National Exams December 2016
04-BS-1, Mathematics
3 hours Duration

Notes:

1. If doubt exists as to the interpretation of any question, the candidate is urged to include a clear statement of any assumptions made along with their answer.
2. Any APPROVED CALCULATOR is permitted. This is a CLOSED BOOK exam. However, candidates are permitted to bring ONE AID SHEET written on both sides.
3. Any five questions constitute a complete paper. Only the first five questions as they appear in your answer book will be marked.
4. All questions are of equal value.

Marking Scheme:

1. 20 marks
2. 20 marks
3. 20 marks
4. (a) 6 marks, (b) 14 marks
5. (a) 10 marks, (b) 10 marks
6. 20 marks
7. 20 marks
8. 20 marks

1. Find the general solution of the differential equation $x^2y'' - 2xy' + 2y = (1-2x)x^3e^{-2x}$.
Note that $'$ denotes differentiation with respect to x.

2. Find the general solution, $x(t)$, of the differential equation $x'' + 4x = 3\cos 2t + 4\cos 3t$.
Note that $'$ denotes differentiation with respect to t.

3. Find the maximum and minimum values of $f(x,y,z) = 4x+y^2+2z^2$ over the ellipsoid $x^2+3y^2+z^2 = 2$.

4. Let $x = \begin{pmatrix} 2 \\ 0 \\ -1 \\ 0 \end{pmatrix}$ and $A = \begin{pmatrix} 1 & 1 & 6 & -1 \\ -1 & 2 & -2 & 1 \\ 1 & -1 & 0 & 1 \\ 1 & 1 & 2 & 2 \end{pmatrix}$

   (a) Show that x is an eigenvector of A and find the associated eigenvalue.

   (b) Show that 3 is an eigenvalue of A and find an associated eigenvector.

5. Let $f(x,y,z) = x^2+y^2+z^2+2y-3x$, and let $g(x,y,z) = 3x+y^2-z^2$.

   (a) Find an equation for the tangent plane to the surface $g(x,y,z) = 9$ at the point $(3,-1,1)$.

   (b) Find the line tangent to the intersection of the surfaces $f(x,y,z) = 0$ and $g(x,y,z) = 9$ at the point $(3,-1,1)$.

6. Evaluate the surface integral $\iint_S \mathbf{F}\cdot dS$ where $\mathbf{F}(x,y,z) = xz\mathbf{i} - 2y\mathbf{j} + 3x\mathbf{k}$ and S is the surface of the region bounded above by the paraboloid $z = 4-x^2-y^2$ and below by the plane $z = 0$.

7. Find the work done by the field $\mathbf{F}(x,y,z) = x^2\mathbf{i} + y\mathbf{j} - z\mathbf{k}$ in moving a particle from the point $(0,2,0)$ to the point $(3\pi,0,2)$ along the path $x = 6t$, $y = 2\cos t$, $z = 2\sin t$.

8. Evaluate the surface integral $\iint_S x^2yz\,dA$ where S is the portion of the cylinder $x^2+y^2 = 4$ with $0 \le z \le 4$ and $y \ge 0$.