National Exams May 2017

16-Mec-A3, SYSTEM ANALYSIS AND CONTROL

3 hours duration

Notes:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.

2. Candidates may use a Casio **or** Sharp approved calculator. This is a **closed book** exam. No aids other than semi-log graph papers are permitted.

3. Any four (4) questions constitute a complete paper. Only the first four (4) questions as they appear in your answer book will be marked.

4. All questions are of equal value.

1) For a second-order system with transfer function

$$G(s) = \frac{3}{s^2 + 2s - 3}$$

determine the following:

(a) The DC gain;
(b) The final value to a step input.

2) Suppose that unity feedback is to be applied around the listed open-loop systems. Use Routh's stability criterion to determine whether the resulting closed-loop systems will be stable.

(a) $KG(s) = \frac{2(s+4)}{s^2(s+1)}$

(b) The transfer function of a typical tape-drive system is given by;

$$G(s) = \frac{K(s+4)}{s[(s+0.5)(s+1)(s^2+0.4s+4)]}$$

where time is measured in milliseconds. Using Routh's stability criterion, determine the range of K for which this system is stable when the characteristic equation is $1 + G(s) = 0$

3) Consider the second-order system

$$G(s) = \frac{1}{s^2 + 2\zeta s + 1}$$

We would like to add a transfer function of the form $D(s) = K(s+a)/(s+b)$ in series with $G(s)$ in a unity-feedback structure.

(a) Ignoring stability for the moment, what are the constraints on K, a, b so that the system is type 1?

(b) What are the constraints placed on K, a, b so that the system is stable and type 1?

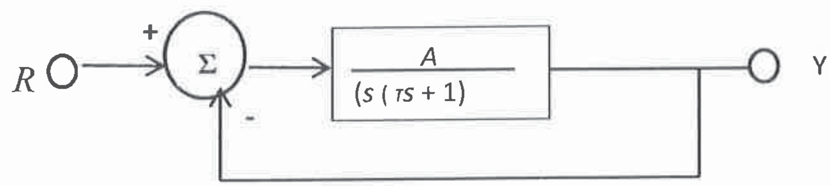
(c) What are the constraints on a and b so that the system is type 1 and remains stable for every positive value for K?

4) (a) The transfer function for the plant in a motor position control is given by

$$G(s) = \frac{A}{s(s+a)}$$

If we were able to select values for both A and a, what would they be to result in a system with $K_v = 20$ and $\zeta = 0.707$, where K_v is the velocity constant, and ζ is the damping ratio?

(b)Consider the system shown below;



What is the system type? Compute the steady-state tracking error due to a ramp input $r(t) = r_0 t\, 1(t)$.

5) For the characteristic equation

$$1 + \frac{K}{s(s+1)(s+5)} = 0$$

(a) Draw the real-axis segments of the corresponding root locus.

(b) Sketch the asymptotes of the locus for $K \rightarrow \infty$

(c) For what value of K are the roots on the imaginary axis?

6) Sketch the asymptotes of the Bode plot magnitude and phase for each of the listed open-loop transfer functions.

(a) $L(s) = \dfrac{1}{s(s+1)(s+5)(s+10)}$

(b) $L(s) = \dfrac{(s+2)}{s(s+1)(s+5)(s+10)}$

Table of Laplace Transforms

$f(t)$	$\mathcal{L}[f(t)] = F(s)$	
1	$\frac{1}{s}$	(1)
$e^{at}f(t)$	$F(s-a)$	(2)
$\mathcal{U}(t-a)$	$\frac{e^{-as}}{s}$	(3)
$f(t-a)\mathcal{U}(t-a)$	$e^{-as}F(s)$	(4)
$\delta(t)$	1	(5)
$\delta(t-t_0)$	e^{-st_0}	(6)
$t^n f(t)$	$(-1)^n \frac{d^nF(s)}{ds^n}$	(7)
$f'(t)$	$sF(s) - f(0)$	(8)
$f^n(t)$	$s^nF(s) - s^{(n-1)}f(0) - \cdots - f^{(n-1)}(0)$	(9)
$\int_0^t f(x)g(t-x)dx$	$F(s)G(s)$	(10)
$t^n \ (n = 0, 1, 2, \ldots)$	$\frac{n!}{s^{n+1}}$	(11)
$t^x \ (x \geq -1 \in \mathbb{R})$	$\frac{\Gamma(x+1)}{s^{x+1}}$	(12)
$\sin kt$	$\frac{k}{s^2+k^2}$	(13)
$\cos kt$	$\frac{s}{s^2+k^2}$	(14)
e^{at}	$\frac{1}{s-a}$	(15)
$\sinh kt$	$\frac{k}{s^2-k^2}$	(16)
$\cosh kt$	$\frac{s}{s^2-k^2}$	(17)
$\frac{e^{at}-e^{bt}}{a-b}$	$\frac{1}{(s-a)(s-b)}$	(18)

$f(t)$	$\mathcal{L}[f(t)] = F(s)$	
$\frac{ae^{at}-be^{bt}}{a-b}$	$\frac{s}{(s-a)(s-b)}$	(19)
te^{at}	$\frac{1}{(s-a)^2}$	(20)
t^ne^{at}	$\frac{n!}{(s-a)^{n+1}}$	(21)
$e^{at}\sin kt$	$\frac{k}{(s-a)^2+k^2}$	(22)
$e^{at}\cos kt$	$\frac{s-a}{(s-a)^2+k^2}$	(23)
$e^{at}\sinh kt$	$\frac{k}{(s-a)^2-k^2}$	(24)
$e^{at}\cosh kt$	$\frac{s-a}{(s-a)^2-k^2}$	(25)
$t\sin kt$	$\frac{2ks}{(s^2+k^2)^2}$	(26)
$t\cos kt$	$\frac{s^2-k^2}{(s^2+k^2)^2}$	(27)
$t\sinh kt$	$\frac{2ks}{(s^2-k^2)^2}$	(28)
$t\cosh kt$	$\frac{s^2-k^2}{(s^2-k^2)^2}$	(29)
$\frac{\sin at}{t}$	$\arctan\frac{a}{s}$	(30)
$\frac{1}{\sqrt{\pi t}}e^{-a^2/4t}$	$\frac{e^{-a\sqrt{s}}}{\sqrt{s}}$	(31)
$\frac{a}{2\sqrt{\pi t^3}}e^{-a^2/4t}$	$e^{-a\sqrt{s}}$	(32)
$\operatorname{erfc}\left(\frac{a}{2\sqrt{t}}\right)$	$\frac{e^{-a\sqrt{s}}}{s}$	(33)