National Exams May 2015
04-BS-1, Mathematics
3 hours Duration

Notes:

1. If doubt exists as to the interpretation of any question, the candidate is urged to include a clear statement of any assumptions made along with their answer.
2. Any APPROVED CALCULATOR is permitted. This is a CLOSED BOOK exam. However, candidates are permitted to bring ONE AID SHEET written on both sides.
3. Any five questions constitute a complete paper. Only the first five questions as they appear in your answer book will be marked.
4. All questions are of equal value.

Marking Scheme:

1. (a) 7 marks, (b) 7 marks, (c) 6 marks
2. 20 marks
3. (a) 6 marks, (b) 8 marks, (c) 6 marks
4. 20 marks
5. 20 marks
6. (a) 10 marks, (b) 10 marks
7. 20 marks
8. 20 marks

1. Find the general solutions of the following differential equations:

   (a) $y' + 2xy = 2xe^{-x^2}$,

   (b) $y' + 2xy^2 = 0$,

   (c) $y'' - 2y' + 3y = 0$.

   Note that in each case, $'$ denotes differentiation with respect to x.

2. Find the general solution, $y(x)$, of the differential equation

$$2x^2y'' - 5xy' - 4y = 3x^4.$$

   Note that $'$ denotes differentiation with respect to x.

3. Consider the matrix

$$A = \begin{pmatrix} 8 & -14 & -6 \\ 4 & -6 & -4 \\ -2 & -2 & 4 \end{pmatrix}$$

   (a) Show that $\begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$ is an eigenvector of A and find the associated eigenvalue.

   (b) Show that 2 is an eigenvalue of A and find an associated eigenvector.

   (c) Using the results of parts (a) and (b), write down two solutions to the linear system $\mathbf{x}' = \mathbf{A}\mathbf{x}$.

4. Find the work done by the field $\mathbf{F}(x, y, z) = x^2\mathbf{i} + y\mathbf{j} - z\mathbf{k}$ in moving a particle from the point $(0, 2, 0)$ to the point $(3\pi, 0, 2)$ along the path $x = 6t$, $y = 2\cos t$, $z = 2\sin t$.

5. Find the equation of the plane tangent to the surface defined implicitly by $xy^2z^3 = 2 + y$ at the point $(x, y, z) = (3, 4, 1/2)$

6. Let P be the plane passing through the three points (2,1,-2), (1,2,0) and (1,0,-1).

   (a) Find an equation representing the plane P.

   (b) Find the line of intersection between the plane P and the plane $x + y - 2z = 3$

7. Find out what type of conic section (e.g., parabola, hyperbola, or ellipse) the following quadratic form represents and transform it to principal axes. (That is, find new variables u and v so that $Q = au^2 + bv^2$.)

$$Q = -2x^2 + 12xy + 7y^2 = 156.$$

8. Let S be the boundary of the region enclosed by the paraboloid $z = x^2 + y^2 - 2$ and the plane $z = 2$ and let

$$\mathbf{F}(x, y, z) = xy^2\mathbf{i} + 2xyz\mathbf{j} - xz^2\mathbf{k}.$$

   Evaluate the surface integral $\iint_S \mathbf{F} \cdot \mathbf{n}\, dA$, where $\mathbf{n}$ is the unit outward normal on S.