National Exams – May 2016

07–Mec-B6 Advanced Fluid Mechanics

3 hours duration

NOTES:

1. If doubt exists as to the interpretation of any question the candidate is urged to submit with the answer paper a clear statement of the assumptions made.
2. Candidates may use any approved Sharp/Casio calculator.
   The exam is OPEN BOOK.
3. Any FIVE (5) out of the 6 questions constitute a complete exam paper for a total of 100 MARKS.
   The first five questions as they appear in the answer book will be marked.
4. Each question is of equal value (20 marks) and question items are marked as indicated.
5. Clarity and organization of the answer are important.

(20) **Question 1**

A large pressurized air reservoir (a) contains air at temperature T_a=100°C and at a constant pressure of P_a=280 kPa. The air passes through a convergent-divergent nozzle from reservoir (a) to another large reservoir (b), as shown in Figure 1. The throat area of the nozzle is A_T=9 cm^2 and the exit area is A_E=36 cm^2.

A mercury manometer reads h = 19 cm between the throat and reservoir (b).

Assume frictional losses are negligible and that the pressurized air density is negligible compared with mercury.

Air properties: γ=1.4, R=287 J/(kg K), C_P=1004.5 J/(kg K)

Mercury properties: ρ=13,550 kg/m^3

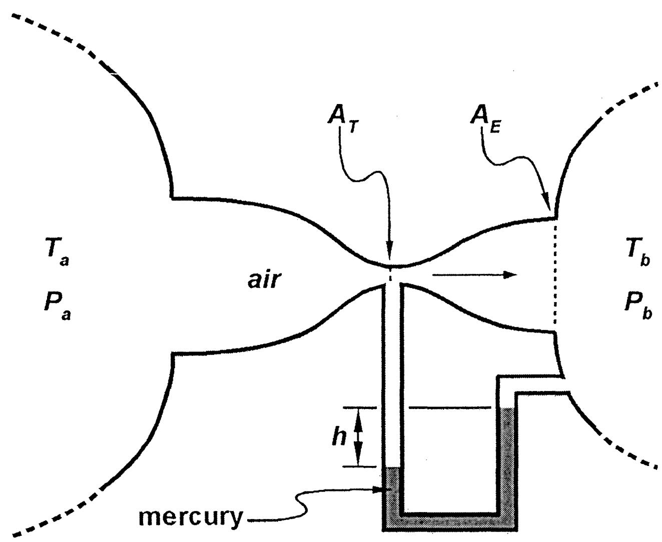

Figure 1: Convergent-divergent nozzle between two large air reservoirs.

(5) (a) Estimate the downstream reservoir pressure.

(5) (b) Is there a normal shock wave in the flow?

(5) (c) If there is a normal shock wave, does it stand in the exit plane or farther upstream?

(5) (d) What would be the mercury manometer reading if the nozzle were operating exactly at supersonic design conditions, i.e. with an *ideal expansion*?

(20) **Question 2**

A long municipal discharge pipe can be modelled as a source of strength m. The pipe is placed at a distance b from the bottom of a deep containment tank as shown in Figure 2. The city engineers want to drain the tank, but do not want to interrupt the discharge during the operation. The drain can be modelled as a sink of strength $-2m$ placed at the centre of the tank directly below the pipe, as shown in the figure. The city supervisor is concerned about the forces generated on the pipe. Your consulting company is asked to estimate these forces using potential flow theory.

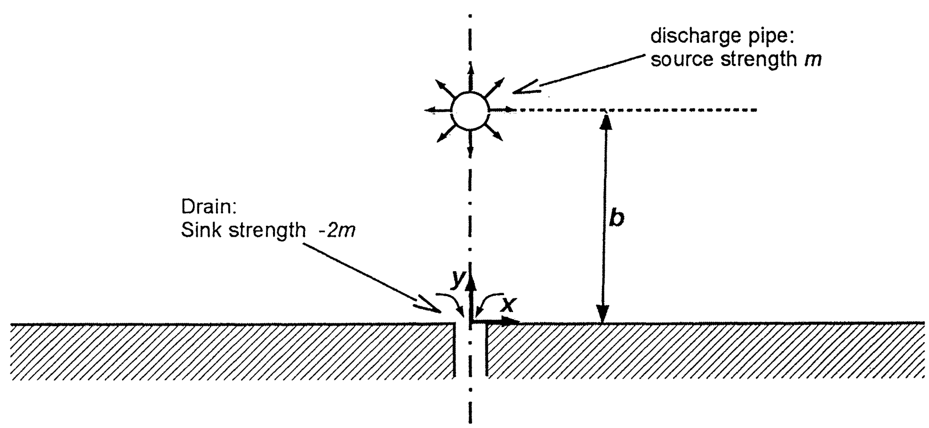

Figure 2: Cross-sectional view of discharge pipe next to a tank bed with a drain.

Assuming that the waste water density is the same as the water in the discharge tank, that the tank bed is flat, that the fluid velocity far from the discharge is negligible, and that the free surface effects can be neglected, determine:

(5) (a) The stream function that will represent this flow.

(5) (b) Verify that the tank bed is correctly simulated.

(5) (c) The velocity distribution along the tank bed.

(5) (d) The forces acting on the discharge pipe.

(20) **Question 3**

Air flows through an insulated constant diameter horizontal duct as shown in Figure 3. The inlet conditions are P_1=680 kPa, T_1=60°C and V_1=110 m/s. The diameter of the pipe is D=13 cm and the flow is "choked" at the exit. Determine the net force of the pipe on the fluid.

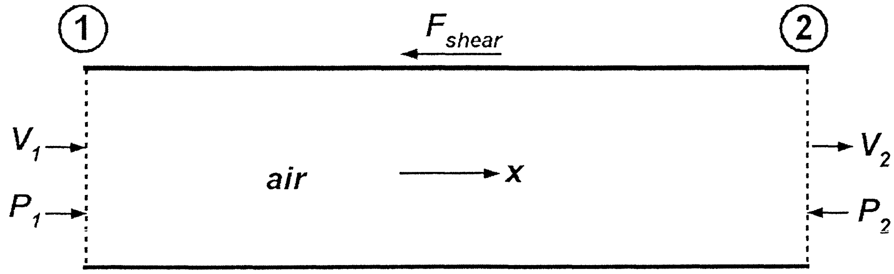

Figure 3: Pipe section with choked flow at the exit.

(20) Question 4

Water flows vertically and at a constant rate in a pipe of length L, as shown in Figure 4. The pipe consists of a circular outer pipe of radius R and a central rod of radius $\delta = R/4$. The walls of the outer pipe are coated with Teflon so that the water does not wet the surface (negligible shear). The inner rod is not coated, so that the water does wet the surface. The walls are non-porous. The inlet and outlet of the pipe are exposed to atmosphere, such that the inlet and outlet static pressures are the same. Assuming that $L/R >> 10$ and that the flow is laminar and axisymmetric, determine the following.

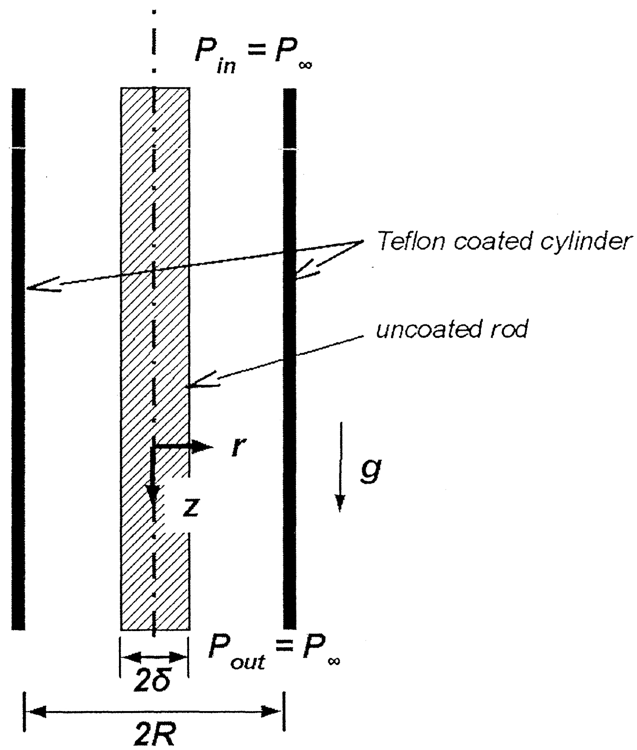

Figure 4: Schematic of vertical pipe with central rod.

(6) (a) A formula for the radial distribution of the radial velocity component v_r.

(7) (b) A formula for the radial distribution of the vertical velocity component v_z.

(7) (c) The force per unit length (magnitude and direction) acting on the inner rod.

(20) **Question 5**

At a sudden contraction in a pipe the diameter changes from D_1 to D_2. The pressure drop, Δp, which develops across the contraction is a function of D_1 and D_2, as well as the velocity U in the larger pipe, and the fluid density ρ and viscosity μ.

(15) (a) Using the Buckingham Pi theorem, and using D_1, U, and μ as repeating variables, determine a suitable set of dimensionless parameters.

(5) (b) Explain why would it be incorrect to include the velocity in the smaller pipe as an additional variable?

(20) **Question 6**

A rectangular cross-section suction wind tunnel is designed with a curved upper wall and a straight bottom wall. We are interested in characterising the boundary layer development along the bottom wall. The shape of the upper wall is designed such that the external (irrotational, or main) flow varies along the tunnel according to:

$$U_0 = A\,x^{1/3}$$

where A is a constant.

The flow can be assumed to be two-dimensional and laminar, and the properties of air to be constant (ρ is density, μ is dynamic viscosity, and $\nu = \mu/\rho$ is kinematic viscosity). The boundary layer thickness is represented by δ and it is assumed that at $x = 0$, $\delta = 0$. The velocity distribution in the boundary layer is assumed to be approximated by:

$$\frac{u}{U_0} = 2\frac{y}{\delta} - \left(\frac{y}{\delta}\right)^2 \qquad \text{such that: } \frac{\delta^*}{\delta} = \frac{1}{3} \quad \text{and} \quad \frac{\theta}{\delta} = \frac{2}{15}$$

where δ^* and θ are the momentum and displacement thickness, respectively.

The boundary layer thickness can be assumed to satisfy: $\delta = B\,x^n$.

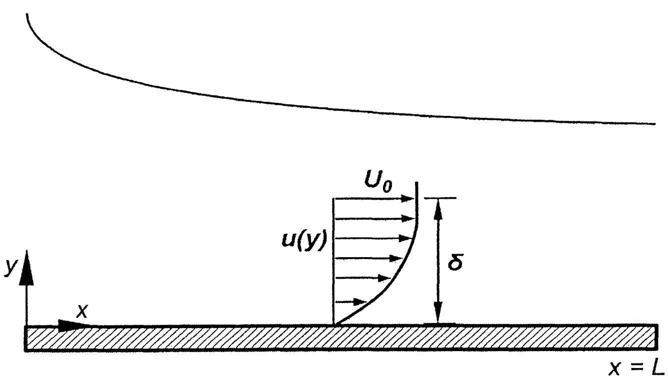

Figure 5: Boundary layer developing over bottom wall of suction wind tunnel.

(5) (a) Find the value of n to satisfy the integral boundary layer equations.

(5) (b) Express δ/x in terms of $\mathrm{Re}_x = \dfrac{U_0\,x}{\nu}$.

(5) (c) What is the local wall shear stress coefficient, $C_{fx} = \dfrac{\tau_w}{\rho U_0^2/2}$, in terms Re_x.

(5) (d) What is the average wall shear stress, $\overline{\tau_w}$, between $x = 0$ and $x = L$?