National Exams – December 2017

16–Mec-A6 Advanced Fluid Mechanics

3 hours duration

NOTES:

1. If doubt exists as to the interpretation of any question the candidate is urged to submit with the answer paper a clear statement of the assumptions made.
2. Candidates may use any approved Sharp/Casio calculator.
   The exam is OPEN BOOK.
3. Any FIVE (5) out of the 6 questions constitute a complete exam paper for a total of 100 MARKS.
   The first five questions as they appear in the answer book will be marked.
4. Each question is of equal value (20 marks) and question items are marked as indicated.
5. Clarity and organization of the answer are important.

(20) **Question 1**

A large pressurized air reservoir (a) contains air at temperature T_a=100°C and at a constant pressure of P_a=270 kPa. The air passes through a convergent-divergent nozzle from reservoir (a) to another large reservoir (b), as shown in Figure 1. The throat area of the nozzle is A_T=9 cm^2 and the exit area is A_E=36 cm^2.

A mercury manometer reads h = 18 cm between the throat and reservoir (b).

Assume frictional losses are negligible and that the pressurized air density is negligible compared with mercury.

Air properties: γ=1.4, R=287 J/(kg K), C_P=1004.5 J/(kg K)

Mercury properties: ρ=13,550 kg/m^3

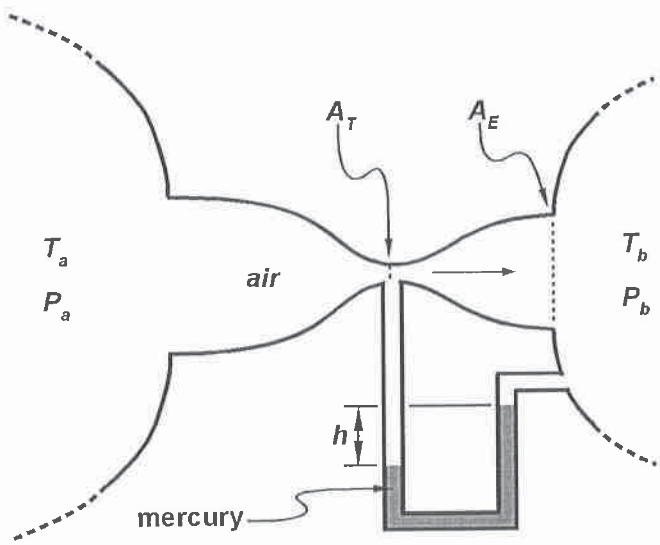

Figure 1: Convergent-divergent nozzle between two large air reservoirs.

(5) (a) Estimate the downstream reservoir pressure.

(5) (b) Is there a normal shock wave in the flow?

(5) (c) If there is a normal shock wave, does it stand in the exit plane or farther upstream?

(5) (d) What would be the mercury manometer reading if the nozzle were operating exactly at supersonic design conditions, i.e. with an *ideal expansion*?

(20) **Question 2**

Consider the ideal flow given by the velocity potential function

$$\phi = -\Gamma \ln r$$

where Γ is a positive constant.

(5) (a) Determine the stream function ψ.

(5) (b) Sketch the equipotential lines and the stream lines of this flow.

(5) (c) Calculate the radial velocity V_r and identify the flow pattern.

(5) (d) Give the physical meaning of the constant $(2\pi\Gamma)$.

(20) **Question 3**

A large reservoir is connected near its base to a 40 m long (L) annulus (radii of $a = 4$ cm and $b = 6$ cm) made of commercial steel ($e = 0.046$ mm). Consider $\rho = 1000$ kg/m^3 and $\nu = \mu/\rho = 1.02 \cdot 10^{-6}$ m^2/s for water. Also, assume $D_{eff}/D_h = 0.670$ in the annulus.

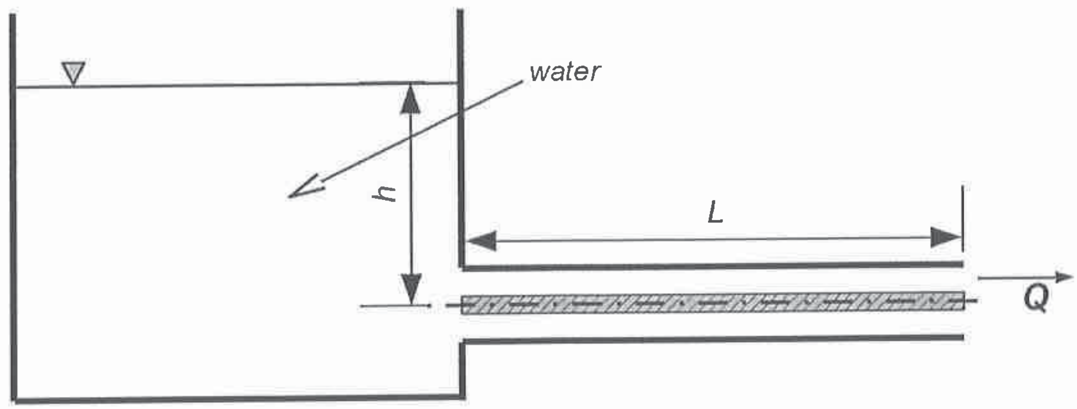

Figure 2: Annular pipe attached to a reservoir.

(15) (a) What should the water level h in the reservoir be to maintain a flow of $Q = 0.01$ m^3/s? Neglect initially the entrance effects at the annulus.

(5) (b) Estimate the effect in this case of a well-designed entrance, compared with a sharp-edged entrance that can be considered to have a loss coefficient of $K = 0.5$.

(20) **Question 4**

Consider the combined gravity-Couette driven flow between two inclined, parallel plates, as depicted in the figure. The lower plate is stationary, while the upper plate moves at constant upward velocity V_{top}. The plates are very long and wide and they are separated by a small gap of length h. The fluid has viscosity μ and density ρ.

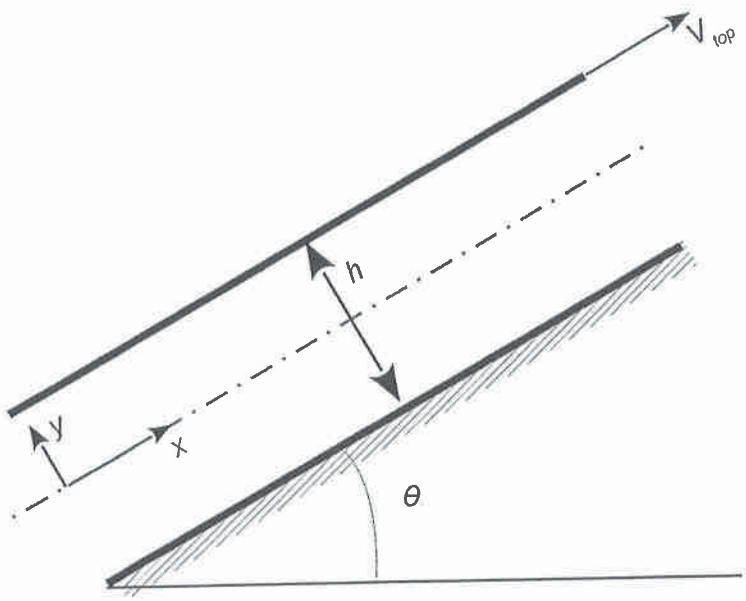

Figure 3: Inclined gravity and Couette driven flow.

(5) (a) What assumptions can be made about this flow scenario?

(5) (b) Write the complete Navier-Stokes equations governing this flow, including known Boundary Conditions.

(5) (c) Simplify and reduce the governing equations to a second order ordinary differential equation (ODE). Justify your assumptions.

(5) (d) Find the solution to this ODE using appropriate Boundary Conditions.

(20) **Question 5**

A student team wants to test a scaled down model of a human-powered submarine to measure the drag coefficient of the vessel. But they do not have access to a water channel, so they will have to use the university's wind tunnel instead.
The actual submarine has a total length of $L = 2.5$ m, a maximum diameter of $D = 1.3$ m, and it is designed to travel through the clear waters of a lake (average temperature 15°C) at a speed of $V = 0.5$ m/s.

The students build a one-eighth scale model of the submarine to test in the wind tunnel. The air in the wind tunnel is at 25°Cand at one standard atmosphere pressure. The scaled model is supported in the middle of the measurement section by a shielded strut, so that the drag force measured by the balance is only due to the model itself. The cross-section of the wind tunnel is much larger than the model.

(15) (a) At what air speed does the wind tunnel need to be run in order to achieve similarity in the flow?

(5) (b) Is your assumption of flow similarity valid for this flow speed? Explain why?

(20) **Question 6**

A rectangular cross-section suction wind tunnel is designed with a curved upper wall and a straight bottom wall. We are interested in characterising the boundary layer development along the bottom wall. The shape of the upper wall is designed such that the external (irrotational, or main) flow varies along the tunnel according to:

$$U_0 = A\, x^{1/3}$$

where A is a constant.

The flow can be assumed to be two-dimensional and laminar, and the properties of air to be constant (ρ is density, μ is dynamic viscosity, and $\nu = \mu/\rho$ is kinematic viscosity). The boundary layer thickness is represented by δ and it is assumed that at $x = 0$, $\delta = 0$. The velocity distribution in the boundary layer is assumed to be approximated by:

$$\frac{u}{U_0} = 2\frac{y}{\delta} - \left(\frac{y}{\delta}\right)^2 \qquad \text{such that: } \frac{\delta^*}{\delta} = \frac{1}{3} \quad \text{and} \quad \frac{\theta}{\delta} = \frac{2}{15}$$

where δ^* and θ are the momentum and displacement thickness, respectively.

The boundary layer thickness can be assumed to satisfy: $\delta = B\, x^n$.

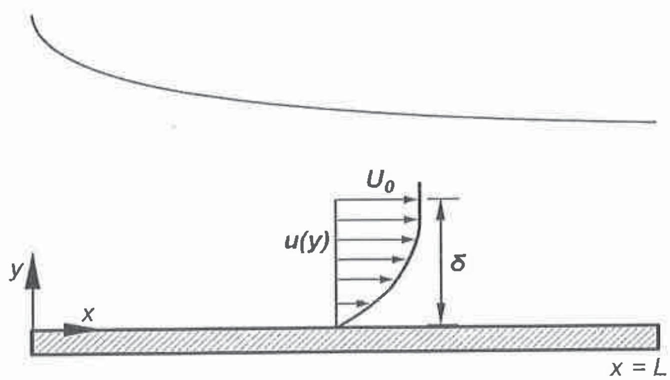

Figure 4: Boundary layer developing over bottom wall of suction wind tunnel.

(5) (a) Find the value of n to satisfy the integral boundary layer equations.

(5) (b) Express δ/x in terms of $\mathrm{Re}_x = \dfrac{U_0\, x}{\nu}$.

(5) (c) What is the local wall shear stress coefficient, $C_{fx} = \dfrac{\tau_w}{\rho U_0^2/2}$, in terms Re_x.

(5) (d) What is the average coefficient $\overline{C_f} = \overline{\tau_w}/(\rho U_0^2/2)$, where $U_0 = A\, L^{1/3}$?