National Exams May 2013

98-Ind-A1

Operations Research

3 hours duration

NOTES:

1 If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.

2 Any non-communicating calculator is permitted. This is an Open Book exam. Note to candidates: You must indicate the type of calculator being used, i.e. write the name and model designation of the calculator, on the first left hand sheet of the exam workbook.

3 There may be more questions than you are able to answer in the allotted time. Although the total value of the questions is 180, any marks achieved will be considered toward the 100 total requirements.

20 1. An airline flies planes on the following route: Vancouver – Calgary – Toronto – New York – Vancouver. The distances are as follows: Vancouver to Calgary, 687 km, Calgary to Toronto, 2690 km, Toronto to New York 571 km, and New York to Vancouver 3910 km. At each stop the plane may purchase Vancouver it is 0.88 $/l, in Calgary 0.15 $/l, in Toronto 0.95 $/l, and in New York 1.05 $/l. The plane's fuel tank holds 12000 l, and for safety, the plane must carry at least 600 l. upon landing. The amount of fuel used per km on each leg of the flight is [1 + (average amount of fuel in tank on leg of flight/2000)]. Assume that this average amount of fuel is [(1/2) (amount of fuel in tank at start of leg + amount of fuel in tank at end of leg)].

   Formulate an LP that can be used to minimize the fuel cost incurred in completing the schedule. Do not solve.

2. Consider the following LP problem

   Max $2x_1+3x_2$

   Subject to

   $x_1+2x_2 \leq 6$

   $2x_1+x_2 \leq 8;$

   $x_i \geq 0,\ i = 1, 2.$

10 a. Use the Simplex Method to find the optimal solution.

10 b. For what range of coefficient "3" in the objective function does the solution remain optimal?

20 3. A catering company must meet its daily demand for clean napkins by either buying new napkins, using regular laundry service which requires a full day turnaround, or special overnight service. Supplied with the demand for each of the next 5 days, and the costs for new napkins, regular laundry service, and overnight laundry service you have been asked to develop a model which will minimize the cost of procuring the napkins over the next 5 days. The company currently has no napkins in usable condition, clean or dirty, on the premises or at the laundry.

   Formulate this problem as a Minimum Cost Network Flow model.

20 4. An electric utility is considering 5 possible locations to build additional power plants over the next 20 years. The cost of building a plant at each site, the annual operating cost and the energy/year that can be provided at each site are given, as well as the total energy requirement for each year of the planning horizon. Assume that at most one plant can be built at each site, that at most one plant can be placed into service in a given year, and that it can produce its full energy contribution starting the year it is placed in service. The company can currently generate 500,000 MWh per year, using its existing resources. Develop an integer programming model to determine the expansion plan that will minimize overall construction and operating costs over the 20 years.

20 5. A job shop has 4 jobs (A, B, C & D) that must be processed on a single machine. The processing times are respectively 2, 4, 6 and 8 days. The due dates for each job are respectively 4, 14, 10 and 16 days from now. For each day a job is late the shop must compensate the customer \$100. Use dynamic programming to determine how the jobs should be sequenced to minimize the total overdue penalty costs.

20 6. An oil company is installing an oil pipeline from an oil field to a refinery. The pipeline requires the welding of 1000 seams, to be carried out by the company's own welders. Defective seams result in leaks, which must be reworked at a cost of \$1,200 per seam. It is estimated from past experience that 5% of the seams will be defective with probability 0.30, or 10% will be defective with probability 0.50, or 20% will be defective with probability 0.20. The company can also hire an expert cleanup team of welders at a one-time cost of \$130,000, who would check all of the welds done by the company welders and repair them as required.

   a. Based on an expected value criterion, should the company bring in the expert clean-up team to check and rework the welds, or repair the welds as they occur?

   b. The company can also improve its information about the quality of its own welders on this job, by x-ray inspection of a randomly selected completed weld at a cost of \$2,000. Is it worthwhile to carry out this inspection?

20 7. Robert Blue is trying to find a parking place near his favorite restaurant. He is approaching the restaurant from the west, and his goal is to park as nearby as possible. The available parking places are pictured in Figure 2. Robert is nearsighted and cannot see ahead: he can only see whether the space he is at now is empty. When Robert arrives at an empty space, he must decide whether to park there or to continue to look for a closer space. Once he passes a space, he cannot return to it. Robert estimates that the probability that space t is empty is pt. If he does not end up with a parking space, he is embarrassed and incurs a cost M (M is a big positive number). If he does park in space t he incurs a cost |t|. Show how Robert can use *dynamic* programming to develop a parking strategy that minimizes his expected cost.

FIGURE 2
Location of Parking Places

$$\rightarrow \boxed{-T}\ \boxed{1-T}\ \boxed{2-T} \cdots \boxed{-2}\ \boxed{-1}\ \boxed{0}\ \boxed{1}\ \boxed{2} \cdots \boxed{T}$$

0 = Restaurant

20 8. A tool crib has exponential inter-arrival and service times and services a very large group of mechanics. The mean time between arrivals is 4 minutes. It takes 3 minutes on the average for tool-crib attendant to service a mechanic. The attendant is paid $10 per hour and the mechanic is paid $15 per hour. Would it be advisable to have a second tool-crib attendant?

20 9. As manager of the Western Wheat Cooperative you have one week to arrange delivery of an order for 6000 tons of wheat, stored among 5 silo locations, to arrive at the port of Rotterdam no later than 3 weeks hence. You will be ready to transport the wheat from the silos starting the morning of day 7, first by rail to one of 4 ports on the *Great Lakes, and from there by ship*. There is only one ship sailing for Rotterdam within the next three weeks at each port. The amount of wheat at each silo is provided, as well as the rail cost/ton and transport time from each silo to each port. Also each ship departure date, expected arrival date in Rotterdam, ship capacity and shipping cost per ton to Rotterdam are known. To simplify describing the mathematical model, you obtained the following specific travel times.

	Port A	Port B	Port C	Port D
Days From Silo 1 to	5	2	1	4
Days From Silo 2 to	2	1	6	2
Days From Silo 3 to	1	4	2	3
Days From Silo 4 to	6	3	5	1
Days From Silo 5 to	4	1	3	4
Ship Departure Day	10	8	11	9
Ship Arrival Day	19	17	21	17

The trains deliver the goods at each port early in the morning and the ships depart late in the day, providing plenty of time to load the ships. There is an incentive for early delivery in $/ton/day, and an overnight storage costs at each port in $/ton/day. Provide an LP model deriving the optimal shipping plan.