98-Phys-A7, Optics, December 2013

3 hours duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate should include in the answer clear statements of the interpretation and any assumptions made.

2. This is a CLOSED BOOK EXAM.

3. Candidates may use one of two calculators, the Casio or Sharp approved models.

4. Answers to question 1 plus any four of questions 2 to 6 constitute a complete exam paper.

5. Answer question 1 in the space provided on the exam paper.

6. The first four questions as they appear in the answer book will be marked.

7. Each question is of equal value. Question 1 is mandatory.

1. A phrase, or a diagram and a phrase, is all that is required in most cases. [20 marks total, one mark for each letter]

a) This examination is about light and optics. Define *light*.

b) Define *optics*.

c) Define *geometrical optics*.

d) Define *physical optics*.

e) What problems with *geometrical optics* led to the development of *physical optics*?

f) Define *plane of incidence*.

g) State the *law of reflection*.

h) State the *law of refraction*.

i) Complete any two rows of the following table:

		wavelength range (nm)	frequency range (Hz)
	UV light		
	red light		
	blue light		
	IR light		

j) Transverse plane waves are solutions to Maxwell's equations for linear, homogeneous, isotropic, stationary media with $\rho_f = 0$ and $J_f = 0$. Define *transverse plane wave*.

k) Is analysis in terms of monochromatic plane waves restrictive?

l) How is the ray of geometrical optics related to the plane wave of physical optics?

m) What is the speed of propagation of the wave $f(x, t) = A\exp(\alpha x)\sin(2\pi(at + bx))$?

n) In m) above, if t represents time and x represents distance, what is the physical interpretation of the parameter a.

o) In m) above, if t represents time and x represents distance, what is the physical interpretation of the parameter b.

p) In m) above, given that the vacuum wavelength is λ_o, what is the refractive index of the material in which the wave propagates?

q) What is the difference between *interference* and *diffraction*?

r) Write an equation for the *Poynting vector* and give the SI units for the quantity the Poynting vector represents.

s) What is *Fraunhoffer* diffraction?

t) What is *Fresnel* diffraction?

2. [20 marks total]

a) Using ray tracing, find the image of an extended object that is placed 1.5 focal lengths in front of: i) a perfect thin plano convex lens; [3 marks]

ii) a perfect thin plano concave lens. [3 marks]

b) Using ray tracing, find the image of an extended virtual object that is 1.5 focal lengths behind i) a perfect thin plano convex lens; [3 marks]

ii) a perfect thin plano concave lens. [3 marks]

c) A telescope contains a graticule in the common focal plane of the objective and the eyepiece. The graticule is a (mostly) transparent piece of glass on which a scale has been etched. Assume that the focal length of the objective is 20 cm and the focal length of the eye piece is 5 cm.

i) If the telescope is focused on a wall that is 30 m away, how much of the wall falls between mm marks on the graticule? [3 marks]

ii) Write the system matrix that could be used to find the location of the image. Leave the system matrix as a product of the individual matrices. [3 marks].

iii) The system matrix is used with a column vector of length 2. State clearly what the two elements of the column vector represent. [2 marks]

3. [20 marks total]

a) Design a 1/2 m monochromator to resolve in first order the sodium D lines at 589.6 nm and 590.0 nm Assume that the input is obtained from an extended source and that all optics are diffraction limited. Draw an engineering diagram of your design. Specify all relevant dimensions and parts, and justify with calculations the values for the dimensions. [16 marks]

b) Use ray tracing to show how spherical aberration and chromatic aberration would affect the performance of your design. [4 marks]

4. [20 marks total]

a) List the possible unique polarizations of light and describe the effect of a linear polarizer and quarter wave plate on each of the unique polarizations that you list. [6 marks]

b) What is the minimum thickness required to produce a quarter wave plate at 600 nm. Assume that the principal refractive indices of the plate are 1.5 and 1.5005. [4 marks]

c) Assume air and water with a refractive index of 1.33 Sketch carefully *R* and *T* for both polarizations as a function of the angle of incidence for internal reflection. Label the salient features and calculate numerical values for the salient features on both the vertical and horizontal axes. [4 marks]

d) Assume air and water with a refractive index of 1.33 Sketch carefully R and T for both polarizations as a function of the angle of incidence for external reflection. Label the salient features and calculate numerical values for the salient features on both the vertical and horizontal axes. [4 marks]

e) Use your results to explain the conditions wherein polarizing sun glasses are most effective. State the orientation of the transmission axis of the sun glasses under normal conditions of use. [2 marks]

5. [20 marks total]

a) A fibre has an attenuation loss of 1.5 dB/km due to Rayleigh scattering for light of 900 nm. Determine the attenuation owing to Rayleigh scattering for light of 1500 nm. For Rayleigh scattering, the loss is proportional to the fourth power of wavelength. [3 marks]

b) Determine the lengths and transit times for the longest and shortest trajectories for propagation in a step-index fibre that is 1 km long. Assume a core index of 1.46 and a cladding index of 1.45. [6 marks]

c) The refractive index for a glass fibre as a function of wavelength is given as

$$n(\lambda) = \frac{S_o}{8}\left(\lambda^2 - \frac{\lambda_o^4}{\lambda^2}\right)$$

where λ_o is the wavelength of zero dispersion.

i) What are the units of S_o? [1 mark]
ii) Explain the concept of zero dispersion and calculate the dispersion for this fibre. [2 marks].
iii) Calculate the group refractive index for the fibre. [4 marks].
iv) Calculate the difference in transit times for two different wavelengths for propagation over a distance of L km. [4 marks]

6. [20 marks total]

a) Explain, using a sketch and the Huygens-Fresnel Principle, how the intensity distribution in the focal plane of a lens is calculated. Assume a circular lens with $f/\# = 5$, focal length of 10 cm, and illumination from a point source 20 cm in front of the lens. Give the dimensions of salient features in the focal plane. [10 marks]

b) Two slits are illuminated by light that consists of 500 nm light plus light of an unknown wavelength. The light from the slits is allowed to impinge on a screen. The fourth minimum of the 500 nm light coincides with the third maximum of the light with the unknown wavelength. What is the unknown wavelength? [3 marks]

c) An anti-reflecting layer is to be applied to a substrate with refractive index of 1.50 at a wavelength of 600 nm. Specify the thickness and refractive index for no reflection of normally incident light at 600 nm. [3 marks]

d) Light of wavelength λ is normally incident on and reflected from a moving mirror. Assume that the speed of the mirror is small and along the direction of the light. What is the shift in the wavelength and frequency of the reflected light, relative to the incident light? [4 marks]

Question 6 d) is the last question. Some formulae follow.

$$E(x,y,z) = \frac{ik}{2\pi}\frac{e^{ikz}}{z} e^{i\frac{k}{2z}(x^2+y^2)} \iint E(x_a, y_a, 0) e^{i\frac{k}{2z}(x_a^2+y_a^2)} e^{-i\frac{k}{z}(x\,x_a + y\,y_a)}\, dx_a\, dy_a$$

The field in the neighbourhood of the focus of a circular lens of radius a is given in the usual paraxial approximation as

$$E(u,v) = \int_0^1 J_0(2\pi v \rho_a)\, e^{-i\pi u \rho_a^2} \rho_a\, d\rho_a$$

with $\rho_a = \sqrt{(x_a^2 + y_a^2)}$, $\quad u = \frac{1}{\lambda}\frac{a^2}{q\,(q+\Delta)}\Delta$, $\quad v = \frac{1}{\lambda}\frac{a\,\sqrt{(x^2+y^2)}}{(q+\Delta)}$.

$J_0(0) = 1$; $J_0(2.4048) = 0$; $J_0(5.5201) = 0$; $J_0(8.6537) = 0$; $J_0(11.7915) = 0$;

$\gamma = \frac{1}{2} k D \sin(\theta)$. The zeros for $J_1(\gamma)$ occur for γ = 0, 3.832, 5.136, 7.016, 8.417, 10.173, 11.620, 13.324, . . .

$$\frac{d}{dx} x^n J_n(x) = x^n J_{n-1}(x)$$

$$(1 + \epsilon)^\xi = 1 + \frac{\xi}{1!}\epsilon + \frac{\xi \times (\xi - 1)}{2!}\epsilon^2 + \ldots$$

zone plate radii $R_m = (m\, r_o\, \lambda)^{0.5}$

The intensity in the far-field as a function of the angle θ_m from the normal of a diffraction grating of N lines, line spacing of a, and line width of b, for illumination with a plane wave with $k = 2\pi/\lambda$ and an angle of incidence of θ_i is

$$I(\theta) = I_o \left[\frac{\sin(\beta)}{\beta}\right]^2 \left[\frac{\sin(N\alpha)}{\sin(\alpha)}\right]^2$$

$$\beta = \frac{kb}{2}\left(\sin(\theta_i) + \sin(\theta_m)\right)$$

$$\alpha = \frac{ka}{2}\left(\sin(\theta_i) + \sin(\theta_m)\right)$$

For a blazed grating, $2\theta_b = \theta_i - \theta_m$

The resolution R and the dispersion D for a grating with N lines and order m are

$$R = \frac{\lambda}{\Delta\lambda} = mN \qquad D = \frac{m}{a\cos(\theta)} .$$

double angle formulae:

$$\begin{aligned}
\sin(A+B) &= \sin(A)\sin(B) + \cos(A)\cos(B) \\
\sin(A-B) &= \sin(A)\sin(B) - \cos(A)\cos(B) \\
\cos(A+B) &= \cos(A)\cos(B) - \sin(A)\sin(B) \\
\cos(A-B) &= \cos(A)\cos(B) + \sin(A)\sin(B)
\end{aligned}$$

$$\begin{aligned}
\sin(A) + \sin(B) &= 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) \\
\sin(A) - \sin(B) &= 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right) \\
\cos(A) + \cos(B) &= 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) \\
\cos(A) - \cos(B) &= 2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)
\end{aligned}$$

$$v_p = \frac{\omega}{k} \qquad v_g = \frac{d\omega}{dk}$$

The translation, refraction at a spherical interface, thin lens, and spherical mirror matrices are listed below.

$$\begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 \\ \frac{n_1 - n_2}{R n_2} & \frac{n_1}{n_2} \end{bmatrix} \quad \begin{bmatrix} 1 & 0 \\ \frac{-1}{f} & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 \\ \frac{-2}{R} & 1 \end{bmatrix}$$

THE END