

National Exams December 2019

17-Ind-B1, Applied Probability & Statistics

3 hours duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
2. This is a closed book exam. One of two calculators is permitted – any Casio or Sharp approved model.
3. Candidates are permitted to have an aid sheet consisting of one 8.5” x 11.0” sheet of paper. Writing is permitted on both sides of the paper.
4. This exam consists of three sections (A→C). Within each section, candidates will be given a choice of questions to answer. Please read the instructions for each section carefully. A breakdown of questions and marks is as follows:

Section A: Do 2 of 4 Questions. Total marks: 20

Section B: Do 2 of 3 Questions. Total marks: 40

Section C: Do 1 of 2 Questions. Total marks: 20

Exam: 5 Questions. Total marks: 100

4. The value of each question is listed in the exam. Remember to check the instructions for each section. DO NOT ATTEMPT TO DO ALL QUESTIONS.
5. Statistical tables are provided. If, for any reason, a value is missing, please note this and use an approximate value.

Section A – Basic Concepts: Complete two of the following four questions. This section is worth a total of 30 marks. Do not attempt all questions.

1. A function $f(x)$ given below:

$$f(x) = \begin{cases} \alpha x & 0 < x < 3 \\ 3 - 2x & 3 \leq x \leq 5 \\ 0 & \textit{elsewhere} \end{cases}$$

- What value must α be for $f(x)$ to be a valid probability function? Why?
- Find the mean of the function, under the assumption that it is a valid pdf, using the value for α you discovered in part a.
- Find the variance of the function using the value for α you discovered in part a.

10 Marks

2. The IE Department at Northern Utopia Technical School (NUTS) went through a major curriculum change about 4 years ago. The faculty at NUTS surmise that on average, student marks have risen by about 6 percent under the new curriculum. To test this value, assume that the following cumulative grades for six students have been drawn at random:

Old Curriculum	New Curriculum
74.4	72.7
62.3	76.7
65.3	78.2
78.4	71.1
63.9	71.2
66.3	74.2

- Determine, using $\alpha = 0.05$, whether the variances from the two populations are the same.
- Using this information, develop a hypothesis test to test whether the difference in grades between the old and new curriculum is actually 6%. Use $\alpha = 0.05$. Please state any assumptions that you have made in developing your hypothesis test.
- Provide a 95% confidence interval for the average marks under the old curriculum.
- Does the confidence interval in (c) tell us whether the data point 78.4 could be considered an anomaly under the old curriculum?

10 Marks

3. An IE would like to validate her simulation model against a sample of data from a real-world system. She has five months of production data showing average job time in system (in minutes). She sets her simulation to run for a period of 1 simulated month and makes five independent replications. The following data is obtained.

Actual	Simulation
104	120
107	67
94	115
110	100
107	138

- a) Are the variances of these two samples the same? Use $\alpha = 0.05$.
b) Test the hypothesis:

$$H_0: \mu_1 = \mu_2$$
$$H_1: \mu_1 \neq \mu_2$$

for this dataset at the $\alpha = 0.05$ level and state any assumptions you make.

10 Marks

4. If x and y have the following joint probability function:

x	y			$p_x(x)$
	0	1	2	
0	0.05	0.10	0.20	0.35
1	0.05	0.15	0.05	0.25
2	0.25	0.10	0.05	0.40
$p_y(y)$	0.35	0.35	0.30	

- a) Find ρ_{xy}
b) Are x and y independent?

10 Marks

Section B – Intermediate Concepts: Complete **two** of the following **three** questions. This section is worth a total of 30 marks. Do not attempt all questions.

1. The final class averages for 10 industrial engineering students taking OR I and Statistics has been compiled from historical records:

OR I	Statistics
86	80
75	81
69	75
74	81
90	92
94	95
83	80
86	81
71	76
65	72

- Assuming OR I to be the independent variable (i.e. x), find a regression line relating Statistics marks (y) to OR I marks.
- Test for the significance of the regression using $\alpha = 0.05$. To aid you in your calculations, you may assume that SST is 460.1.
- Test for the significance of the constant in the regression, using $\alpha = 0.05$.
- Estimate the correlation between a student's statistics mark and their OR I mark.
- If a student scores 60 in their OR I class, calculate the range of likely values for their Statistics mark. Again, use $\alpha = 0.05$.

20 Marks

2. Historically, the median wait time for hip surgery in Nova Scotia has been 18 months (note – medians are used to measure waits to eliminate possible outliers in the data). Over the past 12 months, the following median wait times for hip surgery has been recorded as:

Month	1	2	3	4	5	6	7	8	9	10	11	12
Wait	15	19	16	17	18	12	21	17	14	19	17	16

- Using the sign test, determine ($\alpha = 0.05$) if there is any evidence to suggest that the median wait time for hip surgery has changed over the past 12 months.
- If the probability that wait time exceeds 20 months is known from historical records to be 2% and wait time from month-to-month is assumed to be independent, what is the probability that exactly two months in a year exceeds 21 months?

- c. What is the probability that at least 2 months in a year will record wait times in excess of 20 months?

20 Marks

3. An IE is evaluating the work methods at four workstations in a plant that produces hops (a type of coniferous plant that is used in the brewing of beer). Each of the four workstations involves packing (i.e. gathering the hops into a Kevlar bag, weighing, and sealing). Each of the operators has been issued with the same standard procedure for packing. However, management feels that the resulting packing times are not the same. A sample of three cycles was collected at each workstation. The resulting times (in seconds) are listed below:

WS 1	WS 2	WS 3	WS 4
25.6	25.2	20.8	31.6
24.3	28.6	26.7	29.8
27.9	24.7	22.2	34.3

Please note that the variance for entire sample is 9.51

- Perform an analysis of variance at $\alpha = 0.05$ level of significance.
- Determine, using Bartlett's test, if the cycle time variances is constant between the workstations. Assume a significance level of 0.05. Comment on the results of this test with respect to your answer in part (a).
- Use Tukey's test to determine which, if any, of the workstations has a significantly different cycle time. Use an $\alpha = 0.05$.
- An alternate method to part (c) might be to conduct a pair-wise comparison of means for each possible combination of workstations. How many tests would be required to complete a full pair-wise comparison? What would the confidence level have to be set at for each of the comparisons to ensure an overall confidence of $\alpha = 0.05$.

20 Marks

Section C – Advanced Concepts: Complete one of the following two questions. This section is worth a total of 20 marks. Do not attempt all questions.

1. After running several marathons, a runner has become convinced that her performance (as measured in time to complete the race) is a function of three elements: x_1 – total number of kilometres *100 run in the six weeks prior to the race; x_2 – the longest single run completed during the six weeks prior to the race; and x_3 – the expected temperature on race day. To test this hypothesis, the runner has collected the following dataset:

x_1 Total km (*100)	x_2 Max Dist	x_3 Temp	y Time (Hrs)
4.6	27	11	4.3
4.5	31	10	3.8
4.0	30	25	3.5
4.1	25	22	3.6
4.4	28	14	3.5
4.2	32	17	3.4

The relationship between completion time and (x_1, x_2, x_3) is assumed to be linear in the region of interest.

- a. Build the $(X'X)$ matrix.
- b. A partial table of $(X'X)^{-1}$ is given below, where * , ** , and *** remain to be determined:

3186.5127	*	-10.0932	-22.9764
-585.1664	108.4063	1.7075	4.2294
-10.0932	1.7075	0.0562	**
-22.9764	***	0.0685	0.1706

- c. Assume that $(X'X)^{-1}(X'Y)$ is:

-16.29491
4.1060
0.0093
0.1245

Calculate SSE, SSR, and SST.

- d. Complete the ANOVA table for this regression model ($\alpha = 0.05$) and determine whether the linear model is a significant predictor of y .
- e. Determine if any of the coefficients in your model is significant at the 95% level.
- f. Based on your analysis, is the model a good predictor of race finishing time?

20 Marks

- 2) The following dataset is part of a 2^3 designed experiment to determine the effects of cutting speed (A), tool geometry (B), and cutting angle (C) on the life of a machine tool. Two levels of each factor are chosen, and duplicates were run at each design point.

Run	A	B	C	Rep 1	Rep 2
1	-1	-1	-1	32	31
2	1	-1	-1	15	43
3	-1	1	-1	35	34
4	1	1	-1	35	47
5	-1	-1	1	44	45
6	1	-1	1	40	37
7	-1	1	1	60	50
8	1	1	1	39	41

- a. Complete the rest of the design matrix for the four interactions
- b. Compute the contrast and mean effects for the A factor, the AB factor, and the ABC factor (see the contrast table below)
- c. Calculate the sum of squares for the A factor, the AB factor, and the ABC factor. See the Sum of Squares table below.
- d. Assuming the updated contrast and Sum of Squares, calculate the ANOVA table for this model. Assume $\alpha = 0.05$.

Contrasts

A	B	C	AB	AC	BC	ABC
-34	*	84	**	-50	-6	***

Sum of Squares

SS(A)	SS(B)	SS(C)	SS(AB)	SS(AC)	SS(BC)	SS(ABC)	SST
*	182.25	**	0	***	2.25	81	1457

- e. Factor and interaction terms are significant (if any).
- f. Fit a linear regression model to the data.

20 Marks

Areas Under the Normal Curve

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.40	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.30	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.20	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.10	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.00	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.90	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.80	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.70	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.60	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.50	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.40	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.30	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.20	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.10	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.00	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.90	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.80	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.70	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.60	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.50	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.40	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.30	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.20	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.10	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.00	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.90	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.80	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.70	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.60	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.50	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.40	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.30	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.20	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.10	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.00	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Areas Under the Normal Curve

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.00	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.10	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.20	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.30	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.40	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.50	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.60	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.70	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.80	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.90	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.00	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.10	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.20	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.30	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.40	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.50	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.60	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.70	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.80	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.90	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.00	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.10	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.20	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.30	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.40	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.50	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.60	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.70	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.80	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.90	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.00	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.10	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.20	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.30	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.40	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Critical Values of the t-Distribution

v	α						
	0.40	0.30	0.20	0.15	0.10	0.05	0.025
1	0.325	0.727	1.376	1.963	3.078	6.314	12.706
2	0.289	0.617	1.061	1.386	1.886	2.920	4.303
3	0.277	0.584	0.978	1.250	1.638	2.353	3.182
4	0.271	0.569	0.941	1.190	1.533	2.132	2.776
5	0.267	0.559	0.920	1.156	1.476	2.015	2.571
6	0.265	0.553	0.906	1.134	1.440	1.943	2.447
7	0.263	0.549	0.896	1.119	1.415	1.895	2.365
8	0.262	0.546	0.889	1.108	1.397	1.860	2.306
9	0.261	0.543	0.883	1.100	1.383	1.833	2.262
10	0.260	0.542	0.879	1.093	1.372	1.812	2.228
11	0.260	0.540	0.876	1.088	1.363	1.796	2.201
12	0.259	0.539	0.873	1.083	1.356	1.782	2.179
13	0.259	0.538	0.870	1.079	1.350	1.771	2.160
14	0.258	0.537	0.868	1.076	1.345	1.761	2.145
15	0.258	0.536	0.866	1.074	1.341	1.753	2.131
16	0.258	0.535	0.865	1.071	1.337	1.746	2.120
17	0.257	0.534	0.863	1.069	1.333	1.740	2.110
18	0.257	0.534	0.862	1.067	1.330	1.734	2.101
19	0.257	0.533	0.861	1.066	1.328	1.729	2.093
20	0.257	0.533	0.860	1.064	1.325	1.725	2.086
21	0.257	0.532	0.859	1.063	1.323	1.721	2.080
22	0.256	0.532	0.858	1.061	1.321	1.717	2.074
23	0.256	0.532	0.858	1.060	1.319	1.714	2.069
24	0.256	0.531	0.857	1.059	1.318	1.711	2.064
25	0.256	0.531	0.856	1.058	1.316	1.708	2.060
26	0.256	0.531	0.856	1.058	1.315	1.706	2.056
27	0.256	0.531	0.855	1.057	1.314	1.703	2.052
28	0.256	0.530	0.855	1.056	1.313	1.701	2.048
29	0.256	0.530	0.854	1.055	1.311	1.699	2.045
30	0.256	0.530	0.854	1.055	1.310	1.697	2.042
40	0.255	0.529	0.851	1.050	1.303	1.684	2.021
50	0.255	0.528	0.849	1.047	1.299	1.676	2.009
60	0.254	0.527	0.848	1.045	1.296	1.671	2.000
70	0.254	0.527	0.847	1.044	1.294	1.667	1.994
80	0.254	0.526	0.846	1.043	1.292	1.664	1.990
90	0.254	0.526	0.846	1.042	1.291	1.662	1.987
100	0.254	0.526	0.845	1.042	1.290	1.660	1.984
110	0.254	0.526	0.845	1.041	1.289	1.659	1.982
120	0.254	0.526	0.845	1.041	1.289	1.658	1.980
130	0.254	0.526	0.844	1.041	1.288	1.657	1.978
∞	0.253	0.524	0.842	1.036	1.282	1.645	1.960

Critical Values of the t-Distribution

v	α						
	0.02	0.015	0.01	0.0075	0.005	0.0025	0.0005
1	15.895	21.205	31.821	42.433	63.657	127.321	636.619
2	4.849	5.643	6.965	8.073	9.925	14.089	31.599
3	3.482	3.896	4.541	5.047	5.841	7.453	12.924
4	2.999	3.298	3.747	4.088	4.604	5.598	8.610
5	2.757	3.003	3.365	3.634	4.032	4.773	6.869
6	2.612	2.829	3.143	3.372	3.707	4.317	5.959
7	2.517	2.715	2.998	3.203	3.499	4.029	5.408
8	2.449	2.634	2.896	3.085	3.355	3.833	5.041
9	2.398	2.574	2.821	2.998	3.250	3.690	4.781
10	2.359	2.527	2.764	2.932	3.169	3.581	4.587
11	2.328	2.491	2.718	2.879	3.106	3.497	4.437
12	2.303	2.461	2.681	2.836	3.055	3.428	4.318
13	2.282	2.436	2.650	2.801	3.012	3.372	4.221
14	2.264	2.415	2.624	2.771	2.977	3.326	4.140
15	2.249	2.397	2.602	2.746	2.947	3.286	4.073
16	2.235	2.382	2.583	2.724	2.921	3.252	4.015
17	2.224	2.368	2.567	2.706	2.898	3.222	3.965
18	2.214	2.356	2.552	2.689	2.878	3.197	3.922
19	2.205	2.346	2.539	2.674	2.861	3.174	3.883
20	2.197	2.336	2.528	2.661	2.845	3.153	3.850
21	2.189	2.328	2.518	2.649	2.831	3.135	3.819
22	2.183	2.320	2.508	2.639	2.819	3.119	3.792
23	2.177	2.313	2.500	2.629	2.807	3.104	3.768
24	2.172	2.307	2.492	2.620	2.797	3.091	3.745
25	2.167	2.301	2.485	2.612	2.787	3.078	3.725
26	2.162	2.296	2.479	2.605	2.779	3.067	3.707
27	2.158	2.291	2.473	2.598	2.771	3.057	3.690
28	2.154	2.286	2.467	2.592	2.763	3.047	3.674
29	2.150	2.282	2.462	2.586	2.756	3.038	3.659
30	2.147	2.278	2.457	2.581	2.750	3.030	3.646
40	2.123	2.250	2.423	2.542	2.704	2.971	3.551
50	2.109	2.234	2.403	2.519	2.678	2.937	3.496
60	2.099	2.223	2.390	2.504	2.660	2.915	3.460
70	2.093	2.215	2.381	2.494	2.648	2.899	3.435
80	2.088	2.209	2.374	2.486	2.639	2.887	3.416
90	2.084	2.205	2.368	2.480	2.632	2.878	3.402
100	2.081	2.201	2.364	2.475	2.626	2.871	3.390
110	2.078	2.199	2.361	2.471	2.621	2.865	3.381
120	2.076	2.196	2.358	2.468	2.617	2.860	3.373
130	2.075	2.194	2.355	2.465	2.614	2.856	3.367
∞	2.054	2.170	2.327	2.433	2.576	2.808	3.291

Critical Values of the Chi-Squared Distribution

v	α									
	0.995	0.99	0.98	0.975	0.95	0.90	0.80	0.75	0.70	0.50
1	0.000	0.000	0.001	0.001	0.004	0.016	0.064	0.102	0.148	0.455
2	0.010	0.020	0.040	0.051	0.103	0.211	0.446	0.575	0.713	1.386
3	0.072	0.115	0.185	0.216	0.352	0.584	1.005	1.213	1.424	2.366
4	0.207	0.297	0.429	0.484	0.711	1.064	1.649	1.923	2.195	3.357
5	0.412	0.554	0.752	0.831	1.145	1.610	2.343	2.675	3.000	4.351
6	0.676	0.872	1.134	1.237	1.635	2.204	3.070	3.455	3.828	5.348
7	0.989	1.239	1.564	1.690	2.167	2.833	3.822	4.255	4.671	6.346
8	1.344	1.646	2.032	2.180	2.733	3.490	4.594	5.071	5.527	7.344
9	1.735	2.088	2.532	2.700	3.325	4.168	5.380	5.899	6.393	8.343
10	2.156	2.558	3.059	3.247	3.940	4.865	6.179	6.737	7.267	9.342
11	2.603	3.053	3.609	3.816	4.575	5.578	6.989	7.584	8.148	10.341
12	3.074	3.571	4.178	4.404	5.226	6.304	7.807	8.438	9.034	11.340
13	3.565	4.107	4.765	5.009	5.892	7.042	8.634	9.299	9.926	12.340
14	4.075	4.660	5.368	5.629	6.571	7.790	9.467	10.165	10.821	13.339
15	4.601	5.229	5.985	6.262	7.261	8.547	10.307	11.037	11.721	14.339
16	5.142	5.812	6.614	6.908	7.962	9.312	11.152	11.912	12.624	15.338
17	5.697	6.408	7.255	7.564	8.672	10.085	12.002	12.792	13.531	16.338
18	6.265	7.015	7.906	8.231	9.390	10.865	12.857	13.675	14.440	17.338
19	6.844	7.633	8.567	8.907	10.117	11.651	13.716	14.562	15.352	18.338
20	7.434	8.260	9.237	9.591	10.851	12.443	14.578	15.452	16.266	19.337
21	8.034	8.897	9.915	10.283	11.591	13.240	15.445	16.344	17.182	20.337
22	8.643	9.542	10.600	10.982	12.338	14.041	16.314	17.240	18.101	21.337
23	9.260	10.196	11.293	11.689	13.091	14.848	17.187	18.137	19.021	22.337
24	9.886	10.856	11.992	12.401	13.848	15.659	18.062	19.037	19.943	23.337
25	10.520	11.524	12.697	13.120	14.611	16.473	18.940	19.939	20.867	24.337
26	11.160	12.198	13.409	13.844	15.379	17.292	19.820	20.843	21.792	25.336
27	11.808	12.879	14.125	14.573	16.151	18.114	20.703	21.749	22.719	26.336
28	12.461	13.565	14.847	15.308	16.928	18.939	21.588	22.657	23.647	27.336
29	13.121	14.256	15.574	16.047	17.708	19.768	22.475	23.567	24.577	28.336
30	13.787	14.953	16.306	16.791	18.493	20.599	23.364	24.478	25.508	29.336

Critical Values of the Chi-Squared Distribution

v	α									
	0.3	0.25	0.2	0.1	0.05	0.025	0.02	0.01	0.005	0.001
1	1.074	1.323	1.642	2.706	3.841	5.024	5.412	6.635	7.879	10.828
2	2.408	2.773	3.219	4.605	5.991	7.378	7.824	9.210	10.597	13.816
3	3.665	4.108	4.642	6.251	7.815	9.348	9.837	11.345	12.838	16.266
4	4.878	5.385	5.989	7.779	9.488	11.143	11.668	13.277	14.860	18.467
5	6.064	6.626	7.289	9.236	11.070	12.833	13.388	15.086	16.750	20.515
6	7.231	7.841	8.558	10.645	12.592	14.449	15.033	16.812	18.548	22.458
7	8.383	9.037	9.803	12.017	14.067	16.013	16.622	18.475	20.278	24.322
8	9.524	10.219	11.030	13.362	15.507	17.535	18.168	20.090	21.955	26.124
9	10.656	11.389	12.242	14.684	16.919	19.023	19.679	21.666	23.589	27.877
10	11.781	12.549	13.442	15.987	18.307	20.483	21.161	23.209	25.188	29.588
11	12.899	13.701	14.631	17.275	19.675	21.920	22.618	24.725	26.757	31.264
12	14.011	14.845	15.812	18.549	21.026	23.337	24.054	26.217	28.300	32.909
13	15.119	15.984	16.985	19.812	22.362	24.736	25.472	27.688	29.819	34.528
14	16.222	17.117	18.151	21.064	23.685	26.119	26.873	29.141	31.319	36.123
15	17.322	18.245	19.311	22.307	24.996	27.488	28.259	30.578	32.801	37.697
16	18.418	19.369	20.465	23.542	26.296	28.845	29.633	32.000	34.267	39.252
17	19.511	20.489	21.615	24.769	27.587	30.191	30.995	33.409	35.718	40.790
18	20.601	21.605	22.760	25.989	28.869	31.526	32.346	34.805	37.156	42.312
19	21.689	22.718	23.900	27.204	30.144	32.852	33.687	36.191	38.582	43.820
20	22.775	23.828	25.038	28.412	31.410	34.170	35.020	37.566	39.997	45.315
21	23.858	24.935	26.171	29.615	32.671	35.479	36.343	38.932	41.401	46.797
22	24.939	26.039	27.301	30.813	33.924	36.781	37.659	40.289	42.796	48.268
23	26.018	27.141	28.429	32.007	35.172	38.076	38.968	41.638	44.181	49.728
24	27.096	28.241	29.553	33.196	36.415	39.364	40.270	42.980	45.559	51.179
25	28.172	29.339	30.675	34.382	37.652	40.646	41.566	44.314	46.928	52.620
26	29.246	30.435	31.795	35.563	38.885	41.923	42.856	45.642	48.290	54.052
27	30.319	31.528	32.912	36.741	40.113	43.195	44.140	46.963	49.645	55.476
28	31.391	32.620	34.027	37.916	41.337	44.461	45.419	48.278	50.993	56.892
29	32.461	33.711	35.139	39.087	42.557	45.722	46.693	49.588	52.336	58.301
30	33.530	34.800	36.250	40.256	43.773	46.979	47.962	50.892	53.672	59.703

Critical Values of the F Distribution

		$f_{0.05(v1,v2)}$									
		v1									
v2	1	2	3	4	5	6	7	8	9	10	
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54	241.88	
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	
50	4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13	2.07	2.03	
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	

Critical Values of the F Distribution

		$f_{0.05(v1, v2)}$									
		v1									
v2	10	12	15	20	24	30	40	60	120	∞	
1	241.88	243.91	245.95	248.01	249.05	250.10	251.14	252.20	253.25	254.30	
2	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50	
3	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53	
4	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63	
5	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.37	
6	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67	
7	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23	
8	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93	
9	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71	
10	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54	
11	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.41	
12	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30	
13	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21	
14	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13	
15	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07	
16	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01	
17	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96	
18	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92	
19	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88	
20	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84	
21	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81	
22	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78	
23	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76	
24	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73	
25	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71	
26	2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69	
27	2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67	
28	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65	
29	2.18	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64	
30	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62	
40	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51	
50	2.03	1.95	1.87	1.78	1.74	1.69	1.63	1.58	1.51	1.44	
60	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39	
120	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.26	
∞	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.01	

Critical Values of the F Distribution

		$f_{0.01(v1,v2)}$									
		v1									
v2	1	2	3	4	5	6	7	8	9	10	
1	4052.18	4999.50	5403.35	5624.58	5763.65	5858.99	5928.36	5981.07	6022.47	6055.85	
2	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39	99.40	
3	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35	27.23	
4	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66	14.55	
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16	10.05	
6	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	
7	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	
8	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	
9	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	
14	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	3.94	
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	
17	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68	3.59	
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31	
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21	
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	
25	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22	3.13	
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09	
27	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15	3.06	
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03	
29	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09	3.00	
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	
50	7.17	5.06	4.20	3.72	3.41	3.19	3.02	2.89	2.78	2.70	
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	
∞	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	

Critical Values of the F Distribution

		$f_{0.01}(v_1, v_2)$									
		v1									
v2		10	12	15	20	24	30	40	60	120	∞
1		6055.85	6106.32	6157.28	6208.73	6234.63	6260.65	6286.78	6313.03	6339.39	6365.55
2		99.40	99.42	99.43	99.45	99.46	99.47	99.47	99.48	99.49	99.50
3		27.23	27.05	26.87	26.69	26.60	26.50	26.41	26.32	26.22	26.13
4		14.55	14.37	14.20	14.02	13.93	13.84	13.75	13.65	13.56	13.46
5		10.05	9.89	9.72	9.55	9.47	9.38	9.29	9.20	9.11	9.02
6		7.87	7.72	7.56	7.40	7.31	7.23	7.14	7.06	6.97	6.88
7		6.62	6.47	6.31	6.16	6.07	5.99	5.91	5.82	5.74	5.65
8		5.81	5.67	5.52	5.36	5.28	5.20	5.12	5.03	4.95	4.86
9		5.26	5.11	4.96	4.81	4.73	4.65	4.57	4.48	4.40	4.31
10		4.85	4.71	4.56	4.41	4.33	4.25	4.17	4.08	4.00	3.91
11		4.54	4.40	4.25	4.10	4.02	3.94	3.86	3.78	3.69	3.60
12		4.30	4.16	4.01	3.86	3.78	3.70	3.62	3.54	3.45	3.36
13		4.10	3.96	3.82	3.66	3.59	3.51	3.43	3.34	3.25	3.17
14		3.94	3.80	3.66	3.51	3.43	3.35	3.27	3.18	3.09	3.01
15		3.80	3.67	3.52	3.37	3.29	3.21	3.13	3.05	2.96	2.87
16		3.69	3.55	3.41	3.26	3.18	3.10	3.02	2.93	2.84	2.75
17		3.59	3.46	3.31	3.16	3.08	3.00	2.92	2.83	2.75	2.65
18		3.51	3.37	3.23	3.08	3.00	2.92	2.84	2.75	2.66	2.57
19		3.43	3.30	3.15	3.00	2.92	2.84	2.76	2.67	2.58	2.49
20		3.37	3.23	3.09	2.94	2.86	2.78	2.69	2.61	2.52	2.42
21		3.31	3.17	3.03	2.88	2.80	2.72	2.64	2.55	2.46	2.36
22		3.26	3.12	2.98	2.83	2.75	2.67	2.58	2.50	2.40	2.31
23		3.21	3.07	2.93	2.78	2.70	2.62	2.54	2.45	2.35	2.26
24		3.17	3.03	2.89	2.74	2.66	2.58	2.49	2.40	2.31	2.21
25		3.13	2.99	2.85	2.70	2.62	2.54	2.45	2.36	2.27	2.17
26		3.09	2.96	2.81	2.66	2.58	2.50	2.42	2.33	2.23	2.13
27		3.06	2.93	2.78	2.63	2.55	2.47	2.38	2.29	2.20	2.10
28		3.03	2.90	2.75	2.60	2.52	2.44	2.35	2.26	2.17	2.07
29		3.00	2.87	2.73	2.57	2.49	2.41	2.33	2.23	2.14	2.04
30		2.98	2.84	2.70	2.55	2.47	2.39	2.30	2.21	2.11	2.01
40		2.80	2.66	2.52	2.37	2.29	2.20	2.11	2.02	1.92	1.81
50		2.70	2.56	2.42	2.27	2.18	2.10	2.01	1.91	1.80	1.68
60		2.63	2.50	2.35	2.20	2.12	2.03	1.94	1.84	1.73	1.60
120		2.47	2.34	2.19	2.03	1.95	1.86	1.76	1.66	1.53	1.38
∞		2.32	2.18	2.04	1.88	1.79	1.70	1.59	1.47	1.32	1.01

Critical Values for Bartlett's Test

$b_k(0.01;n)$									
n	Number of populations, k								
	2	3	4	5	6	7	8	9	10
3	0.1411	0.1672							
4	0.2843	0.3165	0.3475	0.3729	0.3937	0.4110			
5	0.3984	0.4304	0.4607	0.4850	0.5046	0.5207	0.5343	0.5458	0.5558
6	0.4850	0.5149	0.5430	0.5653	0.5832	0.5975	0.6100	0.6204	0.6293
7	0.5512	0.5787	0.6045	0.6248	0.6410	0.6542	0.6652	0.6744	0.6824
8	0.6031	0.6282	0.6518	0.6704	0.6851	0.6970	0.7069	0.7153	0.7225
9	0.6445	0.6676	0.6892	0.7062	0.7197	0.7305	0.7395	0.7471	0.7536
10	0.6783	0.6996	0.7195	0.7352	0.7475	0.7575	0.7657	0.7726	0.7786
11	0.7063	0.7260	0.7445	0.7590	0.7703	0.7795	0.7871	0.7935	0.7990
12	0.7299	0.7483	0.7654	0.7789	0.7894	0.7980	0.8050	0.8109	0.8160
13	0.7501	0.7672	0.7832	0.7958	0.8056	0.8135	0.8201	0.8256	0.8303
14	0.7674	0.7835	0.7985	0.8103	0.8195	0.8269	0.8330	0.8382	0.8426
15	0.7825	0.7977	0.8118	0.8229	0.8315	0.8385	0.8443	0.8491	0.8532
16	0.7958	0.8101	0.8235	0.8339	0.8421	0.8486	0.8541	0.8586	0.8625
17	0.8076	0.8211	0.8338	0.8436	0.8514	0.8576	0.8627	0.8670	0.8707
18	0.8181	0.8309	0.8429	0.8523	0.8596	0.8655	0.8704	0.8745	0.8780
19	0.8275	0.8397	0.8512	0.8601	0.8670	0.8727	0.8773	0.8811	0.8845
20	0.8360	0.8476	0.8586	0.8671	0.8737	0.8791	0.8835	0.8871	0.8903

Values for Tukey's Test
Upper Percentage Points of the Studentized Range Distribution: Values of $q(0.05, k, \nu)$

Degrees of Freedom ν	Number of Treatments k									
	2	3	4	5	6	7	8	9	10	
1	18.00	27.00	32.80	37.20	40.50	43.10	45.10	47.10	49.10	
2	6.09	5.33	9.80	10.89	11.73	12.43	13.03	13.54	13.99	
3	4.50	5.91	6.83	7.51	8.04	8.47	8.85	9.18	9.46	
4	3.93	5.04	5.76	6.29	6.71	7.06	7.35	7.60	7.83	
5	3.64	4.60	5.22	5.67	6.99	6.80	6.58	6.33	6.03	
6	3.46	4.34	4.90	5.30	6.49	6.32	6.12	5.90	5.63	
7	3.34	4.16	4.68	5.06	6.16	6.00	5.82	5.61	5.36	
8	3.26	4.04	4.53	4.89	5.92	5.77	5.60	5.40	5.17	
9	3.20	3.95	4.41	4.76	5.74	5.59	5.43	5.24	5.02	
10	3.15	3.88	4.33	4.65	5.60	5.46	5.30	5.12	4.91	
11	3.11	3.82	4.26	4.57	5.49	5.35	5.20	5.03	4.82	
12	3.08	3.77	4.20	4.51	5.39	5.27	5.12	4.95	4.75	
13	3.06	3.73	4.15	4.45	5.32	5.19	5.05	4.88	4.69	
14	3.03	3.70	4.11	4.41	5.25	5.13	4.99	4.83	4.64	
15	3.01	3.67	4.08	4.37	5.20	5.08	4.94	4.78	4.59	
16	3.00	3.65	4.05	4.33	5.15	5.03	4.90	4.74	4.56	
17	2.98	3.63	4.02	4.30	5.11	4.99	4.86	4.70	4.52	
18	2.97	3.61	4.00	4.28	5.07	4.96	4.82	4.67	4.49	
19	2.96	3.59	3.98	4.25	5.04	4.92	4.79	4.65	4.47	
20	2.95	3.58	3.96	4.23	5.01	4.90	4.77	4.62	4.45	
24	2.92	3.53	3.90	4.17	4.92	4.81	4.68	4.54	4.37	
30	2.89	3.49	3.85	4.10	4.82	4.72	4.60	4.46	4.30	
40	2.86	3.44	3.79	4.04	4.73	4.63	4.52	4.39	4.23	
60	2.83	3.40	3.74	3.98	4.65	4.55	4.44	4.31	4.16	
120	2.80	3.36	3.68	3.92	4.56	4.47	4.36	4.24	4.10	
∞	2.77	3.31	3.63	3.86	4.47	4.39	4.29	4.17	4.03	

Wilcoxon Rank-Sum Test

One Tailed Test at $\alpha = 0.025$ or Two-Tailed Test at $\alpha = 0.05$

n_1	n_2																	
	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1																		
2						0	0	0	0	1	1	1	1	1	2	2	2	2
3			0	1	1	2	2	3	3	4	4	5	5	6	6	7	7	8
4		0	1	2	3	4	4	5	6	7	8	9	10	11	11	12	13	13
5			2	3	5	6	7	8	9	11	12	13	14	15	17	18	19	20
6				5	6	8	10	11	13	14	16	17	19	21	22	24	25	27
7					8	10	12	14	16	18	20	22	24	26	28	30	32	34
8						13	15	17	19	22	24	26	29	31	34	36	38	41
9							17	20	23	26	28	31	34	37	39	42	45	48
10								23	26	29	33	36	39	42	45	48	52	55
11									30	33	37	40	44	47	51	55	58	62
12										37	41	45	49	53	57	61	65	69
13											45	50	54	59	63	67	72	76
14												55	59	64	67	74	78	83
15													64	70	75	80	85	90
16														75	81	86	92	98
17															87	93	99	105
18																99	106	112
19																	113	119
20																		127

One Tailed Test at $\alpha = 0.05$ or Two-Tailed Test at $\alpha = 0.1$

n_1	n_2																	
	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1																		0
2						0	0	1	1	1	1	2	2	3	3	3	4	4
3		0	0	1	2	2	3	4	4	5	5	6	7	7	8	9	9	10
4			1	2	3	4	5	6	7	8	9	10	11	12	14	15	16	17
5				4	5	6	8	9	11	12	13	15	16	18	19	20	22	23
6					7	8	10	12	14	16	17	19	21	23	25	26	28	30
7						11	13	15	17	19	21	24	26	28	30	33	35	37
8							15	18	20	23	26	28	31	33	36	39	41	44
9								21	24	27	30	33	36	39	42	45	48	51
10									27	31	34	37	41	44	48	51	55	58
11										34	38	42	46	50	54	57	61	65
12											42	47	51	55	60	64	68	72
13												51	56	61	65	70	75	80
14													61	66	71	77	82	87
15														72	77	83	88	94
16															83	89	95	101
17																96	102	109
18																	109	116
19																		123
20																		138