National Exams – May 2019

16-Mec-A6 Advanced Fluid Mechanics

3 hours duration

NOTES:

1. If doubt exists as to the interpretation of any question the candidate is urged to submit with the answer paper a clear statement of the assumptions made.
2. Candidates may use any approved Sharp/Casio calculator.
   The exam is OPEN BOOK.
3. Any FIVE (5) out of the 6 questions constitute a complete exam paper for a total of 100 MARKS.
   The first five questions as they appear in the answer book will be marked.
4. Each question is of equal value (20 marks) and question items are marked as indicated.
5. Clarity and organization of the answer are important.

(20) **Question 1**

A large pressurized air reservoir (a) contains air at temperature T_a=100°C and at a constant pressure of P_a=300 kPa. The air passes through a convergent-divergent nozzle from reservoir (a) to another large reservoir (b), as shown in Figure 1. The throat area of the nozzle is A_T=9 cm^2 and the exit area is A_E=31.5 cm^2.

A mercury manometer reads h = 15 cm between the throat and reservoir (b).

Assume frictional losses are negligible and that the pressurized air density is negligible compared with mercury.

Air properties: γ=1.4, R=287 J/(kg K), C_P=1004.5 J/(kg K)

Mercury properties: ρ=13,550 kg/m^3

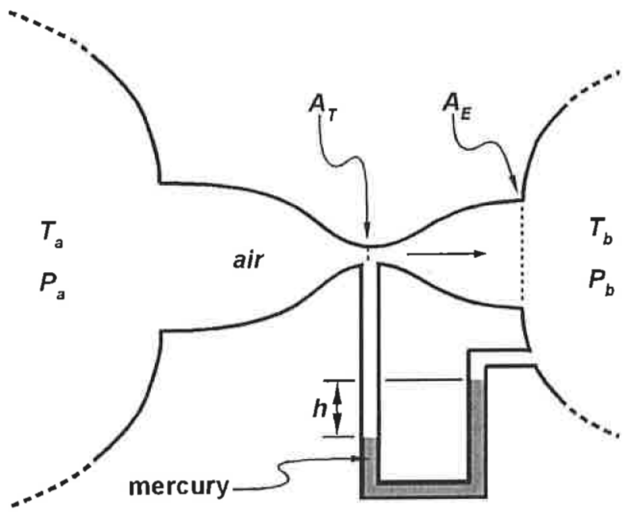


Figure 1: Convergent-divergent nozzle between two large air reservoirs.

(5) (a) Estimate the downstream reservoir pressure.

(5) (b) Is there a normal shock wave in the flow?

(5) (c) If there is a normal shock wave, does it stand in the exit plane or farther upstream?

(5) (d) What would be the mercury manometer reading if the nozzle were operating exactly at supersonic design conditions, i.e. with an *ideal expansion*?

(20) **Question 2**

Consider the ideal flow given by the velocity potential function

$$\phi = -\Gamma \ln r$$

where Γ is a positive constant.

(5) (a) Determine the stream function ψ.

(5) (b) Sketch the equipotential lines and the stream lines of this flow.

(5) (c) Calculate the radial velocity V_r and identify the flow pattern.

(5) (d) Give the physical meaning of the constant $(2\pi\Gamma)$.

(20) **Question 3**

Water flows at a steady rate of Q=15 l/s through a flanged curved pipe. The water enters horizontally and exits at an angle of α=30° relative to the horizontal plane. The inlet and exit internal pipe diameters are D=10 cm and d=5 cm, respectively.

The velocity profiles in the pipe and at the exit may be assumed uniform.

At the exit, a free jet condition may be assumed.

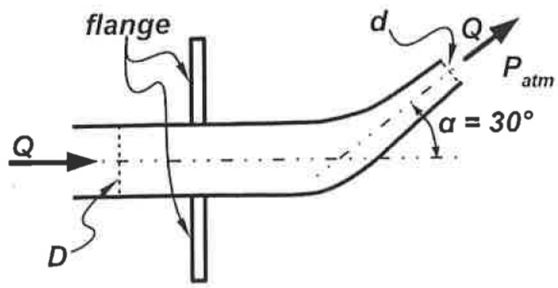


Figure 2: Schematic of curved pipe with flange.

(20) (a) Neglecting frictional losses and gravitational effects, estimate the forces (magnitude and direction, or F_{R_x} and F_{R_y}) acting on the flange.

(20) **Question 4**

Water rotates as a rigid body about the vertical (z) axis in a spinning cylindrical container. The constant rotation rate is ω rad/s and gravity points in the negative z-direction.

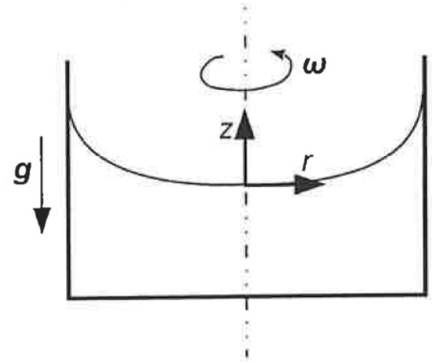


Figure 3: Schematic of spinning cylindrical container.

(6) (a) Simplify the Navier-Stokes equations in r and in z directions. Explain why the viscous stresses can be neglected.

(10) (b) Integrate the simplified inviscid Navier-Stokes equations in r and z directions to derive an expression for the pressure p as a function of r and z everywhere in the fluid.

(4) (c) From your solution to (b), show that the shape of the surface satisfies $z_{\text{surface}} = \omega^2 r^2/2g$, where g is the gravitational acceleration.

(20) **Question 5**

A student team wants to test a scaled down model of a human-powered submarine to measure the drag coefficient of the vessel. But they do not have access to a water channel, so they will have to use the university's wind tunnel instead.
The actual submarine has a total length of $L = 2.5$ m, a maximum diameter of $D = 1.3$ m, and it is designed to travel through the clear waters of a lake (average temperature 15°C) at a speed of $V = 0.5$ m/s.

The students build a one-eighth scale model of the submarine to test in the wind tunnel. The air in the wind tunnel is at 25°Cand at one standard atmosphere pressure. The scaled model is supported in the middle of the measurement section by a shielded strut, so that the drag force measured by the balance is only due to the model itself. The cross-section of the wind tunnel is much larger than the model.

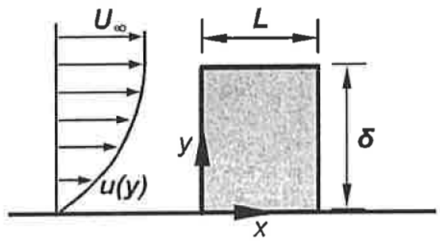
(15) (a) At what air speed does the wind tunnel need to be run in order to achieve similarity in the flow?

(5) (b) Is your assumption of flow similarity valid for this flow speed? Explain why?

(20) **Question 6**

A thin, flat, two-sided plate of length L and height δ is attached orthogonally to a wall and oriented parallel to an approaching boundary layer flow, i.e. the plate is parallel to the x, y-plane. Assume that the boundary layer flow is fully turbulent and that the velocity profile follows a one-seventh power law:

$$u(y) = U_\infty \left(\frac{y}{\delta}\right)^{1/7}$$



(15) (a) Derive a formula for the drag coefficient of this plate.

(5) (b) How does this drag compare against the drag on the same plate immersed on a uniform stream with speed U_∞?