

National Exams December 2013

07-Mec-A3, SYSTEM ANALYSIS AND CONTROL

3 hours duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
2. Candidates may use a Casio or Sharp approved calculator. This is a closed book exam. No aids other than semi-log graph papers are permitted.
3. Any four questions constitute a complete paper. Only the first four (4) questions as they appear in your answer book will be marked.
4. All questions are of equal value.

Question 1:

a) Calculate the unit step response of the system

$$G(s) = \frac{54}{(2s + 6)(s^2 + 3s + 9)}$$

b) A system is given by its transfer function

$$G(s) = \frac{5(1 - 0.4s)}{(s + 1)(0.2s + 1)}$$

What are the time constants of the components of its transient response, and how long does it take for the transient to decay almost completely?

Question 2:

For the system of Fig. 1 with $G(s) = 1/[s(s + 1)(s + 4)]$, use the Routh-Hurwitz criterion to find the limit on K for stability. At this limit, determine the position of the system poles on the imaginary axis of the s -plane.

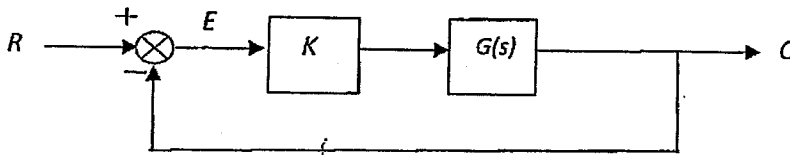


Figure 1

Question 3:

For the system in Figure 2 with $G(s) = (s + 1)(s + 3) / [s(s + 2)(s + 4)]$:

- What is the system type number?
- What is the gain of the loop gain function?
- What are the steady-state errors following unit step and unit ramp inputs?

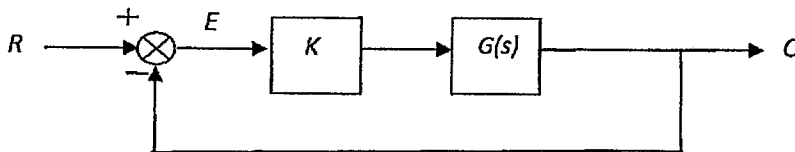


Figure 2

Question 4:

- For the motor position servo in Fig. 3 without the rate or velocity feedback ($K_g = 0$), find K for a system damping ratio 0.5 and the corresponding steady-state error following unit ramp inputs.
- Still with $K_g = 0$, what value of K will give a steady-state unit ramp following error of 0.1, and what is corresponding damping ratio?
- With K as in part (b), what value of K_g will give a system damping ratio 0.5? How does the steady-state error compare with that in part (b)?

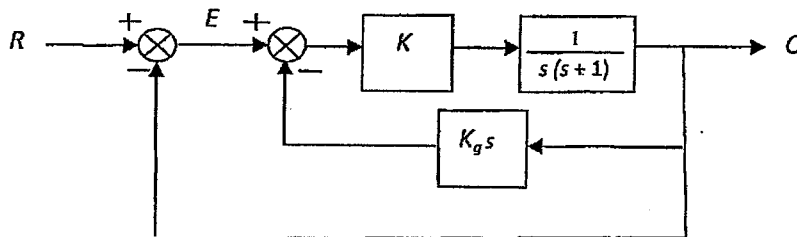


Figure 3

Question 5:

For a system with loop gain function

$$G = \frac{K(s^2 + 4s + 8)}{s^2(s - 1)}$$

sketch the loci and find the range of values of K for which the system is stable.

Question 6:

For a system with loop gain function

$$G(s) = \frac{20}{(s + 5)(0.1s + 1)(0.025s + 1)}$$

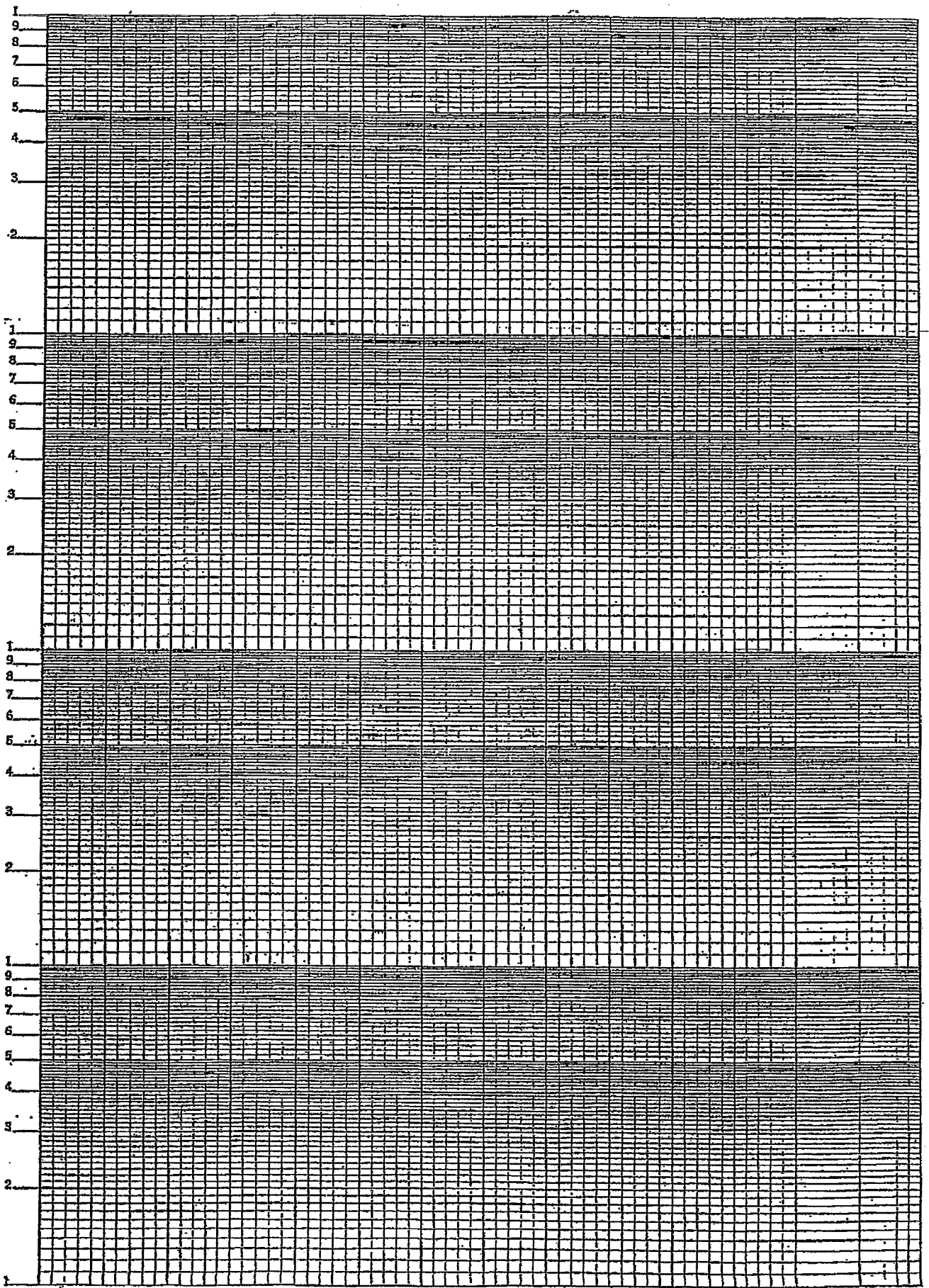
- (a) Plot the asymptotic Bode magnitude plot.
- (b) Determine the phase margin and gain margin.

Laplace Transform Table

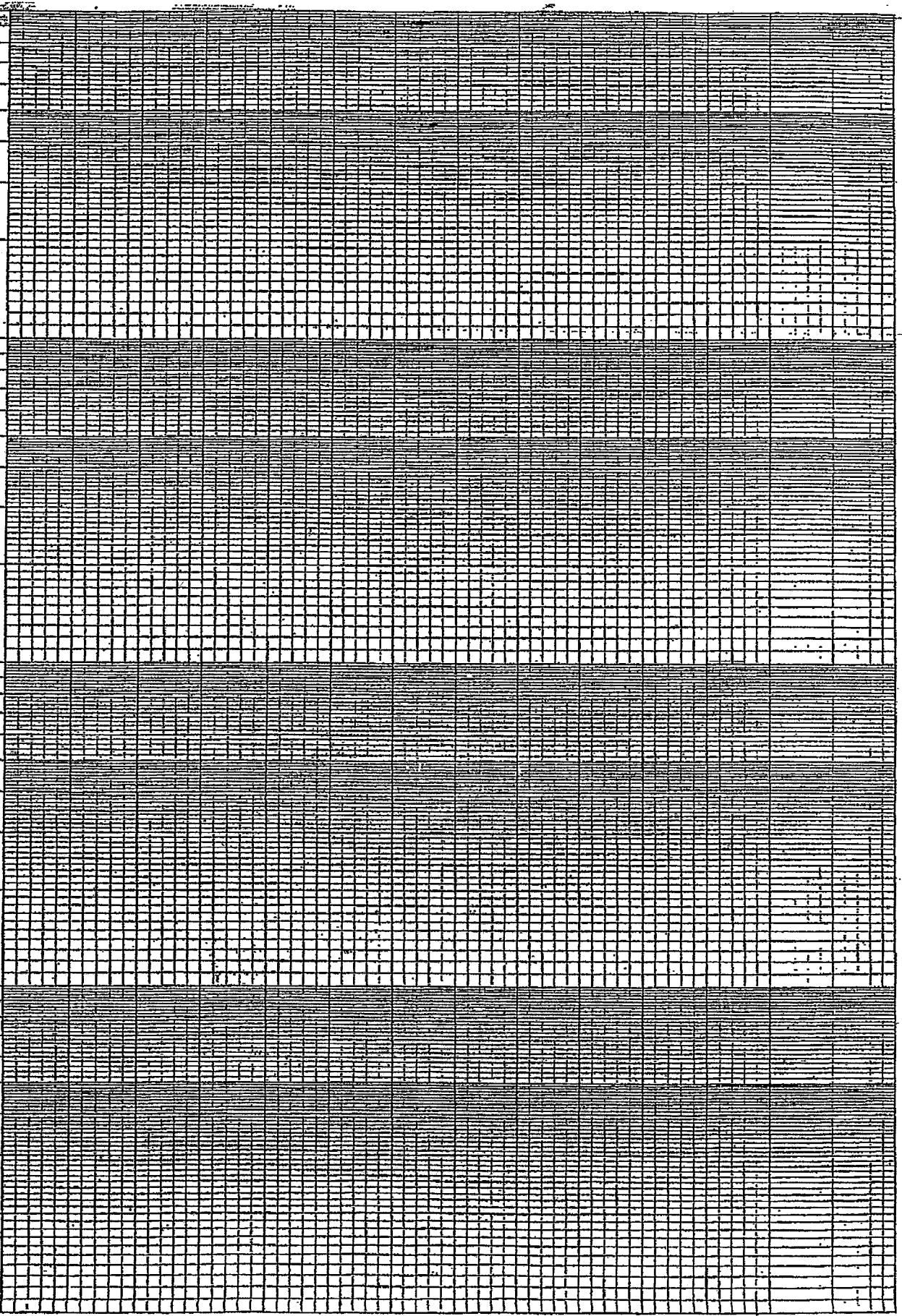
Laplace Transform $F(s)$	Time Function $f(t)$
1	Unit-impulse function $\delta(t)$
$\frac{1}{s}$	Unit-step function $u(t)$
$\frac{1}{s^2}$	Unit-ramp function t
$\frac{n!}{s^{n+1}}$	t^n ($n =$ positive integer)
$\frac{1}{s + \alpha}$	$e^{-\alpha t}$
$\frac{1}{(s + \alpha)^2}$	$t e^{-\alpha t}$
$\frac{n!}{(s + \alpha)^{n+1}}$	$t^n e^{-\alpha t}$ ($n =$ positive integer)
$\frac{1}{(s + \alpha)(s + \beta)}$	$\frac{1}{\beta - \alpha} (e^{-\alpha t} - e^{-\beta t})$ ($\alpha \neq \beta$)
$\frac{s}{(s + \alpha)(s + \beta)}$	$\frac{1}{\beta - \alpha} (\beta e^{-\beta t} - \alpha e^{-\alpha t})$ ($\alpha \neq \beta$)
$\frac{1}{s(s + \alpha)}$	$\frac{1}{\alpha} (1 - e^{-\alpha t})$
$\frac{1}{s(s + \alpha)^2}$	$\frac{1}{\alpha^2} (1 - e^{-\alpha t} - \alpha t e^{-\alpha t})$
$\frac{1}{s^2(s + \alpha)}$	$\frac{1}{\alpha^2} (\alpha t - 1 + e^{-\alpha t})$
$\frac{1}{s^3(s + \alpha)^2}$	$\frac{1}{\alpha^2} \left[t - \frac{1}{\alpha} + \left(t + \frac{2}{\alpha} \right) e^{-\alpha t} \right]$

Laplace Transform Table (continued)

Laplace Transform $F(s)$	Time Function $f(t)$
$\frac{s}{(s + \alpha)^2}$	$(1 - \alpha t)e^{-\alpha t}$
$\frac{\omega_n^2}{s^2 + \omega_n^2}$	$\sin \omega_n t$
$\frac{s}{s^2 + \omega_n^2}$	$\cos \omega_n t$
$\frac{\omega_n^2}{s(s^2 + \omega_n^2)}$	$1 - \cos \omega_n t$
$\frac{\omega_n^2(s + \alpha)}{s^2 + \omega_n^2}$	$\omega_n \sqrt{\alpha^2 + \omega_n^2} \sin(\omega_n t + \theta)$ where $\theta = \tan^{-1}(\omega_n/\alpha)$
$\frac{\omega_n}{(s + \alpha)(s^2 + \omega_n^2)}$	$\frac{\omega_n}{\alpha^2 + \omega_n^2} e^{-\alpha t} + \frac{t}{\sqrt{\alpha^2 + \omega_n^2}} \sin(\omega_n t - \theta)$ where $\theta = \tan^{-1}(\omega_n/\alpha)$
$\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$	$\frac{\omega_n}{\sqrt{1 - \zeta^2}} e^{-\zeta\omega_n t} \sin \omega_n \sqrt{1 - \zeta^2} t \quad (\zeta < 1)$
$\frac{\omega_n^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}$	$1 - \frac{1}{\sqrt{1 - \zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_n \sqrt{1 - \zeta^2} t + \theta)$ where $\theta = \cos^{-1} \zeta \quad (\zeta < 1)$
$\frac{s\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$	$\frac{-\omega_n^2}{\sqrt{1 - \zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_n \sqrt{1 - \zeta^2} t - \theta)$ where $\theta = \cos^{-1} \zeta \quad (\zeta < 1)$
$\frac{\omega_n^2(s + \alpha)}{s^2 + 2\zeta\omega_n s + \omega_n^2}$	$\omega_n \sqrt{\frac{\alpha^2 - 2\alpha\zeta\omega_n + \omega_n^2}{1 - \zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_n \sqrt{1 - \zeta^2} t + \theta)$ where $\theta = \tan^{-1} \frac{\omega_n \sqrt{1 - \zeta^2}}{\alpha - \zeta\omega_n} \quad (\zeta < 1)$
$\frac{\omega_n^2}{s^2(s^2 + 2\zeta\omega_n s + \omega_n^2)}$	$t - \frac{2\zeta}{\omega_n} + \frac{1}{\omega_n^2 \sqrt{1 - \zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_n \sqrt{1 - \zeta^2} t + \theta)$ where $\theta = \cos^{-1}(2\zeta^2 - 1) \quad (\zeta < 1)$



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Semi-Logarithmic
4 Cycles x 10 to the Inch