

**NATIONAL EXAMINATIONS**

**December 2017**

**16-MEC-B6 FLUID MACHINERY**

**Three hours duration**

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**Notes to Candidates**

1. This is a **Closed Book** examination.
2. Examination consists of two Sections: **Section A is Calculative (5 questions)** and **Section B is Descriptive (3 questions)**.
3. **Do four (4) questions (including all parts of each question) from Section A (Calculative) and two (2) questions from Section B (Descriptive)**.
4. **Six questions constitute a complete paper.** (Total 60 marks).
5. **All questions are of equal value.** (Each 10 marks).
6. Read the entire question before commencing the calculations and take note of hints or recommendations given. Note that Question 1 is on two pages.
7. If doubt exists as to the interpretation of any question, the candidate is urged to submit, with the answer paper, a clear statement of any assumptions made.
8. If any initial parts of a multi-part question cannot be solved the remaining parts may be worked by making appropriate assumptions for the first parts from the technical data given.
9. Candidates may use one of the approved **Casio** or **Sharp** calculators.
10. **Reference data** for particular questions are given in the Attachments on pages 10 to 17. **All pages from which data has been obtained or on which answers have been written are to be returned with the answer booklet to show any working. Candidate's names must be on these sheets.**
11. **Reference formulae and constants** are given on pages 18 to 22.
12. **Drawing Instruments** (scale ruler, protractor and sharp pencil) are required for vector diagrams. While calculation of velocities by trigonometric ratios with reference to a sketch is acceptable it is longer and more time consuming.

**SECTION A CALCULATIVE QUESTIONS**

**Show all steps in the calculations and state the units for all intermediate and final answers.**

**QUESTION 1 HYDRO TURBINES****PART I KAPLAN TURBINE EFFICIENCY**

Refer to the Examination Paper Attachments Page 10 **Kaplan Turbine** for illustrative purposes only.

Hydro turbines of the Kaplan type are installed at Mactaquac on the Saint John River. In order to determine the efficiency of the Mactaquac turbines the following hypothetical measurements are considered:

Turbine-generator speed	112.5 rev/min
Generator electrical output	110 MW
Water flow rate	354 m <sup>3</sup> /s
Inlet pipe diameter (not in picture)	6.4 m
Outlet pipe diameter (bottom of picture)	7.0 m
Inlet water pressure	226 kPa gauge
Outlet water pressure	-4.5 m H <sub>2</sub> O

The elevation of the outlet pressure measuring point is 5.0 m below that of the inlet pressure measuring point.

Determine the following:

- Water flow velocities at inlet and outlet of the turbine. (1)
- Hydraulic power produced by the water (input to turbine-generator). (3)
- Electrical power output and efficiency of turbine-generator. (1)

( 5 marks )

**This Question is continued on the next page .....**

### Question 1 Continued

#### PART II TURBINE SETTING

Refer to the following Examination Paper Attachments:

Page 11 **Churchill Falls Hydro Plant** for illustrative purposes only.

Page 12 **Critical Cavitation Parameter** for Thoma parameter.

Churchill Falls Hydro Power Station has the following technical parameters:

Turbine-generator design original output	483.2 MW
Speed of turbine-generator	200 rev/min
Net head on turbine	312.4 m
Turbine runner diameter	5.82 m

- (a) Calculate the specific speed of the turbine. (2)
- (b) From the graph determine the Thoma cavitation parameter  $\sigma$ . (1)
- (c) Calculate the setting (maximum elevation) of the turbine runner relative to the downstream water level based on the critical cavitation parameter (Thoma coefficient). Assume that the maximum water temperature at this location is 15°C. (1)
- (d) If the turbine elevation (at spiral casing and runner) is 117.3 m and the water level downstream (in surge chamber) is 132.8 m determine the actual setting of the turbine. Compare this with the calculated value obtained in (c) above and comment on the result. (1)

( 5 marks )

[ 10 marks ]

**QUESTION 2 HYDRO TURBINE MODEL**

Refer to the following Examination Paper Attachments:

Page 11 **Churchill Falls Hydro Plant** for illustrative purposes only.

Page 13 **Specific Speed of Turbines** for efficiency.

Technical specifications for the hydro turbines at Churchill Falls Hydro Power Station are as follows:

Original plant capacity	5225 MW (later updated)
Number of generating units	11
Original turbine output	483.2 MW (later updated)
Generator voltage	15 kV
Electrical frequency	60 Hz
Speed of machine	200 rev/min
Design net head	312.4 m
Turbine runner diameter	5.82 m
Turbine runner weight	77.13 Mg

- (a) Calculate the specific speed of the machine. (2)
- (b) From the chart determine the efficiency of the turbine. (1)
- (c) Calculate the volume flow rate through the turbine. (1)

Prior to construction, a model test is required to prove the performance of the prototype machine. Assume that an homologous (scaled to be geometrically identical) model runner 250 mm in diameter is available and can be tested in an instrumented hydraulic system under a head of 20 m. Use the turbine affinity laws or similarity rules to do the following:

- (c) Determine the speed at which the model should run. (2)
- (d) Determine the necessary flow through the model. (1)
- (e) Determine the ideal (no friction) power developed by the model. (1)

Due to scaling differences, the efficiency of the model and the prototype are not identical. The Moody equation allows the hydraulic efficiencies of the model and prototype to be compared.

- (f) Determine the efficiency that should be measured on the model to ensure that the prototype would meet its specified efficiency. (2)

[ 10 marks ]

**QUESTION 3 CURTIS TYPE IMPULSE TURBINE**

Refer to the Examination Paper Attachments Page 14 **Steam Turbine Velocity Diagram** (one stage only) for nomenclature of velocities and angles

Steam exits the nozzles and enters the first stage moving blades of a velocity compounded two stage (Curtis) impulse turbine at 1411 m/s. The nozzle angle is  $20^\circ$  and the fixed blade exit angle is the same as its inlet angle, that is, the fixed blades are symmetrical. The moving blades of both stages are also symmetrical but with different angles. Assume zero fluid friction in nozzles and blades.

- (a) Determine a blade velocity to give optimum work (minimum exit kinetic energy). (1)
- (b) Draw to scale the velocity diagrams for the two stages. (4)
- (c) Determine all the actual and relative steam velocities and blade angles and show them on the diagrams. (1)
- (d) Calculate the work done by each stage, in kJ/kg of steam. (2)
- (e) Calculate the total power output for a steam flow of 100 kg/s. (1)
- (f) Calculate the blade efficiency (1)

*Note: The scale drawing should be to a large enough scale for accurate measurements (a scale of 1 mm = 10 m/s is suggested). While calculation of velocities by trigonometric ratios is acceptable it is longer and more time consuming.*

[ 10 marks ]

**QUESTION 4 PUMP DESIGN**

Refer to the Examination Paper Attachments Page 15 **Pump Velocity Diagram**.

A preliminary analysis for a particular centrifugal pump to deliver water under ambient conditions has yielded the following parameters:

Impeller inlet radius	100 mm
Impeller outlet radius	180 mm
Impeller inlet width	50 mm (in axial direction)
Impeller outlet width	30 mm (in axial direction)
Rotational speed	1720 rev/min
Volumetric flow rate	0.25 m <sup>3</sup> /s
Delivery head	40 m

As a next step in the design of the pump the impeller blade angles need to be determined.

- Calculate the power required by the pump assuming ideal conditions (no friction). (1)
- Calculate the mechanical torque required on the shaft. (1)
- Calculate the blade inlet and outlet tangential velocities. (2)
- Calculate the water inlet and outlet radial velocities neglecting blade thickness. (2)
- Calculate the tangential water velocity at the outlet assuming pure radial flow at the inlet. (2)
- Draw to scale the inlet and outlet velocity diagrams and determine the inlet and outlet blade angles. (2)

*Note: The velocity vector diagrams must be drawn sufficiently large to obtain a suitably accurate answer (10 mm = 5 m/s minimum). While calculation of velocities by trigonometric ratios is acceptable it is longer and more time consuming.*

[ 10 marks ]

**QUESTION 5 COMPRESSOR FIRST STAGE**

Refer to the Examination Paper Attachments Page 16 **Acacia and Port Rex Power Stations** and Page 17 **Compressor Velocity Diagram**.

Each power station has three units. Each unit has an output of 60 MW and is powered by twin back to back gas turbines driving a common electrical generator. The diagram and specifications on Page 14 are for one gas turbine only.

Consider the first stage of the compressor (N1 rotor) which has the following approximate parameters:

Rotor hub diameter at inlet	$D_1 = 480 \text{ mm}$
Blade tip diameter at inlet	$D_2 = 1120 \text{ mm}$
Inlet guide vane outlet angle	$\alpha_1 = 30^\circ$
First stage moving blade outlet angle	$\beta_2 = 40^\circ$

Assume that the plant is operating under the following conditions:

Rotational speed	$N = 6800 \text{ rev/min}$
Air flow at inlet to compressor	$M = 136 \text{ kg/s}$
Air temperature at inlet to compressor	$T_1 = 15^\circ\text{C} \quad (288^\circ\text{K})$

Assume that ideal conditions prevail (no friction losses).

- Calculate the mean blade velocity  $U$  and air inlet axial velocity  $C_{x1}$ .  
(2)
- Draw to scale (see note below) the velocity diagrams at the first stage moving blade inlet and outlet and measure the absolute velocities  $C_1$  and  $C_2$  and relative velocities  $W_1$  and  $W_2$ .  
(3)
- Determine the work done in kJ/kg and power input to the first stage in kW.  
(1)
- Determine the enthalpy rise and hence temperature rise in the first stage.  
(3)
- Determine the pressure ratio of the first stage assuming isentropic conditions.  
(1)

*Note: the scale drawing should be to a large enough scale for accurate measurements (a scale of  $10 \text{ m/s} = 4 \text{ mm}$  is suggested). While calculation of velocities by trigonometric ratios is acceptable it is longer and more time consuming.*

[ 10 marks ]

## SECTION B DESCRIPTIVE QUESTIONS

*Note that each five mark part of each question requires a full page answer with complete explanations with sketches, if appropriate, to support the explanation.*

### QUESTION 6 TURBINE BLADE CHARACTERISTICS

#### PART I IMPULSE AND REACTION

Explain the difference between an impulse turbine and a reaction turbine. In particular refer to the changes in velocity in both the fixed and moving blades. Clarify how the forces developed are created and how they influence the transfer of energy from the fluid to the blades. If appropriate, show the difference between impulse and reaction in velocity diagrams for an axial flow gas or steam turbine.

( 5 marks )

#### PART II OPTIMUM BLADE EFFICIENCY

With respect to a Pelton turbine show graphically in a sketch (efficiency versus blade velocity) how and explain why the efficiency varies with turbine blade velocity (wheel rotational speed) when the water jet velocity remains constant. Consider the whole range of possibilities from a blade velocity of zero to a blade velocity equal to that of the jet. If appropriate, draw velocity diagrams to illustrate the explanation.

( 5 marks )

[ 10 marks ]

### QUESTION 7 GAS TURBINE NUMBER OF STAGES

Refer to the Examination Paper Attachments Page 16 **Acacia and Port Rex Power Stations**.

The cross section of the gas turbines shows the number of stages in the compressor and the number of stages in the turbine. Explain why the compressor has many more stages than the turbine. Give the reasons for selecting the appropriate number of stages for both the compressor and the turbine. In particular explain what requires the compressor to have a certain minimum number of stages and why the turbine must have multiple stages.

*Answer the question by considering the limiting conditions for each part (compressor and turbine) of the machine.*

[ 10 marks ]



## QUESTION 8 PUMP CHARACTERISTICS

### PART II PUMP SETTING

Consider the setting of pumps relative to the free liquid surface on the suction side of the pump. State and explain what factors need to be taken into account in determining the maximum elevation of the pump relative to liquid surface.

(5 marks)

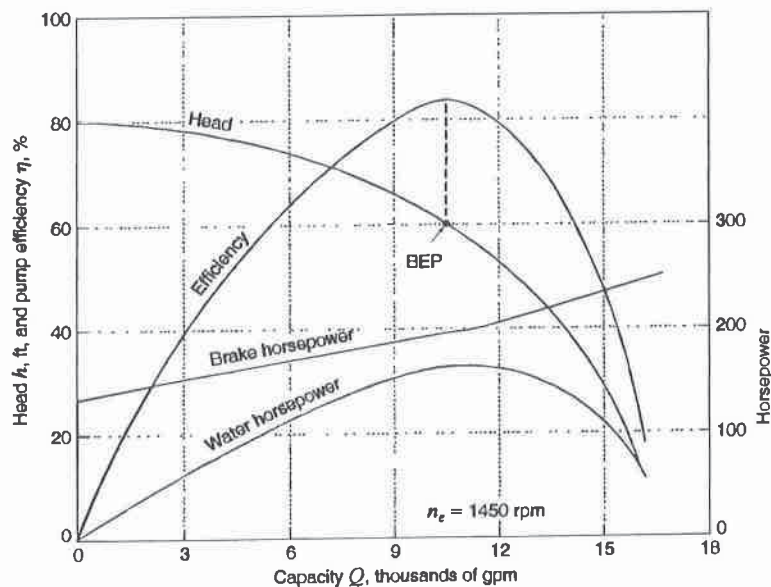
### PART II CENTRIFUGAL PUMPS

With reference to the figure below explain the following:

- Why the hydraulic efficiency (water horsepower) rises from zero to a peak and then declines towards zero
- Why the difference between the hydraulic power (water horsepower) and the mechanical power (brake horsepower) decreases to a low value and then increases to a value greater than the initial value.

( 5 marks )

[ 10 marks ]



Characteristic curves for a typical mixed-flow centrifugal pump.

**EXAMINATION PAPER ATTACHMENTS**

**QUESTION 1 PART I KAPLAN TURBINE**

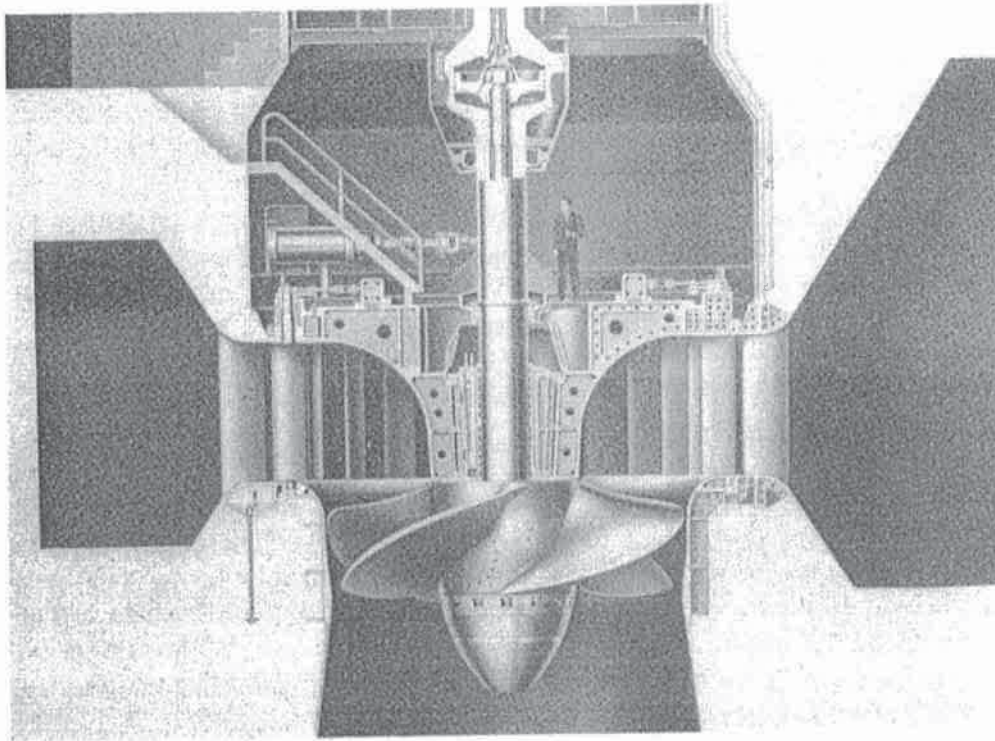
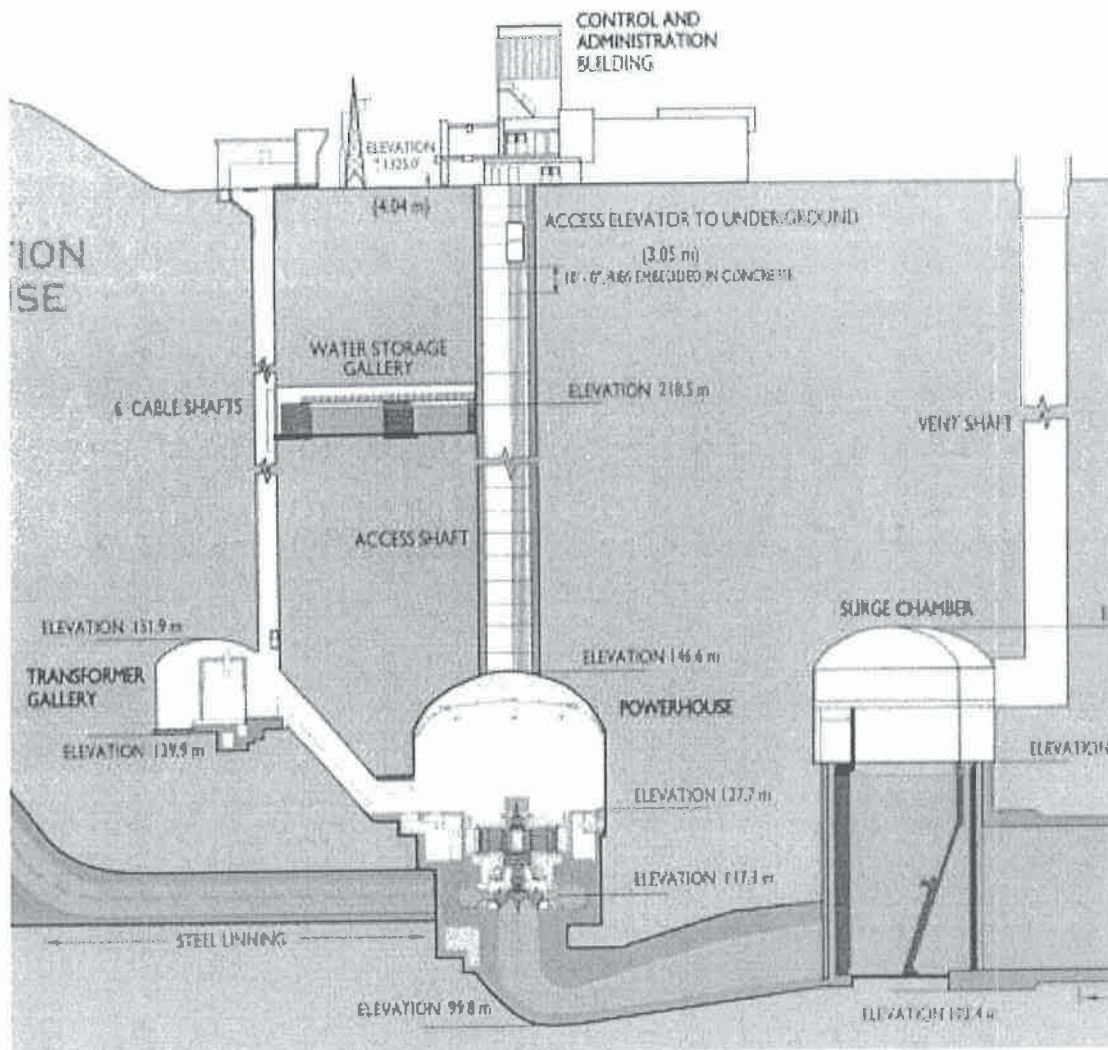


Figure 16.7 Kaplan turbine at Watts Bar Dam. 42,000 hp at 94.7 rpm under a head of 52 ft.

### EXAMINATION PAPER ATTACHMENTS

#### QUESTION 1 PART II CHURCHILL FALLS HYDRO PLANT

Part cross section of Churchill Falls Hydro Power Station showing powerhouse and surge chamber.

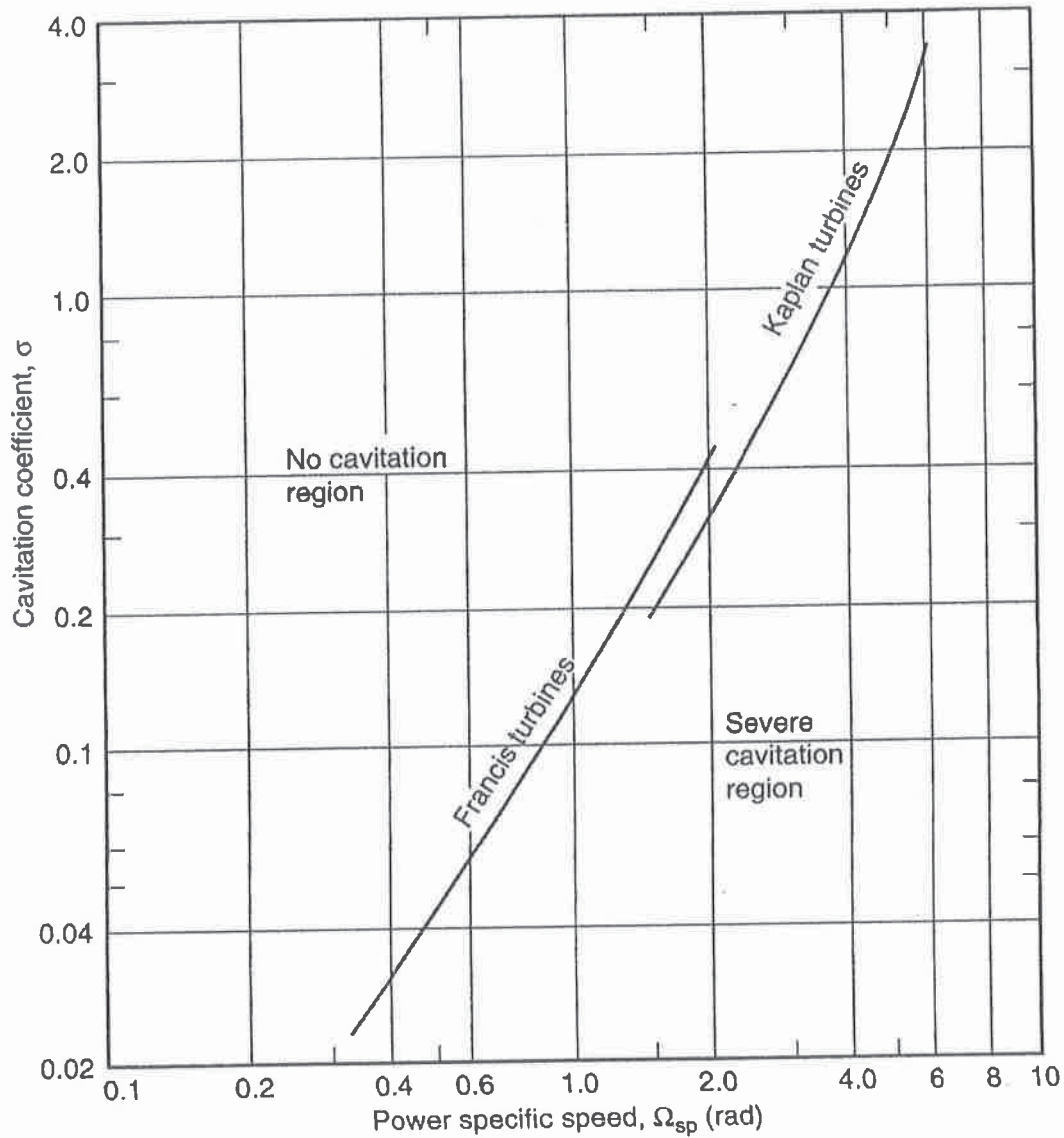


*Note: This diagram is for orientation purposes only. Reproduction may be poor so no information is required to be taken directly from the drawing.*

EXAMINATION PAPER ATTACHMENTS

NAME .....

QUESTION 1 PART II CRITICAL CAVITATION PARAMETER

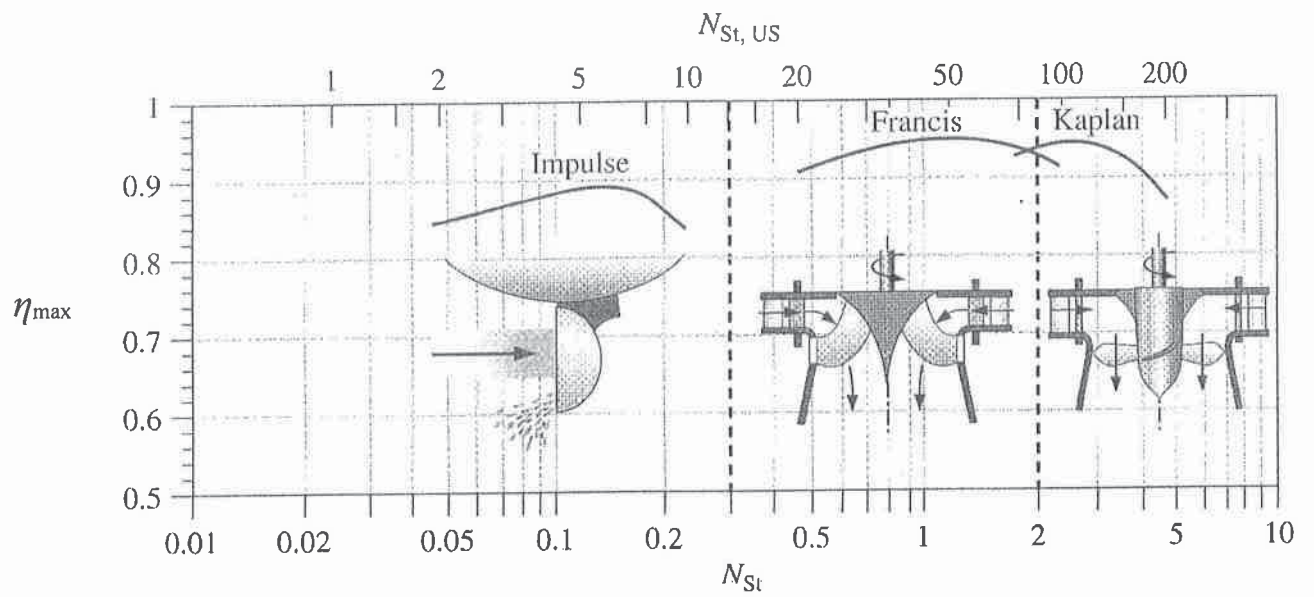


Variation of critical cavitation parameter with non-dimensional specific speed in SI units for Francis and Kaplan turbines

EXAMINATION PAPER ATTACHMENTS

QUESTION 2 SPECIFIC SPEED OF TURBINES

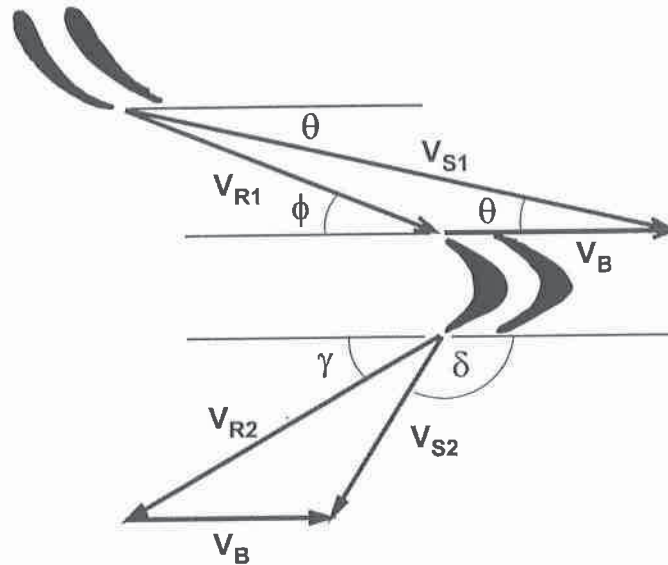
Maximum efficiency  $\eta_{max}$  of different types of hydro turbines versus specific speed  $N_{St}$  in SI units (bottom scale)



EXAMINATION PAPER ATTACHMENTS

QUESTION 3 STEAM TURBINE VELOCITY DIAGRAM

Nomenclature for velocity vectors and angles

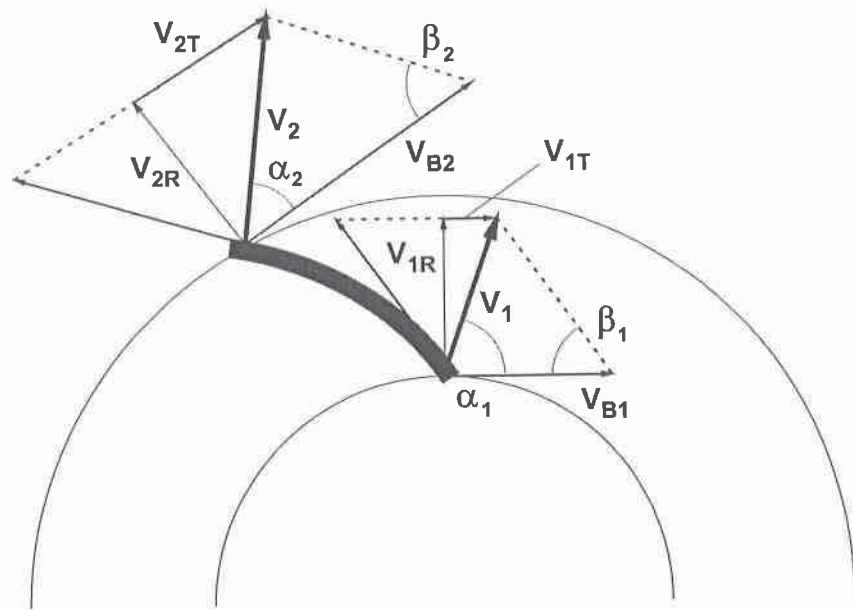


- $V_{S1}$  Absolute steam velocity entering moving blades
- $V_{R1}$  Relative steam velocity entering moving blades
- $V_B$  Moving blade velocity
- $V_{R2}$  Relative steam velocity leaving moving blades
- $V_{S2}$  Absolute steam velocity leaving moving blades

EXAMINATION PAPER ATTACHMENTS

QUESTION 4 PUMP VELOCITY DIAGRAM

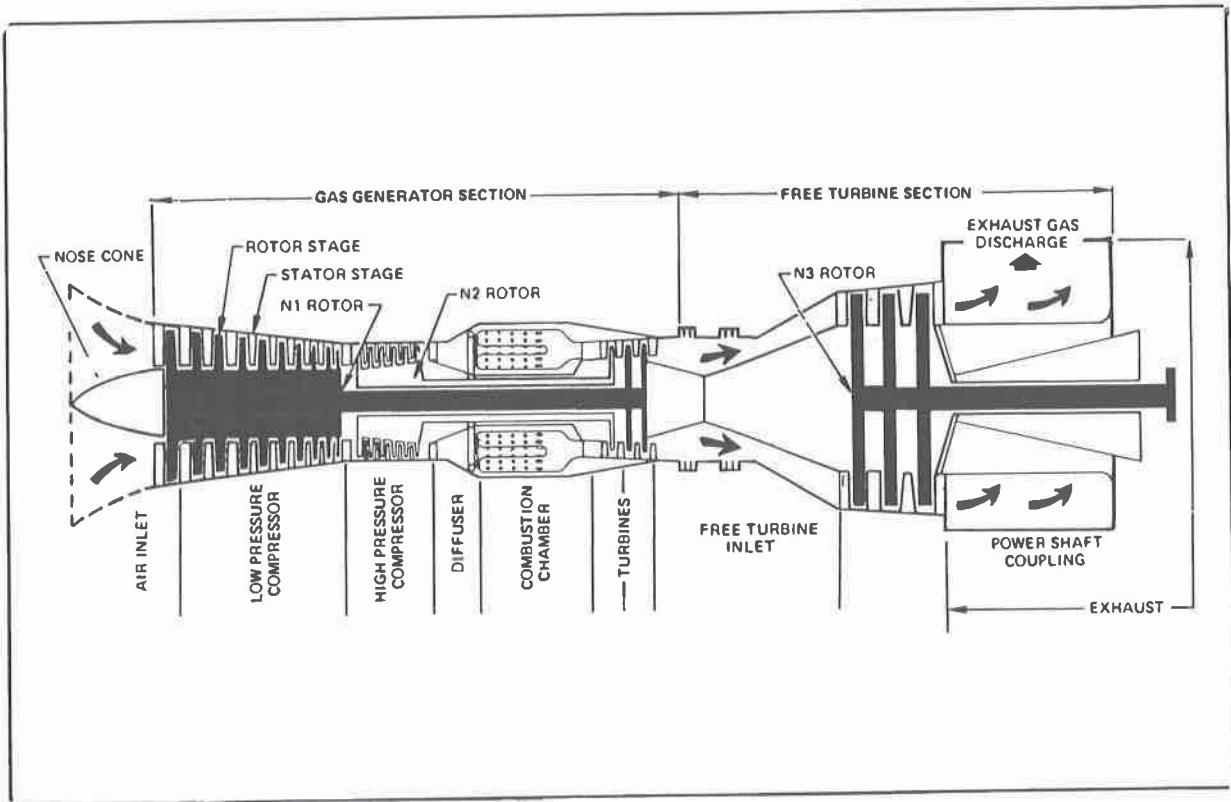
Nomenclature for velocity vectors and angles



- $V_1$  Absolute water velocity at inlet
- $V_{B1}$  Blade velocity at inlet
- $V_{1R}$  Radial water velocity at inlet
- $V_{1T}$  Tangential water velocity at inlet
  
- $V_2$  Absolute water velocity at outlet
- $V_{B2}$  Blade velocity at outlet
- $V_{2R}$  Radial water velocity at outlet
- $V_{2T}$  Tangential water velocity at outlet

EXAMINATION PAPER ATTACHMENTS

QUESTION 5 & QUESTION 7 ACACIA AND PORT REX POWER STATIONS



Technical Specifications

Peak Load

Base Load

Net Output	(kW)	60 860	57 100
Heat Rate	(kJ/kWh)	11 791	11 887
Speed - N1 Rotor	(rev/min)	6 805	6 640
Speed - N2 Rotor	(rev/min)	8 395	8 320
Speed - Power Turbine	(rev/min)	3 000	3 000
Temperature - Gas Generator Turbine Inlet	(°C)	1 077	1 043
Temperature - Power Turbine Inlet	(°C)	682	657
Temperature - Power Turbine Exhaust	(°C)	483	467
Exhaust Gas Flow Rate	(kg/s)	278	272
Gas Generator Pressure Ratio		14.1	13.6

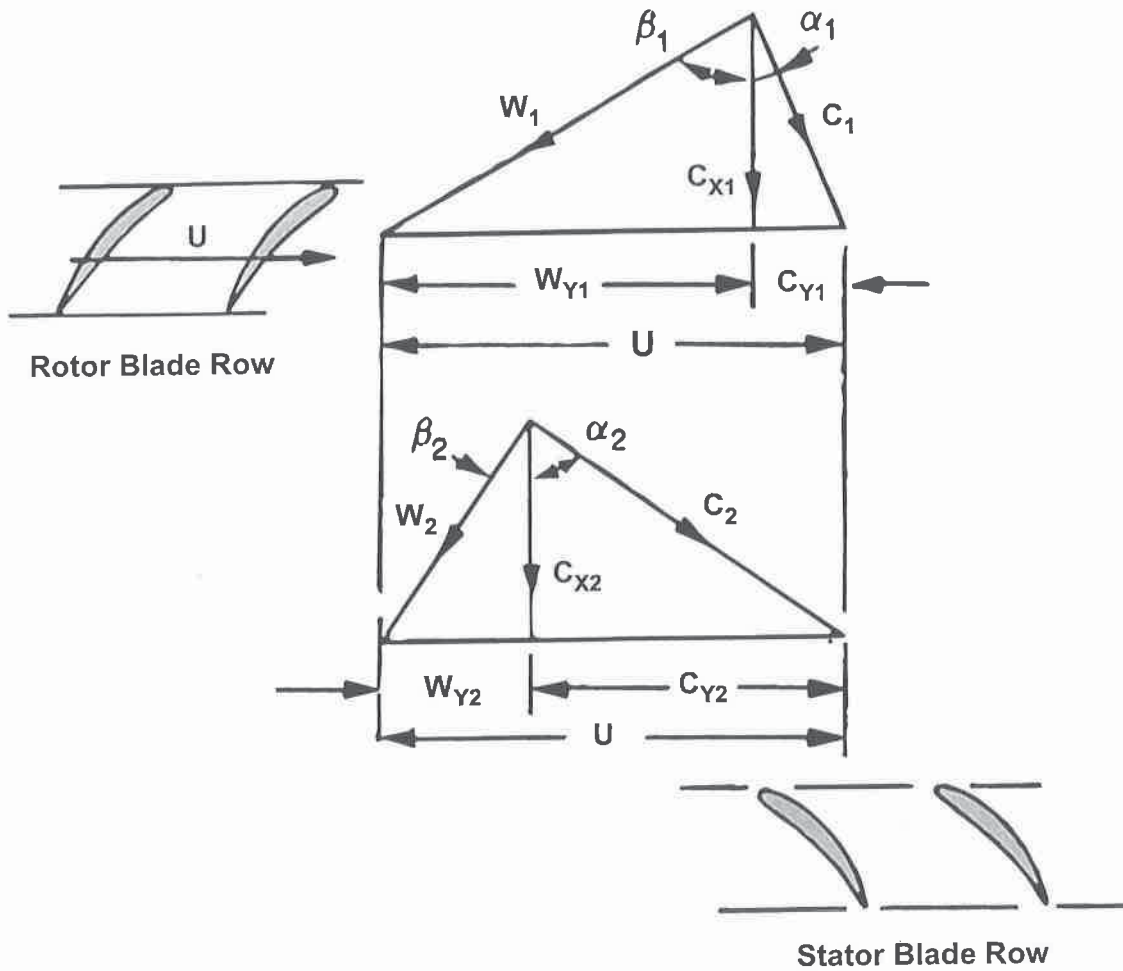
- N1 Low Speed Compressor and Turbine
- N2 High Speed Compressor and Turbine

Inlet Air Conditions 15°C

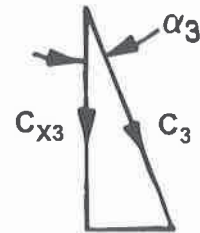


EXAMINATION PAPER ATTACHMENTS

QUESTION 5 COMPRESSOR VELOCITY DIAGRAM



- $U$  Blade velocity
- $C_1$  Rotor blade absolute inlet velocity
- $W_1$  Rotor blade relative inlet velocity
- $C_2$  Rotor blade absolute outlet velocity
- $W_2$  Rotor blade relative outlet velocity
- $C_3$  Stator blade absolute outlet velocity



## EXAMINATION REFERENCE MATERIAL

## NOMENCLATURE FOR REFERENCE EQUATIONS (SI UNITS)

a	Sonic velocity	m/s
A	Flow area, Surface area	m <sup>2</sup>
c <sub>p</sub>	Specific heat at constant pressure	J/kg°C
c <sub>v</sub>	Specific heat at constant volume	J/kg°C
b	Width	m
C	Velocity	m/s
D	Diameter	m
E	Energy	J
F	Force	N
g	Gravitational acceleration	m/s <sup>2</sup>
h	Specific enthalpy	J/kg
h	System head	m
h <sub>L</sub>	Head loss	m
H	Pump or turbine head	m
k	Ratio of specific heats	
L	Length	m
m	Mass	kg
M	Mass flow rate	kg/s
M <sub>A</sub>	Mach number	
N	Rotational speed	rev/s
N <sub>s</sub>	Specific Speed	
p	Pressure	Pa (N/m <sup>2</sup> )
P	Power	W (J/s)
q	Heat transferred	J/kg
Q	Heat	J
Q	Flow rate	m <sup>3</sup> /s
r	Radius	m
R	Specific gas constant	J/kg K
s	Entropy	J/kg K
T	Temperature	K
u	Specific internal energy	J/kg
U	Internal Energy	J
U	Velocity	m/s
v	Specific volume	m <sup>3</sup> /kg
V	Velocity	m/s
w	Specific work	J/kg
W	Work	J
W	Velocity	m/s
x	Length	m

$z$	Elevation	m
$\alpha$	Pump blade angle	°
$\alpha$	Compressor blade angle	°
$\beta$	Pump blade angle	°
$\beta$	Compressor blade angle	°
$\gamma$	Turbine blade angle	°
$\phi$	Turbine blade angle	°
$\delta$	Turbine blade angle	°
$\eta$	Efficiency	°
$\theta$	Nozzle angle	
$\mu$	Dynamic viscosity	Ns/m <sup>2</sup>
$\nu$	Kinematic viscosity	m <sup>2</sup> /s
$\rho$	Density	kg/m <sup>3</sup>
$\sigma_c$	Critical cavitation parameter	
$T$	Thrust	N
$\tau$	Torque	Nm
$\phi$	Peripheral velocity factor	
$\omega$	Rotational speed	rad/s
$\Omega$	Heat transfer rate	J/s

### GENERAL CONSTANTS

*Use unless otherwise specified*

Acceleration due to gravity:	$g = 9.81 \text{ m/s}^2$
Atmospheric pressure:	$p_{\text{atm}} = 100 \text{ kPa}$
Water vapour pressure:	$p_{\text{vapour}} = 1.71 \text{ kPa}$ (at 15°C)
Water vapour pressure:	$p_{\text{vapour}} = 2.34 \text{ kPa}$ (at 20°C)
Density of water:	$\rho_{\text{water}} = 1000 \text{ kg/m}^3$
Density of air:	$\rho_{\text{air}} = 1.21 \text{ kg/m}^3$ (at 15°C)
Density of air:	$\rho_{\text{air}} = 1.19 \text{ kg/m}^3$ (at 20°C)
Specific heat of air:	$c_p = 1.005 \text{ kJ/kg}^\circ\text{C}$
Specific heat of air:	$c_v = 0.718 \text{ kJ/kg}^\circ\text{C}$
Specific heat of water:	$c_p = 4.19 \text{ kJ/kg}^\circ\text{C}$

## GENERAL REFERENCE EQUATIONS

### Basic Thermodynamics

First Law:	$dE = \delta Q - \delta W$
Enthalpy:	$h = u + pv$
Continuity:	$\rho VA = \text{constant}$
Potential Energy:	$E_{PE} = mgz$
Kinetic Energy:	$E_{KE} = V^2/2$
Internal Energy:	$E_{IN} = U$
Flow Work:	$w = \Delta(pv)$
Energy Equation:	$zg + V^2/2 + u + pv + \Delta w + \Delta q = \text{constant}$

### Ideal Gas Relationships

Gas Law:	$pv = RT$
Specific Heat at Constant Pressure:	$c_p = \Delta h / \Delta T$
Specific Heat at Constant Volume:	$c_v = \Delta u / \Delta T$
Specific Gas Constant:	$R = c_p - c_v$
Ratio of Specific Heats	$k = c_p / c_v$
Isentropic Relations:	$p_1/p_2 = (v_2/v_1)^k = (T_1/T_2)^{k/(k-1)}$

## FLUID MACHINERY REFERENCE EQUATIONS

### Fluid Mechanics

Pressure	$p = \rho gh$
Continuity Equation	$\rho_1 V_1 A_1 = \rho_2 V_2 A_2 = M$
Bernoulli's Equation	$p_1/\rho g + z_1 + V_1^2/2g = p_2/\rho g + z_2 + V_2^2/2g$
Momentum Equation	$F = p_1 A_1 - p_2 A_2 - \rho VA(V_2 - V_1) \quad (\text{one dimensional})$

### Energy Equation

Pump and Turbine	$p_1/\rho g + z_1 + V_1^2/2g + w_{in}/g = p_2/\rho g + z_2 + V_2^2/2g + w_{out}/g$
Pipe Flow	$p_1/\rho g + z_1 + V_1^2/2g = p_2/\rho g + z_2 + V_2^2/2g + h_L$

### Compressible Flow

Mach Number	$M_A = V/a$
Sonic Velocity	$a = [kRT]^{1/2}$

**Steam Turbines**

Nozzle Equation:	$h_1 - h_2 = (V_2^2 - V_1^2) / 2$
Work:	$w = [(V_1^2_{\text{absolute}} - V_2^2_{\text{absolute}}) + (V_2^2_{\text{relative}} - V_1^2_{\text{relative}})] / 2$
Work:	$w = (V_{S1} \cos \theta - V_{S2} \cos \delta) V_{\text{blade}}$
Power:	$P = wM$
Force on Blades:	$F = M (V_{S1} \cos \theta - V_{S2} \cos \delta)$
Power to Blades:	$P = M (V_{S1} \cos \theta - V_{S2} \cos \delta) V_B$
Power to Blades:	$P = M [(V_{S1}^2 - V_{S2}^2) + (V_{R2}^2 - V_{R1}^2)] / 2$

**Gas Turbines**

State Equation:	$pv = RT$	
Isentropic Equation:	$(T_2/T_1) = (p_2/p_1)^{(k-1)/k}$	
Enthalpy Change:	$h_1 - h_2 = c_p(T_1 - T_2)$	(ideal gas)
Nozzle Equation:	$h_1 - h_2 = (V_2^2 - V_1^2) / 2$	
Work:	$w = (C_1 \sin \alpha_1 + C_2 \sin \alpha_2) U$	
Work:	$w = [(C_1^2 - C_2^2) + (W_2^2 - W_1^2)] / 2$	
Power:	$P = wM$	

**Compressors**

Work	$w = U(C_{Y2} - C_{Y1})$
Rotor Enthalpy Change	$h_1 + \frac{1}{2}W_1^2 = h_2 + \frac{1}{2}W_2^2$
Stator Enthalpy Change	$h_2 + \frac{1}{2}C_2^2 = h_3 + \frac{1}{2}C_3^2$
Isentropic Equation	$(T_3/T_1) = (p_3/p_1)^{(k-1)/k}$

**Hydraulic Machines**

Similarity Equations:	$Q_M/Q_P = (\omega_M/\omega_P) (D_M/D_P)^3$
	$H_M/H_P = (\omega_M/\omega_P)^2 (D_M/D_P)^2$
	$P_M/P_P = (\rho_M/\rho_P) (\omega_M/\omega_P)^3 (D_M/D_P)^5$
Pump Specific Speed:	$N_s = \omega Q^{1/2} / (gH)^{3/4}$
Turbine Specific Speed:	$N_s = \omega P^{1/2} / [\rho^{1/2} (gH)^{5/4}]$
Critical Cavitation Parameter:	$\sigma = \{[(p_{\text{atmosphere}} - p_{\text{vapour}}) / \rho g] - \Delta z\} / H$
Moody Efficiency Relationship:	$\eta_P = 1 - (1 - \eta_M) (D_M/D_P)^{1/4} (H_M/H_P)^{1/10}$
Power:	$P = \rho g Q H$

**Pumps**

Hydraulic Torque:	$\tau = \rho Q (r_2 V_2 \tau - r_1 V_1 \tau)$
Hydraulic Torque:	$\tau = \rho Q (r_2 V_2 \cos \alpha_2 - r_1 V_1 \cos \alpha_1)$
Power:	$P = 2\pi N \tau$
Net Positive Suction Head:	$NPSH = [(p_{\text{atmosphere}} - p_{\text{vapour}}) / \rho g] - \Delta z - h_L$
Peripheral Velocity Factor:	$\phi = V_{B2} / (2gh)^{1/2}$
Critical Cavitation Parameter:	$\sigma_C = NPSH / H$
Approximate Moody Efficiency:	$(1 - \eta_P) / (1 - \eta_M) \approx (D_M / D_P)^{1/5}$

**Jet Propulsion**

Thrust:	$T = M(V_{\text{jet}} - V_{\text{aircraft}})$
Thrust Power:	$T V_{\text{aircraft}} = M(V_{\text{jet}} - V_{\text{aircraft}}) V_{\text{aircraft}}$
Jet Power:	$P = M(V_{\text{jet}}^2 - V_{\text{aircraft}}^2) / 2$
Propulsion Efficiency:	$\eta_P = 2V_{\text{aircraft}} / (V_{\text{jet}} + V_{\text{aircraft}})$

**Wind Turbines**

Maximum Ideal Power:	$P_{\text{max}} = 8 \rho A V_1^3 / 27$
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