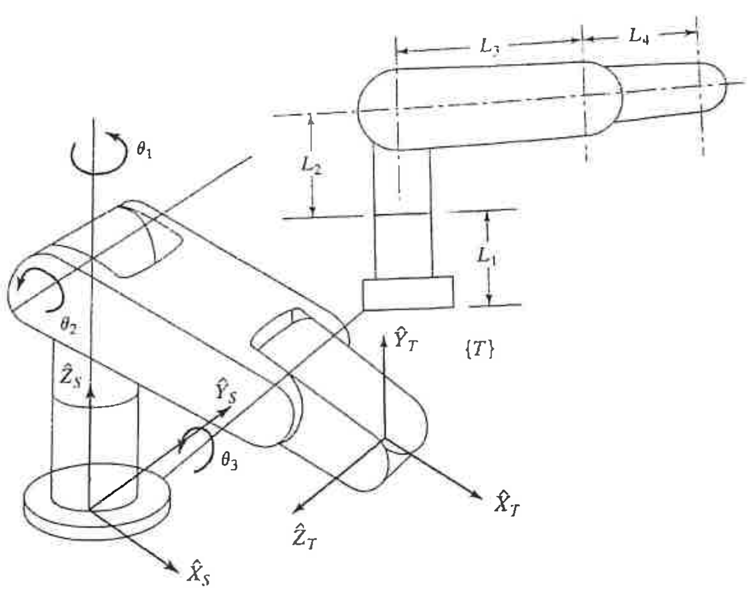

Figure 3

a) Assign link frames {0} through {3} for this arm-that is, sketch the arm, showing the attachments of the frames. **(5)**
b) Derive the DH table for the manipulator. **(5)**

c) Drive the transformation matrices ${}^{0}_{1}T, {}^{1}_{2}T, and\ {}^{2}_{3}T$. **(10)**

(20)
7. For the manipulator shown in Figure 4, derive the Jacobian using velocity propagation way from base to tip. Write it in terms of a frame {4} located at the tip of the hand and having the same orientation as frame {3}.

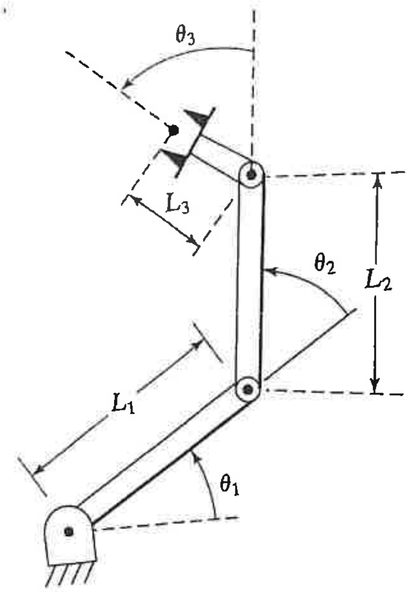

Figure 4