04-CHEM-B1, TRANSPORT PHENOMENA

May 2016

3 hours duration

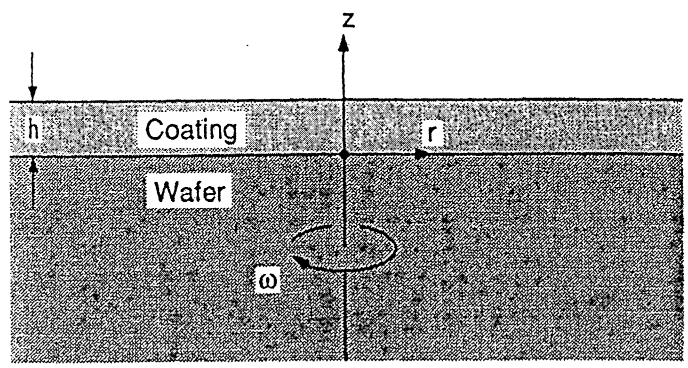
NOTES

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
2. The examination is an **open book exam.** One textbook of your choice with notations listed on the margins etc., but no loose notes are permitted into the exam.
3. Candidates may use any **non-communicating** calculator.
4. All problems are worth 25 points. **One problem** from **each** of sections A, B, and C must be attempted. A **fourth** problem from **any section** must also be attempted.
5. **Only the first four** questions as they appear in the answer book will be marked.
6. State all assumptions clearly.

Section A: Fluid Mechanics

A1. The simplest patient infusion system is that of gravity flow from an intravenous (IV) bag. A 500 ml IV bag containing an aqueous solution is connected to a vein in the forearm of a patient. Venous pressure in the forearm is 0 mm Hg (gage pressure). The IV bag is placed on a stand such that the entrance to the tube leaving the IV bag is exactly 1 meter above the vein into which the IV fluid enters. The length of the IV bag is 30 cm. The IV is fed through an 18-gage tube (internal diameter = 0.953 mm) and the total length of the tube is 2 meters. Calculate the flow rate of the IV fluid, and estimate the time needed to empty the IV bag.

A2. The spin coating process is used to coat a thin film of photoresist (organic polymer) on the wafer. A viscous solution containing the photoresist material is made to form a thin (~ 1 μm) uniform thin film on the wafer by spinning the wafer as shown in the figure below.



a) **[22 points]** Derive an expression for thickness of the film on the wafer (h) as a function time (t), angular velocity (ω), and the kinematic viscosity (ν) of the photoresist solution.

b) **[3 points]** What is the corresponding equation for thickness of a very thin film of photoresist on the wafer?

Section B: Heat Transfer

B1. Air at 1 atm and 150 ^{o}C enters a pipe of diameter 5.08 cm, and it is heated as it moves through the pipe at a velocity of 8 m/s. Assume constant heat flux per unit length of the pipe, and that the pipe wall temperature is 20 ^{o}C above air temperature.

a) [18 points] Determine the heat transfer per unit length of the pipe.

b) [7 points] Calculate the bulk temperature rise through a 2-meter length of pipe.

DATA:

Thermal conductivity of air = 3.52×10^{-2} W/m ^{o}C
Viscosity of air = 2.38×10^{-5} kg/m.s
Specific heat capacity of air = 1.017 kJ/kg ^{o}C
Molecular weight of air = 29 g/mol

B2. Derive the differential energy balance equation in spherical coordinates for heating a sphere of radius r, if the temperature variation with angular position is zero.

Section C: Mass Transfer

C1. For rate of mass transfer from a cylinder rotating in a liquid bath, the Sherwood number (Sh) is given by the equation

$$\mathbf{Sh = k_d / U_r = 0.079\ (Re)^{-0.3}\ (Sc)^{-0.644}}$$

where k_d is the mass transfer coefficient between cylinder and liquid, U_r is the peripheral speed of rotation, Schmidt number (Sc) = ν/D_{AB}, Reynold's number (Re) = dU/ν, and kinematic viscosity (ν) = μ/ρ.

In an experiment, a 1.5-cm diameter by 5.0-cm long iron cylinder was rotated at a speed of 77 rpm in a melt of carbon-saturated iron at 1275 °C. The rate of dissolution, expressed as decrease of rod radius with time, was – 5.57 x 10^{-3} cm/s. For the dissolution of iron in carbon-saturated iron, the iron mass concentration driving force (ΔX_{Fe}) was estimated to be 0.385. Assuming the densities of the iron rod and the carbon-saturated iron melt are equal, compare the experimental rate of dissolution with that predicted by the above correlation.

<u>DATA</u>:

Diffusivity of iron in the carbon-saturated iron melt at 1275 °C = 9 x 10^{-5} cm^2/s
Kinematic viscosity of iron melt at 1275 °C = 1.41 x 10^{-2} cm^2/s

C2. During class break, a guy comes and cleans the blackboards. He leaves a thin film of water whose thickness is 1 mm. Water evaporates from that surface into the surrounding air, whose humidity is 10%. We want to calculate how long it takes for the water to evaporate completely. Assume a diffusional distance between the water surface and ambient air of 1 cm. At 25 °C, the vapor pressure of water is 0.0313 atm, and the diffusion coefficient of water vapor in air is 0.282 cm^2/s.

a) [3 points] Define the problem, draw coordinates, and state assumptions.

b) [13 points] Calculate the steady-state flux of water from the surface.

c) [2 points] Obtain an expression to relate the flux to the disappearance of liquid water.

d) [7 points] Calculate the time required for the water film to evaporate and the chalkboard to dry completely.

APPENDIX A

Summary of the Conservation Equations

Table A.1 The Continuity Equation

$$\frac{\partial \rho}{\partial t}+(\nabla \cdot \rho \vec{u})=0$$	(1.1)
Rectangular coordinates (*x*, *y*, *z*) $$\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x}(\rho u_x)+\frac{\partial}{\partial y}(\rho u_y)+\frac{\partial}{\partial z}(\rho u_z)=0$$	(1.1a)
Cylindrical coordinates (*r*, *θ*, *z*) $$\frac{\partial \rho}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}(\rho r u_r)+\frac{1}{r}\frac{\partial}{\partial \theta}(\rho u_\theta)+\frac{\partial}{\partial z}(\rho u_z)=0$$	(1.1b)
Spherical coordinates (*r*, *θ*, *ϕ*) $$\frac{\partial \rho}{\partial t}+\frac{1}{r^2}\frac{\partial}{\partial r}(\rho r^2 u_r)+\frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\rho u_\theta \sin\theta)+\frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}(\rho u_\phi)=0$$	(1.1c)

Table A.2 The Navier-Stokes equations for Newtonian fluids of constant ρ and μ

	$$\frac{\partial \vec{u}}{\partial t}+(\vec{u}\cdot\nabla)\vec{u}=-\frac{1}{\rho}\nabla P+\vec{g}+\nu(\nabla^2\vec{u})$$	(A2)
Rectangular coordinates (*x*, *y*, *z*)		
x-component	$$\frac{\partial u_x}{\partial t}+u_x\frac{\partial u_x}{\partial x}+u_y\frac{\partial u_x}{\partial y}+u_z\frac{\partial u_x}{\partial z}=-\frac{1}{\rho}\frac{\partial P}{\partial x}+g_x+\nu\left(\frac{\partial^2 u_x}{\partial x^2}+\frac{\partial^2 u_x}{\partial y^2}+\frac{\partial^2 u_x}{\partial z^2}\right)$$	(A2a)
y-component	$$\frac{\partial u_y}{\partial t}+u_x\frac{\partial u_y}{\partial x}+u_y\frac{\partial u_y}{\partial y}+u_z\frac{\partial u_y}{\partial z}=-\frac{1}{\rho}\frac{\partial P}{\partial y}+g_y+\nu\left(\frac{\partial^2 u_y}{\partial x^2}+\frac{\partial^2 u_y}{\partial y^2}+\frac{\partial^2 u_y}{\partial z^2}\right)$$	(A2b)
z-component	$$\frac{\partial u_z}{\partial t}+u_x\frac{\partial u_z}{\partial x}+u_y\frac{\partial u_z}{\partial y}+u_z\frac{\partial u_z}{\partial z}=-\frac{1}{\rho}\frac{\partial P}{\partial z}+g_z+\nu\left(\frac{\partial^2 u_z}{\partial x^2}+\frac{\partial^2 u_z}{\partial y^2}+\frac{\partial^2 u_z}{\partial z^2}\right)$$	(A2c)

Cylindrical coordinates (r, θ, z)

r-component

$$\frac{\partial u_r}{\partial t}+u_r\frac{\partial u_r}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta}+u_z\frac{\partial u_r}{\partial z}-\frac{u_\theta^2}{r}=-\frac{1}{\rho}\frac{\partial P}{\partial r}+g_r+\nu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial (r u_r)}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u_r}{\partial \theta^2}-\frac{2}{r^2}\frac{\partial u_\theta}{\partial \theta}+\frac{\partial^2 u_r}{\partial z^2}\right] \quad \text{(A2d)}$$

θ-component

$$\frac{\partial u_\theta}{\partial t}+u_r\frac{\partial u_\theta}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta}+u_z\frac{\partial u_\theta}{\partial z}+\frac{u_r u_\theta}{r}=-\frac{1}{\rho r}\frac{\partial P}{\partial \theta}+g_\theta+\nu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial (r u_\theta)}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u_\theta}{\partial \theta^2}+\frac{2}{r^2}\frac{\partial u_r}{\partial \theta}+\frac{\partial^2 u_\theta}{\partial z^2}\right] \quad \text{(A2e)}$$

z-component

$$\frac{\partial u_z}{\partial t}+u_r\frac{\partial u_z}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_z}{\partial \theta}+u_z\frac{\partial u_z}{\partial z}=-\frac{1}{\rho}\frac{\partial P}{\partial z}+g_z+\nu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u_z}{\partial \theta^2}+\frac{\partial^2 u_z}{\partial z^2}\right] \quad \text{(A2f)}$$

Spherical coordinates (r, θ, ϕ)

r-component

$$\frac{\partial u_r}{\partial t}+u_r\frac{\partial u_r}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta}+\left(\frac{u_\phi}{r\sin\theta}\right)\frac{\partial u_r}{\partial \phi}-\frac{u_\theta^2}{r}-\frac{u_\phi^2}{r}=-\frac{1}{\rho}\frac{\partial P}{\partial r}+g_r+\nu\left[\frac{1}{r^2}\frac{\partial^2}{\partial r^2}\left(r^2 u_r\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial u_r}{\partial \theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 u_r}{\partial \phi^2}\right] \quad \text{(A2g)}$$

θ-component

$$\frac{\partial u_\theta}{\partial t}+u_r\frac{\partial u_\theta}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta}+\left(\frac{u_\phi}{r\sin\theta}\right)\frac{\partial u_\theta}{\partial \phi}+\frac{u_r u_\theta}{r}-\frac{u_\phi^2}{r}\cot\theta=-\frac{1}{\rho r}\frac{\partial P}{\partial \theta}+g_\theta+\nu\left[\begin{array}{l}\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial u_\theta}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial \theta}\left(u_\theta\sin\theta\right)\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 u_\theta}{\partial \phi^2}\\+\frac{2}{r^2}\frac{\partial u_r}{\partial \theta}-\frac{2\cot\theta}{r^2\sin\theta}\frac{\partial u_\phi}{\partial \phi}\end{array}\right] \quad \text{(A2h)}$$

ϕ-component

$$\frac{\partial u_\phi}{\partial t}+u_r\frac{\partial u_\phi}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_\phi}{\partial \theta}+\frac{u_\phi}{r\sin\theta}\frac{\partial u_\phi}{\partial \phi}+\frac{u_r u_\phi}{r}+\frac{u_\theta u_\phi}{r}\cot\theta=-\frac{1}{\rho r\sin\theta}\frac{\partial P}{\partial \phi}+g_\varphi+\nu\left[\begin{array}{l}\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial u_\phi}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial \theta}\left(u_\phi\sin\theta\right)\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 u_\phi}{\partial \phi^2}\\+\frac{2}{r^2\sin\theta}\frac{\partial u_r}{\partial \phi}+\frac{2\cot\theta}{r^2\sin\theta}\frac{\partial u_\theta}{\partial \phi}\end{array}\right] \quad \text{(A2i)}$$

Table A.3 The Energy Equation for Incompressible Media

$$\rho c_P\left[\frac{\partial T}{\partial t}+(\vec{u}\cdot\nabla)(T)\right]=[\nabla\cdot k\nabla T]+\dot{T}_G$$	(A3)
Rectangular coordinates (x, y, z) $$\rho c_P\left[\frac{\partial T}{\partial t}+u_x\frac{\partial T}{\partial x}+u_y\frac{\partial T}{\partial y}+u_z\frac{\partial T}{\partial z}\right]=\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{T}_G$$	(A3a)
Cylindrical coordinates (r, θ, z) $$\rho c_P\left[\frac{\partial T}{\partial t}+u_r\frac{\partial T}{\partial r}+\frac{u_\theta}{r}\frac{\partial T}{\partial \theta}+u_z\frac{\partial T}{\partial z}\right]=\frac{1}{r}\frac{\partial}{\partial r}\left(rk\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \theta}\left(k\frac{\partial T}{\partial \theta}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{T}_G$$	(A3b)
Spherical coordinates (r, θ, ϕ) $$\rho c_P\left[\frac{\partial T}{\partial t}+u_r\frac{\partial T}{\partial r}+\frac{u_\theta}{r}\frac{\partial T}{\partial \theta}+\frac{u_\phi}{r\sin\theta}\frac{\partial T}{\partial \phi}\right]=$$ $$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2k\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(k\sin\theta\frac{\partial T}{\partial \theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right)+\dot{T}_G$$	(A3c)

Table A4: The continuity equation for species *A* in terms of the molar flux

$$\frac{\partial C_A}{\partial t}=-\left(\nabla\cdot\vec{N}_A\right)+\dot{R}_{A,G}$$	(4.)
Rectangular coordinates (x, y, z) $$\frac{\partial C_A}{\partial t}=-\left(\frac{\partial[N_A]_z}{\partial x}+\frac{\partial[N_A]_y}{\partial y}+\frac{\partial[N_A]_z}{\partial z}\right)+\dot{R}_{A,G}$$	(4a)
Cylindrical coordinates (r, θ, z) $$\frac{\partial C_A}{\partial t}=-\left\{\frac{1}{r}\frac{\partial}{\partial r}[rN_A]_r+\frac{1}{r}\frac{\partial}{\partial \theta}[N_A]_\theta+\frac{\partial}{\partial z}[N_A]_z\right\}+\dot{R}_{A,G}$$	(4b)
Spherical coordinates (r, θ, ϕ) $$\frac{\partial C_A}{\partial t}=-\left\{\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2[N_A]_r\right)+\frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}\left([N_A]_\theta\sin\theta\right)+\frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}[N_A]_\phi\right\}+\dot{R}_{A,G}$$	(4c)

Table A.5: The continuity equation for species *A*

$$\frac{\partial C_A}{\partial t} + (\vec{u}\cdot\nabla)C_A = D_A\nabla^2 C_A + \dot{R}_{A,G}$$	(5)
Rectangular coordinates (*x*, *y*, *z*) $$\frac{\partial C_A}{\partial t} + u_x\frac{\partial C_A}{\partial x} + u_y\frac{\partial C_A}{\partial y} + u_z\frac{\partial C_A}{\partial z} = \frac{\partial}{\partial x}\left(D\frac{\partial C_A}{\partial x}\right) + \frac{\partial}{\partial y}\left(D\frac{\partial C_A}{\partial y}\right) + \frac{\partial}{\partial z}\left(D\frac{\partial C_A}{\partial z}\right) + \dot{R}_{A,G}$$	(5a)
Cylindrical coordinates (*r*, *θ*, *z*) $$\frac{\partial C_A}{\partial t} + u_r\frac{\partial C_A}{\partial r} + \frac{u_\theta}{r}\frac{\partial C_A}{\partial \theta} + u_z\frac{\partial C_A}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\left(rD\frac{\partial C_A}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \theta}\left(D\frac{\partial C_A}{\partial \theta}\right) + \frac{\partial}{\partial z}\left(D\frac{\partial C_A}{\partial z}\right) + \dot{R}_{A,G}$$	(5b)
Spherical coordinates (*r*, *θ*, *ϕ*) $$\frac{\partial C_A}{\partial t} + u_r\frac{\partial C_A}{\partial r} + \frac{u_\theta}{r}\frac{\partial C_A}{\partial \theta} + \frac{u_\phi}{r\sin\theta}\frac{\partial C_A}{\partial \phi} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 D\frac{\partial C_A}{\partial r}\right)$$ $$+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(D\sin\theta\frac{\partial C_A}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial \phi}\left(D\frac{\partial C_A}{\partial \phi}\right) + \dot{R}_{A,G}$$	(5c)