NATIONAL EXAMS
07-Elec-B2 Advanced Control Systems – May 2016

3 hours duration

NOTES:

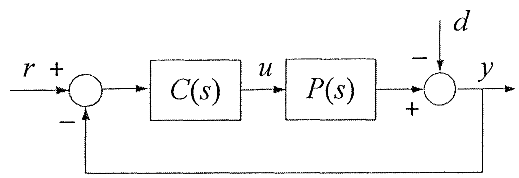
1. If a doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.

2. Candidates may use one of two calculators, a Casio or a Sharp. This is a closed-book examination. Tables of Laplace and z-transforms are attached.

3. Any four questions constitute a complete paper. Only the first four questions as they appear in your answer book will be marked.

4. All questions are of equal value.

1. Consider the control system below with $P(s) = \dfrac{6(1-s)}{s(2+s)(1+s)}$ and $C(s) = K$.



(a) The value of K is increased from zero to a value of K_{max} at which the system exhibits sustained oscillation. What is the value of K_{max} and what is the oscillation frequency?

(b) For $K = K_{max}/2$ determine the phase margin.

(c) Define the tracking error, $e(t) = r(t) - y(t)$. Determine the steady state tracking error when $r(t)$ is a ramp with unit slope and $d(t) = 1$.

(d) Determine the steady state tracking error when $d(t) = 0$, and $r(t) = 4 \sin t$.

2. Consider the system, $P(s) = \dfrac{3(s+\alpha)}{s(s+4)^2}$.

(a) Find a state space model for the system.

(b) Justify the conditions under which the system controllable? observable?

(c) The system input and output are uniformly sampled with a sample period of h and the discrete input is applied to a zero order hold device. Determine the poles of the sampled data system as a function of h. Detailed calculations are not necessary.

3. Consider the system,

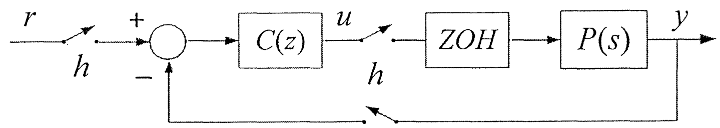
$$\dot{x}(t) = \begin{bmatrix} -1 & 0 & 0 \\ 1 & -2 & -1 \\ 0 & 1 & 0 \end{bmatrix} x + B \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u$$

$$y(t) = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} x$$

Design a controller of the form, $u(t) = Lr(t) - Kx(t)$ such that the closed loop poles are at $s = -5$, $-3 \pm j$ and the DC gain, that relates a constant value of y to a constant value of r, is unity.

4. Consider the sampled data system shown on the right. The input to the ZOH, the set-point, r, and the output, y, are uniformly sampled with a sample period of h. $C(z)$ and $P(s)$ are given by,

$$C(z) = \frac{k_1 z + k_0}{z-1}, \quad P(s) = \frac{1}{s}$$



(a) Determine $C(z)$, i.e., k_0 and k_1, such that the closed loop poles are all located at $z = 0$.

(b) Determine the corresponding discrete closed loop transfer function, $T(z)$, that relates $Y(z)$ to $R(z)$.

(c) Sketch the associated unit step response at $y(t)$, being careful to show the inter-sample behavior.

5. An experiment is conducted on a continuous time system, P, whose output is uniformly sampled with sample period, $h = 1$ second. The discrete input, $u(kh)$, is applied to a zero order hold device which drives the input of P. Measurements for $u(kh)$ and $y(kh)$ appear in the Table. Assume that P is modeled by $P(s) = \dfrac{Ke^{-sT}}{s\tau + 1}$.

kh	$u(kh)$	$y(kh)$
0	0	0
1	1	0
2	1	0
3	1	1.000
4	1	1.750
5	1	2.312
6	1	2.734
7	1	3.051
8	1	3.288

(a) Give a procedure to identify a discrete time model, $G(z)$, that relates $y(kh)$ to $u(kh)$, and identify the parameters.

(b) Now determine the unknown parameters of $P(s)$.

6. Consider the plant, $P(s) = \dfrac{e^{-2s}}{s}$.

(a) Design a proportional feedback controller for the plant such that the gain margin is 8dB.

(b) Determine the associated phase margin.

(c) Determine the steady state output when the set point input is a unit step and sketch, approximately, the unit step response.

Inverse Laplace Transforms	
$F(s)$	$f(t)$
$\dfrac{A}{s+\alpha}$	$Ae^{-\alpha t}$
$\dfrac{C+jD}{s+\alpha+j\beta}+\dfrac{C-jD}{s+\alpha-j\beta}$	$2e^{-\alpha t}\left(C\cos\beta t+D\sin\beta t\right)$
$\dfrac{A}{(s+\alpha)^{n+1}}$	$\dfrac{At^{n}e^{-\alpha t}}{n!}$
$\dfrac{C+jD}{\left(s+\alpha+j\beta\right)^{n+1}}+\dfrac{C-jD}{\left(s+\alpha-j\beta\right)^{n+1}}$	$\dfrac{2t^{n}e^{-\alpha t}}{n!}\left(C\cos\beta t+D\sin\beta t\right)$

Inverse z-Transforms	
$F(z)$	$f(nT)$
$\dfrac{Kz}{z-a}$	Ka^{n}
$\dfrac{\left(C+jD\right)z}{z-re^{j\varphi}}+\dfrac{\left(C-jD\right)z}{z-re^{-j\varphi}}$	$2r^{n}\left(C\cos n\varphi-D\sin n\varphi\right)$
$\dfrac{Kz}{\left(z-a\right)^{r}}$, $r=2,3...$	$\dfrac{Kn\left(n-1\right)...\left(n-r+2\right)}{\left(r-1\right)!a^{r-1}}a^{n}$

Table of Laplace and z-Transforms (*h* denotes the sample period)		
$f(t)$	$F(s)$	$F(z)$
unit impulse	1	1
unit step	$\frac{1}{s}$	$\frac{hz}{z-1}$
$e^{-\alpha t}$	$\frac{1}{s+\alpha}$	$\frac{z}{z-e^{-\alpha h}}$
t	$\frac{1}{s^2}$	$\frac{hz}{(z-1)^2}$
$\cos \beta t$	$\frac{s}{s^2+\beta^2}$	$\frac{z(z-\cos\beta h)}{z^2-2z\cos\beta h+1}$
$\sin \beta t$	$\frac{\beta}{s^2+\beta^2}$	$\frac{z\sin\beta h}{z^2-2z\cos\beta h+1}$
$e^{-\alpha t}\cos \beta t$	$\frac{s+\alpha}{(s+\alpha)^2+\beta^2}$	$\frac{z(z-e^{-\alpha h}\cos\beta h)}{z^2-2ze^{-\alpha h}\cos\beta h+e^{-2\alpha h}}$
$e^{-\alpha t}\sin \beta t$	$\frac{\beta}{(s+\alpha)^2+\beta^2}$	$\frac{ze^{-\alpha h}\sin\beta h}{z^2-2ze^{-\alpha h}\cos\beta h+e^{-2\alpha h}}$
$t\,f(t)$	$-\frac{dF(s)}{ds}$	$-zh\frac{dF(z)}{dz}$
$e^{-\alpha t}f(t)$	$F(s+\alpha)$	$F(ze^{\alpha h})$