National Exams May 2015

04-CHEM-B1, Transport Phenomena

3 hours duration

NOTES

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
2. The examination is an OPEN BOOK EXAM.
3. Candidates may use any **non-communicating** calculator.
4. All problems are worth 25 points. **One problem** from **each** of sections A, B, and C must be attempted. A **fourth** problem from **any section** must also be attempted.
5. **Only the first four** questions as they appear in the answer book will be marked.
6. State all assumptions clearly.

Section A: Fluid Mechanics

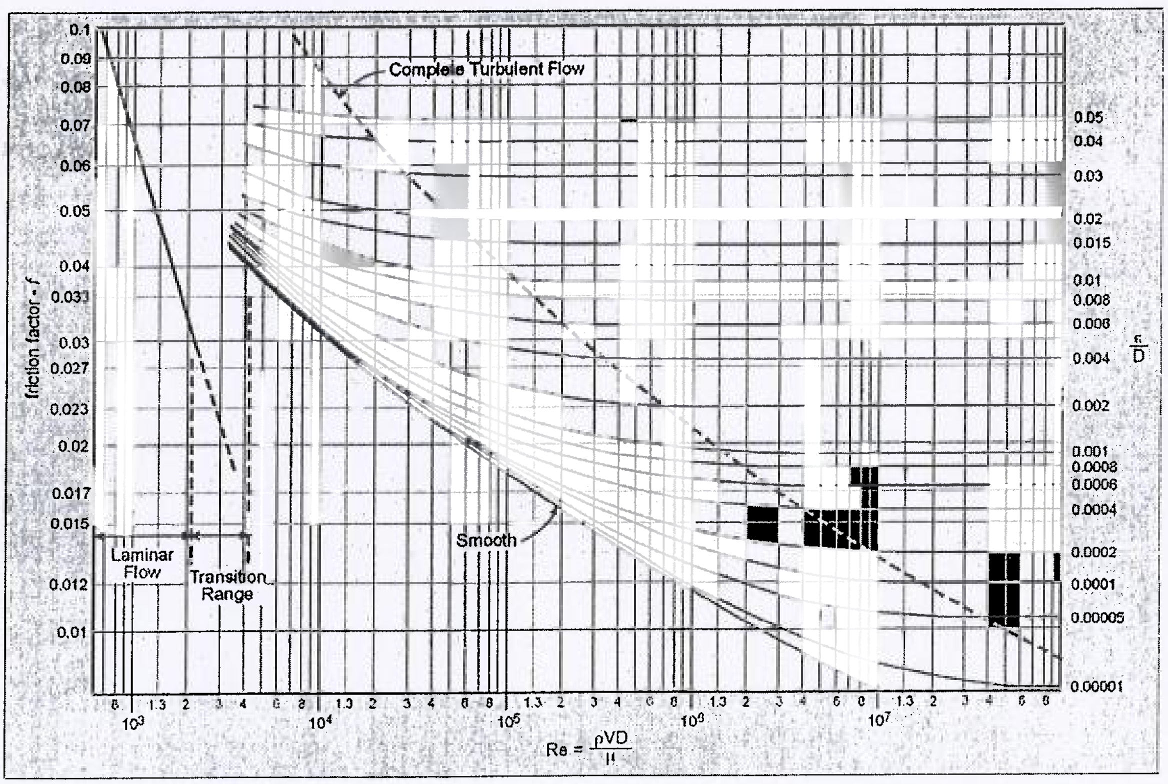
A1. A Newtonian fluid is flowing in laminar flow through a circular horizontal tube of radius R and length L. At the entrance the pressure is p_0 and at the exit the pressure is p_L. You may assume that gravity forces in this system are negligible, the flow is fully developed and that there is no accumulation in the system. Perform a shell momentum balance and derive an expression for the average velocity of this fluid.

A2. Benzene (density = 0.899 g/cm^3, viscosity = 8 x 10^{-4} N.s/m^2) flows steadily through a 150-meter long horizontal pipe of 5.5 cm inside diameter at a flow rate of 15 liters/min. The equivalent roughness for the pipe is 8.5 x 10^{-4} ft.

a) [13 points] Calculate the pressure drop.

b) [10 points] What is the pressure drop if the same amount of kerosene (density = 0.82 g/cm^3, viscosity = 2.5 x 10^{-3} N.s/m^2) is flowing through the same pipe?

c) [2 points] Explain the difference in pressure drop between (a) and (b).



Moody friction factor (f) vs. Reynolds number (Re) for pipes

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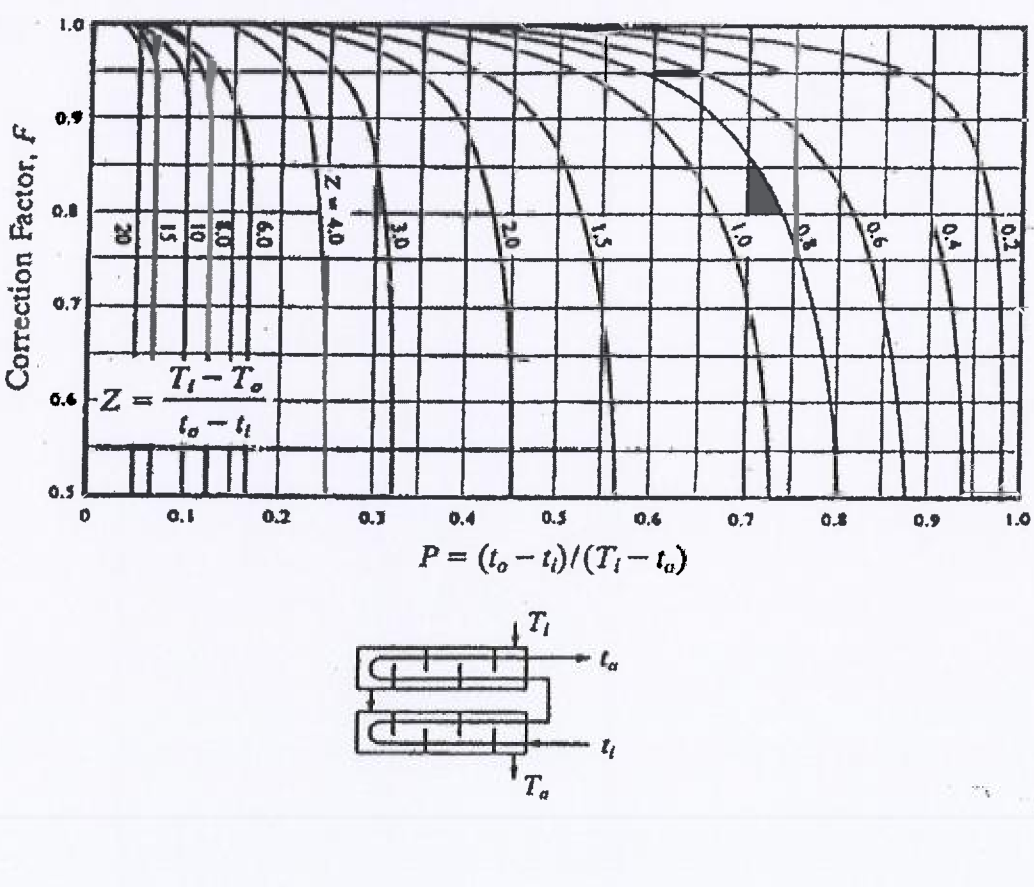
Section B: Heat Transfer

B1. Heat flows through the walls of an annular pipe whose inside and outside radii are R_i and R_o, respectively. The thermal conductivity (k) of the material is related to the temperature by the equation $k = aT^2 + b$, where **a** and □ are constants.

a) [20 points] Derive the temperature profile of the pipe wall.

b) [5 points] Compute the heat loss through the walls of the pipe.

B2. Nitrobenzene (viscosity = 7 x 10^{-4} N.s/m^2, thermal conductivity = 0.15 W/m.K, specific heat capacity = 2.38 J/g.K) at a flow rate of 14,400 kg/hr is to be cooled from 400 K to 315 K by heating a stream of benzene from 305 K to 345 K. Two tubular heat exchangers are available each with 44 cm internal diameter shell fitted with 166 tubes. Each tube is 5 m in length with 19 mm outside diameter and 15 mm inside diameter. The tubes are arranged in two passes on 25 mm square pitch with a baffle spacing of 150 mm. There are two passes on the shell side and the operation is to be countercurrent. With benzene passing through the tubes, the anticipated film coefficient on the tube side is 1000 W/m^{2}K. Assuming true cross-flow prevails in the shell and using the figure below for double shell pass exchangers, what value of scale resistance could be allowed if these heat exchangers were used?



Section C: Mass Transfer

C1. The convection mass transfer coefficient k_c is useful for determining the rate convection from a surface where the concentration is constant into the flowing medium. It relates the local flux to the concentration difference given by $N_A = k_c (C_{As} - C_{A0})$, where C_{As} is the surface concentration and C_{A0} is the far-field or baseline/background concentration.

For laminar flow across a flat plate, the Sherwood number (Sh) is given by the equation

$$Sh = k_c x/D_{AB} = 0.332 (Re_x)^{1/2} (Sc)^{1/3}$$

where Schmidt number (Sc) = ν/D_{AB}, Reynold's number (Re_x) = $v_{max}\, x/\nu$, and kinematic viscosity (ν) = μ/ρ.

Water is flowing through a pipe of radius R at flow rate of Q. One small section of the pipe of length L is made of lead. The solubility of lead salts in water (containing oxygen) at system temperature is S (μM). As you know, lead is toxic and thus, the concentration needs to be minimized. Assume the diffusion boundary layer is much smaller than R, and that flow is laminar throughout. Also assume that the inlet water is free of lead salts.

a) **[7 points]** Derive an expression for the flux as a function of length along the pipe.

b) **[5 points]** Derive an expression for the total amount of lead entering the water per unit time.

c) **[5 points]** Derive an expression for the average lead concentration at the outlet of the pipe.

d) **[8 points]** How much would the concentration of lead change at the outlet when the temperature is increased from 25 °C to 100 °C? The viscosity of water at 25 °C and 100 °C is 9×10^{-4} Pa.s and 2.7×10^{-4} Pa.s, respectively. The solubility of lead salts in water containing oxygen at 100 °C is twice that at 25 °C.

C2. A swimming pool is 80% full. In the time interval t_1, 10% of the water evaporates, dropping the water level from 80% to 70% full. In the time interval t_2, another 10% of water evaporates, dropping its water level from 70% to 60%. Assume the gentle breeze above the pool is constant over time.

a) **[3 points]** Write the assumptions necessary to calculate the time two intervals, t_1 and t_2.

b) **[11 points]** Write the expression for average evaporating flux N_A.

c) **[11 points]** Obtain an expression each for time intervals t_1 and t_2, and calculate the ratio t_2/t_1.

APPENDIX A

Summary of the Conservation Equations

Table A.1 The Continuity Equation

$$\frac{\partial \rho}{\partial t}+(\nabla \cdot \rho \vec{u})=0$$	(1.1)
Rectangular coordinates (x, y, z) $$\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x}(\rho u_x)+\frac{\partial}{\partial y}(\rho u_y)+\frac{\partial}{\partial z}(\rho u_z)=0$$	(1.1a)
Cylindrical coordinates (r, θ, z) $$\frac{\partial \rho}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}(\rho r u_r)+\frac{1}{r}\frac{\partial}{\partial \theta}(\rho u_\theta)+\frac{\partial}{\partial z}(\rho u_z)=0$$	(1.1b)
Spherical coordinates (r, θ, ϕ) $$\frac{\partial \rho}{\partial t}+\frac{1}{r^2}\frac{\partial}{\partial r}(\rho r^2 u_r)+\frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\rho u_\theta \sin\theta)+\frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}(\rho u_\phi)=0$$	(1.1c)

Table A.2 The Navier-Stokes equations for Newtonian fluids of constant ρ and μ

	$$\frac{\partial \vec{u}}{\partial t}+(\vec{u}\cdot\nabla)\vec{u}=-\frac{1}{\rho}\nabla P+\vec{g}+\nu(\nabla^2\vec{u})$$	(A2)
Rectangular coordinates (x, y, z)		
x-component	$$\frac{\partial u_x}{\partial t}+u_x\frac{\partial u_x}{\partial x}+u_y\frac{\partial u_x}{\partial y}+u_z\frac{\partial u_x}{\partial z}=-\frac{1}{\rho}\frac{\partial P}{\partial x}+g_x+\nu\left(\frac{\partial^2 u_x}{\partial x^2}+\frac{\partial^2 u_x}{\partial y^2}+\frac{\partial^2 u_x}{\partial z^2}\right)$$	(A2a)
y-component	$$\frac{\partial u_y}{\partial t}+u_x\frac{\partial u_y}{\partial x}+u_y\frac{\partial u_y}{\partial y}+u_z\frac{\partial u_y}{\partial z}=-\frac{1}{\rho}\frac{\partial P}{\partial y}+g_y+\nu\left(\frac{\partial^2 u_y}{\partial x^2}+\frac{\partial^2 u_y}{\partial y^2}+\frac{\partial^2 u_y}{\partial z^2}\right)$$	(A2b)
z-component	$$\frac{\partial u_z}{\partial t}+u_x\frac{\partial u_z}{\partial x}+u_y\frac{\partial u_z}{\partial y}+u_z\frac{\partial u_z}{\partial z}=-\frac{1}{\rho}\frac{\partial P}{\partial z}+g_z+\nu\left(\frac{\partial^2 u_z}{\partial x^2}+\frac{\partial^2 u_z}{\partial y^2}+\frac{\partial^2 u_z}{\partial z^2}\right)$$	(A2c)

Cylindrical coordinates (r, θ, z)

r-component

$$\frac{\partial u_r}{\partial t}+u_r\frac{\partial u_r}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta}+u_z\frac{\partial u_r}{\partial z}-\frac{u_\theta^2}{r}$$
$$=-\frac{1}{\rho}\frac{\partial P}{\partial r}+g_r+\nu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(ru_r)}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u_r}{\partial \theta^2}-\frac{2}{r^2}\frac{\partial u_\theta}{\partial \theta}+\frac{\partial^2 u_r}{\partial z^2}\right] \quad \text{(A2d)}$$

θ-component

$$\frac{\partial u_\theta}{\partial t}+u_r\frac{\partial u_\theta}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta}+u_z\frac{\partial u_\theta}{\partial z}+\frac{u_r u_\theta}{r}$$
$$=-\frac{1}{\rho r}\frac{\partial P}{\partial \theta}+g_\theta+\nu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(ru_\theta)}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u_\theta}{\partial \theta^2}+\frac{2}{r^2}\frac{\partial u_r}{\partial \theta}+\frac{\partial^2 u_\theta}{\partial z^2}\right] \quad \text{(A2e)}$$

z-component

$$\frac{\partial u_z}{\partial t}+u_r\frac{\partial u_z}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_z}{\partial \theta}+u_z\frac{\partial u_z}{\partial z}$$
$$=-\frac{1}{\rho}\frac{\partial P}{\partial z}+g_z+\nu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u_z}{\partial \theta^2}+\frac{\partial^2 u_z}{\partial z^2}\right] \quad \text{(A2f)}$$

Spherical coordinates (r, θ, ϕ)

r-component

$$\frac{\partial u_r}{\partial t}+u_r\frac{\partial u_r}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta}+\left(\frac{u_\phi}{r\sin\theta}\right)\frac{\partial u_r}{\partial \phi}-\frac{u_\theta^2}{r}-\frac{u_\phi^2}{r}=-\frac{1}{\rho}\frac{\partial P}{\partial r}+g_r$$
$$+\nu\left[\frac{1}{r^2}\frac{\partial^2}{\partial r^2}\left(r^2 u_r\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial u_r}{\partial \theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 u_r}{\partial \phi^2}\right] \quad \text{(A2g)}$$

θ-component

$$\frac{\partial u_\theta}{\partial t}+u_r\frac{\partial u_\theta}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta}+\left(\frac{u_\phi}{r\sin\theta}\right)\frac{\partial u_\theta}{\partial \phi}+\frac{u_r u_\theta}{r}-\frac{u_\phi^2}{r}\cot\theta=-\frac{1}{\rho r}\frac{\partial P}{\partial \theta}+g_\theta$$
$$+\nu\left[\begin{array}{l}\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial u_\theta}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial \theta}\left(u_\theta\sin\theta\right)\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 u_\theta}{\partial \phi^2}\\ +\frac{2}{r^2}\frac{\partial u_r}{\partial \theta}-\frac{2\cot\theta}{r^2\sin\theta}\frac{\partial u_\phi}{\partial \phi}\end{array}\right] \quad \text{(A2h)}$$

ϕ-component

$$\frac{\partial u_\phi}{\partial t}+u_r\frac{\partial u_\phi}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_\phi}{\partial \theta}+\frac{u_\phi}{r\sin\theta}\frac{\partial u_\phi}{\partial \phi}+\frac{u_r u_\phi}{r}+\frac{u_\theta u_\phi}{r}\cot\theta=-\frac{1}{\rho r\sin\theta}\frac{\partial P}{\partial \phi}$$
$$+g_\varphi+\nu\left[\begin{array}{l}\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial u_\phi}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial \theta}\left(u_\phi\sin\theta\right)\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 u_\phi}{\partial \phi^2}\\ +\frac{2}{r^2\sin\theta}\frac{\partial u_r}{\partial \phi}+\frac{2\cot\theta}{r^2\sin\theta}\frac{\partial u_\theta}{\partial \phi}\end{array}\right] \quad \text{(A2i)}$$

Table A.3 The Energy Equation for Incompressible Media

$$\rho c_P\left[\frac{\partial T}{\partial t}+(\vec{u}\cdot\nabla)(T)\right]=[\nabla\cdot k\nabla T]+\dot{T}_G \qquad \text{(A3)}$$

Rectangular coordinates (*x*, *y*, *z*)

$$\rho c_P\left[\frac{\partial T}{\partial t}+u_x\frac{\partial T}{\partial x}+u_y\frac{\partial T}{\partial y}+u_z\frac{\partial T}{\partial z}\right]=\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{T}_G \qquad \text{(A3a)}$$

Cylindrical coordinates (*r*, *θ*, *z*)

$$\rho c_P\left[\frac{\partial T}{\partial t}+u_r\frac{\partial T}{\partial r}+\frac{u_\theta}{r}\frac{\partial T}{\partial \theta}+u_z\frac{\partial T}{\partial z}\right]=\frac{1}{r}\frac{\partial}{\partial r}\left(rk\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \theta}\left(k\frac{\partial T}{\partial \theta}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{T}_G \qquad \text{(A3b)}$$

Spherical coordinates (*r*, *θ*, *ϕ*)

$$\rho c_P\left[\frac{\partial T}{\partial t}+u_r\frac{\partial T}{\partial r}+\frac{u_\theta}{r}\frac{\partial T}{\partial \theta}+\frac{u_\phi}{r\sin\theta}\frac{\partial T}{\partial \phi}\right]=$$
$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2k\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(k\sin\theta\frac{\partial T}{\partial \theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right)+\dot{T}_G \qquad \text{(A3c)}$$

Table A4: The continuity equation for species *A* in terms of the molar flux

$$\frac{\partial C_A}{\partial t}=-(\nabla\cdot\vec{N}_A)+\dot{R}_{A,G} \qquad \text{(4.)}$$

Rectangular coordinates (*x*, *y*, *z*)

$$\frac{\partial C_A}{\partial t}=-\left(\frac{\partial[N_A]_z}{\partial x}+\frac{\partial[N_A]_y}{\partial y}+\frac{\partial[N_A]_z}{\partial z}\right)+\dot{R}_{A,G} \qquad \text{(4a)}$$

Cylindrical coordinates (*r*, *θ*, *z*)

$$\frac{\partial C_A}{\partial t}=-\left\{\frac{1}{r}\frac{\partial}{\partial r}[rN_A]_r+\frac{1}{r}\frac{\partial}{\partial \theta}[N_A]_\theta+\frac{\partial}{\partial z}[N_A]_z\right\}+\dot{R}_{A,G} \qquad \text{(4b)}$$

Spherical coordinates (*r*, *θ*, *ϕ*)

$$\frac{\partial C_A}{\partial t}=-\left\{\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2[N_A]_r\right)+\frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}\left([N_A]_\theta\sin\theta\right)+\frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}[N_A]_\phi\right\}+\dot{R}_{A,G} \qquad \text{(4c)}$$

Table A.5: The continuity equation for species *A*

$$\frac{\partial C_A}{\partial t} + (\vec{u}\cdot\nabla)C_A = D_A\nabla^2 C_A + \dot{R}_{A,G}$$	(5)
Rectangular coordinates (*x, y, z*) $$\frac{\partial C_A}{\partial t} + u_x\frac{\partial C_A}{\partial x} + u_y\frac{\partial C_A}{\partial y} + u_z\frac{\partial C_A}{\partial z} = \frac{\partial}{\partial x}\left(D\frac{\partial C_A}{\partial x}\right) + \frac{\partial}{\partial y}\left(D\frac{\partial C_A}{\partial y}\right) + \frac{\partial}{\partial z}\left(D\frac{\partial C_A}{\partial z}\right) + \dot{R}_{A,G}$$	(5a)
Cylindrical coordinates (*r, θ, z*) $$\frac{\partial C_A}{\partial t} + u_r\frac{\partial C_A}{\partial r} + \frac{u_\theta}{r}\frac{\partial C_A}{\partial \theta} + u_z\frac{\partial C_A}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\left(rD\frac{\partial C_A}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \theta}\left(D\frac{\partial C_A}{\partial \theta}\right) + \frac{\partial}{\partial z}\left(D\frac{\partial C_A}{\partial z}\right) + \dot{R}_{A,G}$$	(5b)
Spherical coordinates (*r, θ, φ*) $$\frac{\partial C_A}{\partial t} + u_r\frac{\partial C_A}{\partial r} + \frac{u_\theta}{r}\frac{\partial C_A}{\partial \theta} + \frac{u_\phi}{r\sin\theta}\frac{\partial C_A}{\partial \phi} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 D\frac{\partial C_A}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(D\sin\theta\frac{\partial C_A}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial \phi}\left(D\frac{\partial C_A}{\partial \phi}\right) + \dot{R}_{A,G}$$	(5c)